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Plenary Presentations

Jacek Blazewicz

Multi-agent based approach for the origins of life hypothesis

Abstract

Multi-agent systems have been used extensively in scheduling, but the methodology has many other applications. One of those appears to be the analysis of the origins of life hypothesis. One of the most recognized hypotheses for the origins of life is the RNA world hypothesis. Laboratory experiments have been conducted to prove some assumptions of that hypothesis. However, despite some successes in the "wet-lab" experiments, we are still far from a complete explanation. Bioinformatics, supported by operations research and in particular by multi-agent approach, appears to provide perfect tools to model and test various scenarios of the origins of life where wet-lab experiments cannot reflect the true complexity of the problem. This paper illustrates some recent advancements in that area and points out possible directions for further research.

Ender Özcan

A Review of Selection Hyper-heuristics: Recent Advances

Abstract

Hyper-heuristics emerged as general purpose optimisation methodologies that search the space of heuristics, rather than candidate solutions directly, for solving computationally difficult problems. The current state-of-the-art in hyper-heuristic development involves designing adaptive search methods that are applicable to instances with different characteristics not only from a single problem domain, but also across multiple domains. A key goal is enabling 'plug-and-play' search components, including data science techniques (e.g., machine learning and statistics) to be applied to optimisation without them having to be re-implemented for every problem domain. Selection hyper-heuristics separate the high level automated search control embedding learning heuristic selection and move acceptance methods from the low level problem domain details. In the last two decades, particularly after the cross-domain heuristic search challenge in 2011, there has been an extremely rapid growth in this area of research, leading to many highly-effective selection hyper-heuristics applied to various problem domains. As a means of achieving generality, the initially proposed interface between the selection hyper-heuristic and domain layers was extremely restrictive allowing no problem specific information flow. However, there is a current trend towards moving away from this type of interface to facilitate more expressive selection hyper-heuristics capable of operating in an information rich environment, whilst still maintaining domain independence of the search control. This talk provides a review of selection hyper-heuristics focusing on the recent advances in the field.

Hoong Chuin Lau

Combining Machine Learning and Optimization for Real-World Scheduling Applications

Abstract

In this Big Data era, data can and should be exploited for more effective resource scheduling. In this talk, I will discuss a framework that combines data analytics, machine learning and optimization to solve real-world complex scheduling problems effectively. I will illustrate with three diverse scheduling problems ranging from crowd logistics to police officer scheduling to maritime traffic coordination, showing how spatial-temporal patterns can be learnt from data (both historical and real-time), and utilized to generate effective schedules.

Papers

MISTA 2017

Customer Order Scheduling on Unrelated Parallel Machines to Minimize Total Weighted Completion Time

Haidong Li · Xiaoyun Xu · Yaping Zhao

Abstract This paper considers a customer order scheduling problem in unrelated parallel machine environment. The objective is to minimize the total weighted completion time of orders. Several optimality properties of this problem are derived, and a computable lower bound of the objective function is established. Due to the NP-completeness of the problem, two heuristic algorithms are proposed. Theoretical analysis shows that the worst-case performances of both algorithms are bounded. Numerical studies are carried out to demonstrate the effectiveness of the lower bound and the proposed heuristics.

1 Introduction

This paper considers a customer order scheduling problem on unrelated parallel machines to minimize total weighted completion time. To be specific, there are n orders $\mathbb{J} = \{1, 2, ..., n\}$ with each one consisting of various different product types $\mathbb{T} = \{1, 2, ..., t\}$. The workload of product type k in order j is $p_{jk}, \forall j \in \mathbb{J}, \forall k \in \mathbb{T}$. The release times of all orders are considered as 0 in this study, and order j has a positive weight $W_j, \forall j \in \mathbb{J}$. Consider a facility with m unrelated machines $\mathbb{M} = \{1, 2, ..., m\}$ in parallel. Each machine can produce all types of products, and the workload of each product type can be split arbitrarily over all machines. The workload of each order can be processed independently on each machine, and preemptions are allowed. Machine i processes product type k at speed v_{ik}, $\forall i \in \mathbb{M}, \forall k \in \mathbb{T}$, which are predetermined and heterogeneous across all the product type-machine pairs. The completion time of order j, denoted as C_j, is the time when all product types of order j have been finished. The

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objective is to schedule the orders on the machines so as to minimize total weighted completion time $\sum W_j C_j$. According to the notation system introduced in [3], this problem is represented as $Rm|O|\sum W_j C_j$, where "O" represents "order".

In recent years, customer order scheduling problem has received an enormous amount of attention in the literature. The concept of customer order scheduling is first introduced by [7]. The authors consider a single machine problem with the objective of minimizing the total completion time of orders, and provide a dynamic programming algorithm for the problem with two product types and a given order processing sequence. For single machine environment, variations of customer order scheduling problems with different objectives have been well explored in the literature [10], [1], [11], [4], [2], [12].

For parallel machine environment, Leung *et al.* [8] provide a thorough review on customer order scheduling problem and classify the problem into three categories: 1) the fully dedicated case, in which each machine can produce one and only one type of product; 2) the fully flexible case, in which all the machines are identical and each machine is capable of producing all the products; 3) the arbitrary case, in which all the machines are unrelated and each machine is capable of producing all the products. For dedicated parallel machine environment, there are several papers dealing with customer order scheduling problem to minimize total weighted completion time. Sung and Yoon [14] show that the problem of minimizing total weighted completion time is NP-hard in the strong sense. They also show that the worst-case performance of the weighted shortest processing time (WSPT) rule for permutation schedules is 2 for the case of two machines. Wang and Cheng [15] establish three heuristics based on WSPT rule and show that all of them have an m worst-case bound for the case of m machines. Leung *et al.* [9] modify the SPTL and ECT heuristics by taking the weights of orders into account and also provide the worst-case bound of these heuristics. For unrelated parallel machine environment, much fewer related works have been found. As concerns the unweighted cases, Yang [17] establishes the complexity of customer order scheduling problem in the unrelated parallel machine environment. He prove the NP-completeness of the problem with two machines to minimize total completion time. Xu *et al.* [16] also consider the customer order scheduling problem to minimize total completion time. They propose three heuristics and show that their worst-case performances are bounded. To the knowledge of the authors, no additional result on the problem in this paper has been reported in the literature.

This paper investigates the customer order scheduling problem on unrelated parallel machines to minimize total weighted completion time of orders. In this study, the scheduling problem is formulated as a Mixed Integer Programming (MIP). A non-trivial lower bound on the objective is established, and two heuristic algorithms are also proposed to solve this problem. The worst-case performance of each algorithm is shown to be bounded. Numerical studies are conducted to demonstrate the performance of the proposed heuristics under various application scenarios.

The remainder of this paper is organized as follows. Section 2.1 formulates the problem $Rm|O|\sum W_j C_j$ as a Mixed Integer Programming (MIP). In Section 2.2, a lower bound on the objective function is established. In Section 3, two heuristics are proposed to solve the problem and their worst-case performance bounds are also constructed. Section 4 presents the numerical study and demonstrates the effectiveness of the proposed heuristics. Concluding remarks are given in Section 5.

2 Theoretical Study

2.1 Mathematical Programme for $Rm|O|\sum W_jC_j$

In this section, this paper formulates the problem $Rm|O|\sum W_jC_j$ as a mathematical programme. To facilitate the formulation, two optimality properties of the studied problem are presented first in the following two lemmas.

Lemma 1 *For $Rm|O|\sum W_jC_j$, in any optimal schedule, all machines must complete all workloads simultaneously.*

Proof The proof is inspired by the optimality analysis in [16]. By contradiction. Suppose that there exists an optimal schedule such that m machines do not complete all workloads simultaneously, and order j is the order with maximum completion time. A feasible schedule can always be constructed by assigning part of the workload of order j on the latest finishing machine to other machines so that the completion time of order j is not increasing. Repeating this procedure till a better schedule is constructed where all m machines complete all workloads simultaneously. A contradiction. □

Lemma 2 *For $Rm|O|\sum W_jC_j$, there exists an optimal schedule in which all machines process the customer orders in the same sequence and without preemptions.*

Proof By contradiction. Suppose that there exists no optimal schedule such that all machines process the customer orders in the same sequence and without preemptions. Let machine i be the latest finishing machine in one optimal schedule π^{opt}, i.e., all other machines finish at the same time as or earlier than machine i. Let order j be the order that finishes last on machine i. For each machine, move all workloads of order j to finish last. This operation will not change the completion time of order j. However, each of the order $l \neq j$ finishes at the same time as or earlier than before. Then, delete the last order and consider only the first $(n-1)$ orders.

By repeating the above operation, a new schedule π^* is then constructed with the objective function no worse than π^{opt}, and all machines in π^* process the customer orders in the same sequence and without preemptions. A contradiction. □

To obtain the optimal schedule described in Lemma 2, three sets of decision variables are defined: (i) x_{ij} for $i, j \in \mathbb{J}$: a binary variable that takes a value of 1 if order i is processed before order j and takes a value of 0 otherwise; (ii) s_{jm} for $j \in \mathbb{J}, m \in \mathbb{M}$: a variable that represents the starting time of order j on machine m; (iii) y_{mtj} for $m \in \mathbb{M}, t \in \mathbb{T}, j \in \mathbb{J}$: a variable that represents the portion of type t in order j processed by machine m. In terms of these variables, a Mixed Integer Programming (MIP) formulation of the problem is established as follows.

$$\textbf{(MIP)} \quad \min \sum_j W_j C_j$$

$$\text{s.t.} \quad x_{ij} + x_{ji} = 1, \forall i \neq j \in \mathbb{J}; \tag{1}$$

$$\sum_m y_{mtj} = 1, \forall t \in \mathbb{T}, \forall j \in \mathbb{J}; \tag{2}$$

$$s_{jm} + M \times (1 - x_{ij}) \geqslant s_{im} + \sum_t y_{mti} \times p_{it}/v_{mt},$$
$$\forall m \in \mathbb{M}, \forall i \neq j \in \mathbb{J}; \tag{3}$$

$$s_{im} + M \times x_{ij} \geqslant s_{jm} + \sum_t y_{mtj} \times p_{jt}/v_{mt},$$
$$\forall m \in \mathbb{M}, \forall i \neq j \in \mathbb{J}; \tag{4}$$

$$C_j \geqslant s_{jm} + \sum_t y_{mtj} \times p_{jt}/v_{mt},$$
$$\forall m \in \mathbb{M}, \forall j \in \mathbb{J}; \tag{5}$$

$$x_{ij} \in \{0, 1\}, \forall i \neq j \in \mathbb{J};$$
$$s_{jm} \geqslant 0, \forall j \in \mathbb{J}, \forall m \in \mathbb{M};$$
$$y_{mtj} \geqslant 0, \forall m \in \mathbb{M}, \forall t \in \mathbb{T}, \forall j \in \mathbb{J}.$$

Constraints (1) mean that one order is processed either before or after another order; Constraints (2) ensure the completion of the workload of each order; Constraints (3) and (4) are a pair of dual constraints to define the start time of each order; Constraints (5) define the orders' completion times.

As an immediate extension of the complexity results in [17], the $Rm|O|\sum W_jC_j$ problem is NP-complete. For small scale problem instances, solving the proposed MIP by optimization softwares such as CPLEX can yield optimal solutions in reasonable time. However, when the size of the problem becomes large, it is still difficult and time-consuming to obtain the optimal solution.

2.2 Lower Bound of $Rm|O|\sum W_jC_j$

Due to the NP-completeness of the problem $Rm|O|\sum W_jC_j$, optimal schedules and their optimal objective function values cannot be obtained in reasonable time for large scale problem instances. In order to evaluate the performances of feasible schedules, it is reasonable to establish a lower bound on the objective function as comparison.

To derive the lower bound, several additional notations are introduced first. Let $Rm|O|C_{\max}$ denote the problem which has the same machine environment with $Rm|O|\sum W_jC_j$ but aims to minimize the completion time of the last finishing order, denoted as $C_{\max}$; solve $Rm|O|C_{\max}$ for each individual order $j \in \mathbb{J}$, then $C^j_{\max}$ denotes the minimum makespan of order j; sort $C^j_{\max}$'s in a nondecreasing order, then $C^{[j]}_{\max}$ denotes the j-th one in the sequence; solve $Rm|O|C_{\max}$ for all n orders, then $C^O_{\max}$ denotes the minimum makespan of n orders.

In addition, let π denote a feasible schedule; $C_j(\pi)$ and $W_j(\pi)$ denote the completion time and the weight value of the j-th order in schedule π, respectively; sort $W_j(\pi)$'s in a nondecreasing order, then $W^{[j]}$ denotes the j-th one in the sequence. With the above notations and definitions, consider the following two lemmas.

Lemma 3 *For any optimal schedule π^{opt} of $Rm|O|\sum W_jC_j$, the following two inequalities must hold:*

$$C_{n-j}(\pi^{opt}) \geqslant C_n(\pi^{opt}) - \sum_{k=n-j+1}^{n} C_{max}^{[k]},$$
$$\forall j \in \{1, 2, \ldots, n-1\}; \tag{6}$$
$$C_j(\pi^{opt}) \geqslant C_{\max}^{[j]}, \forall j \in \{1, 2, \ldots, n\}. \tag{7}$$

Proof The detailed proof can be referred to Theorem 2 and Theorem 3 in [16]. □

Lemma 4 (Rearrangement Inequality from [5]) *Suppose that $x_1 \leqslant x_2 \leqslant \ldots \leqslant x_n$, $y_1 \leqslant y_2 \leqslant \ldots \leqslant y_n$ and $z_1, z_2, \ldots, z_n$ is any rearrangement of $y_1, y_2, \ldots, y_n$. Then*

$$\begin{aligned} x_1y_n + x_2y_{n-1} + \ldots + x_ny_1 &\leqslant x_1z_1 + x_2z_2 + \ldots + x_nz_n \\ &\leqslant x_1y_1 + x_2y_2 + \ldots + x_ny_n. \end{aligned}$$

According to Lemma 3 and Lemma 4, a lower bound of the optimal objective function value is shown in the following theorem.

Theorem 1 *Problem $Rm|O|\sum W_jC_j$ has the following lower bound*

$$\begin{aligned} LB &= \sum_{j=1}^{n-1} W^{[n-j+1]} \times \max\left\{ C_{\max}^{O} - \sum_{k=j+1}^{n} C_{\max}^{[k]}, C_{\max}^{[j]} \right\} + W^{[1]}C_{\max}^{O} \\ &\leqslant OBJ(\pi^{opt}), \end{aligned}$$

where $OBJ(\pi^{opt}) = \sum_{j=1}^{n} W_j(\pi^{opt})C_j(\pi^{opt})$.

Proof It is obvious that for any optimal schedule π^{opt}, $C_n(\pi^{opt}) \geqslant C_{\max}^{O}$. Therefore, according to Lemma 3, it can be obtained that

$$OBJ(\pi^{opt}) \geqslant \sum_{j=1}^{n-1} W_j(\pi^{opt}) \times \max\left\{ C_{\max}^{O} - \sum_{k=j+1}^{n} C_{\max}^{[k]}, C_{\max}^{[j]} \right\} + W_n(\pi^{opt})C_{\max}^{O}.$$

Let $a_j = \max\left\{ C_{\max}^{O} - \sum_{k=j+1}^{n} C_{\max}^{[k]}, C_{\max}^{[j]} \right\}, \forall j \in \{1, 2, \ldots, n-1\}$ and $a_n = C_{\max}^{O}$. It will be shown through Case #1 to Case #3 that the sequence $\{a_j\}_{j=1}^{n}$ is nondecreasing.

Case #1: Suppose that

$$C_{\max}^{O} - \sum_{k=j+1}^{n} C_{\max}^{[k]} \geqslant C_{\max}^{[j]}, \forall j \in \{1, 2, \ldots, n-1\}.$$

Then

$$a_j = C_{\max}^{O} - \sum_{k=j+1}^{n} C_{\max}^{[k]}, \forall j \in \{1, 2, \ldots, n-1\}.$$

It is obvious that the sequence $\{a_j\}_{j=1}^{n-1}$ is nondecreasing.

Case #2: Suppose that

$$C^O_{\max} - \sum_{k=j+1}^{n} C^{[k]}_{\max} < C^{[j]}_{\max}, \forall j \in \{1, 2, \ldots, n-1\}.$$

Then

$$a_j = C^{[j]}_{\max}, \forall j \in \{1, 2, \ldots, n-1\}.$$

It is obvious that the sequence $\{a_j\}_{j=1}^{n-1}$ is nondecreasing.

Case #3: There exists an adjacent number pair $(i, i+1)$ such that

$$C^O_{\max} - \sum_{k=i+1}^{n} C^{[k]}_{\max} < C^{[i]}_{\max}$$

and

$$C^O_{\max} - \sum_{k=i+2}^{n} C^{[k]}_{\max} \geqslant C^{[i+1]}_{\max}.$$

Then,

$$\sum_{k=i+1}^{n} C^{[k]}_{\max} \leqslant C^O_{\max} < \sum_{k=i}^{n} C^{[k]}_{\max}.$$

Thus, for $j \in \{1, 2, \ldots, i\}$, $a_j = C^{[j]}_{\max}$, and the sequence $\{a_j\}_{j=1}^{i}$ is nondecreasing; for $j \in \{i+1, i+2, \ldots, n-1\}$, $a_j = C^O_{\max} - \sum_{k=j+1}^{n} C^{[k]}_{\max}$, and the sequence $\{a_j\}_{j=i+1}^{n-1}$ is nondecreasing. Moreover, it is obvious that

$$a_i = C^{[i]}_{\max} \leqslant C^{[i+1]}_{\max} \leqslant C^O_{\max} - \sum_{k=i+2}^{n} C^{[k]}_{\max} = a_{i+1}.$$

Therefore, the sequence $\{a_j\}_{j=1}^{n-1}$ is nondecreasing.

Concluded from the discussion of the above three cases, the sequence $\{a_j\}_{j=1}^{n-1}$ is nondecreasing. In addition, it is trivial to show that

$$a_n = C^O_{\max} \geqslant \max\left\{C^O_{\max} - C^{[n]}_{\max}, C^{[n-1]}_{\max}\right\} = a_{n-1}.$$

Therefore, the sequence $\{a_j\}_{j=1}^{n}$ is nondecreasing.

Replacing x_j and y_j in Lemma 4 with $W^{[j]}$ and a_j, respectively, Lemma 4 suggests that

$$\sum_{j=1}^{n} W_j(\pi^{opt}) a_j \geqslant \sum_{j=1}^{n} W^{[n-j+1]} a_j.$$

Therefore,

$$OBJ(\pi^{opt}) \geqslant \sum_{j=1}^{n} W_j(\pi^{opt}) a_j \geqslant \sum_{j=1}^{n} W^{[n-j+1]} a_j = LB.$$

□

The tightness of the above lower bound is of great concern in both theoretical and computational studies. In order to demonstrate the tightness of LB, in the following corollary, it is shown that LB equals global optimum under certain mild conditions.

Corollary 1 *When the machine environment is identical (Pm) or uniform (Qm), $LB = OBJ(\pi^{opt})$ if $(C^i_{\max} - C^j_{\max})(W_i - W_j) < 0, \forall i, j \in \mathbb{J}$.*

Proof The proof is an immediate consequence of Theorem 1 and thus omitted for brevity. □

3 Heuristics for $Rm|O|\sum W_jC_j$

For small scale problem instances, solving the MIP in Section 2.1 can yield the optimal solution in reasonable time. When the size of the problem becomes large, obtaining the optimal solution could be very time-consuming. As alternative methods, two heuristics for $Rm|O|\sum W_jC_j$ are proposed to solve the problem. The designs of both heuristics are based on insights from the optimality properties shown in the previous section, and both of them can be implemented quite easily.

3.1 Heuristic $H1$

The first heuristic, named $H1$, is a constructive method. The design of this heuristic is inspired by Corollary 1. To be specific, in order to solve the studied problem with a total of n orders, heuristic $H1$ first proceeds by solving n subproblems individually, each with a single order. In each subproblem, $H1$ minimizes the completion time of that particular order, that is, solves $Rm|O|C_{\max}$ with a single order. It is trivial to show that the starting and ending times on all machines are identical in each of n subproblems, resulting in forming n individual "processing blocks". These blocks are then reassembled together according to the Weighted-Shortest-Processing-Time (WSPT) rule as if they were individual jobs. Formally, heuristic $H1$ is described as follows.

1. Solve subproblem $Rm|O|C_{\max}$ optimally for each single order j individually. Obtain π^j_{sub}'s and their corresponding $C^j_{\max}, \forall j \in \mathbb{J}$. Here π^j_{sub} is the processing block of order j with identical starting and finishing time on all machines.
2. Sort n orders according to the WSPT rule based on values of weight W_j and $C^j_{\max}$.
3. Construct a nondelayed feasible schedule π by combining all processing blocks $\pi^j_{sub}, \forall j \in \mathbb{J}$ in the WSPT sequence.

It is clear that heuristic $H1$ is optimal when the parallel machine environment is identical (Pm) or uniform (Qm). For the arbitrary case where all machines are unrelated (Rm), heuristic $H1$ has the following worst-case performance bound:

Theorem 2 *For $Rm|O|\sum W_jC_j$, the worst-case performance bound for heuristic $H1$ is*

$$\frac{OBJ(\pi^{H1})}{OBJ(\pi^{opt})} \leqslant \frac{(\sum_{j=1}^{n} jW^{[j]}) \times (\sum_{j=1}^{n} C^{[j]}_{\max})}{n \times \sum_{j=1}^{n} W^{[n-j+1]}C^{[j]}_{\max}},$$

where $OBJ(\pi^{H1})$ denotes the objective function value of $H1$.

Proof See Appendix. □

3.2 Heuristic $H2$

The design of $H2$ is inspired by two observations. First, to reduce the completion time of each order, it is generally preferable to assign each product type on the machine which processes it at the fastest rate. Second, from Lemma 1, it seems reasonable to evenly distribute the workloads so that no machine is dominantly busy.

Specifically, heuristic $H2$ starts by arranging orders according to a non-increasing order of weights $W_j, \forall j \in \mathbb{J}$. Then, $H2$ proceeds by calculating the machine efficiency for each product type. The efficiency factor, e_{mt}, is defined as the ratio of the speed of processing type t on machine m to the maximum speed of processing type t on all machines. A list of product types in a non-increasing order of e_{mt} is created for each machine. The workload allocation of each order is determined as follows. Initially all the product types are "unassigned". Once a product type is allocated to a machine, it will be labeled as "assigned". Every time, pick the machine with minimum current completion time and assign it with the next "unassigned" product type in its list. The process continues until all product types in one order have been assigned. Then, reset all the product types and repeat the above steps to assign the next order. Detailed description of heuristic $H2$ is listed in the following.

Heuristic $H2$

Input: $V = [v_{mt}]_{m\times t}$, $P' = [p'_{jt}]_{n\times t}$ where p'_{jt} denotes the workload of type t of the order with weight $W^{[n-j+1]}$;
set $y_{mtj} = 0$ for every m, t and j;
for *every machine* m **do**
  create a list of all product types in non-increasing order of e_{mt}, where $e_{mt} = v_{mt}/(\max_{m\in\mathbb{M}}\{v_{mt}\})$;
  set 0 to C_m, the completion time of product types currently assigned to machine m;
end
set $j = 1$;
while $j \leqslant n$ **do**
  label all product types as "unassigned";
  while *not all product types are assigned* **do**
    find machine m such that C_m is minimal among all machines;
    find the next "unassigned" type t on the list of machine m;
    if *such* t *exists* **then**
      set 1 to y_{mtj}, label product type t as "assigned";
      update C_m by setting $C_m = C_m + p'_{jt}/v_{mt}$;
    else
    end
  end
  update j by setting $j = j + 1$;
end
return $Y^{H2} = [y_{mtj}]_{m\times t\times n}$

Heuristic $H2$ is a greedy algorithm. At each iteration, it assigns workload to the earliest finishing machine. Therefore, in the long run, this greedy algorithm maintains the balance of workloads and prevents the occurrence of excessive long processing time on a certain machine.

However, unlike $H1$ where the workloads of all product types are distributed among multiple machines, $H2$ does not allow workload splitting, that is, the entire workload

of each product type will be processed by one machine only. The "non-splitting workload" feature in heuristic $H2$ is very desirable in many manufacturing practices. For example, in textile industry where products are not allowed to be separated, heuristic $H2$ becomes the only heuristic proposed in this study that is feasible to apply.

Although no workload splitting is allowed in $H2$, the performance of heuristic $H2$ is still bounded, as shown by the following theorem.

Theorem 3 *For $Rm|O|\sum W_jC_j$, the worst-case performance bound for heuristic $H2$ is*

$$\frac{OBJ(\pi^{H2})}{OBJ(\pi^{opt})} \leqslant \frac{\sum_{j=1}^{n} W_j(\pi^{H2}) \times [(\frac{\sum_{k=1}^{j-1} T_k^{\max})}{m} + T_j^{\max}]}{LB},$$

where $OBJ(\pi^{H2})$ denotes the objective function value of $H2$, LB is obtained from Theorem 1 and $T_j^{\max}$ is the sum of the largest processing time of each product type in the j-th order.

Proof See Appendix. □

4 Computational Experiment

4.1 Experiment Design

This experimental study aims to examine the quality of the lower bound as well as the performance of the two proposed heuristics. The scale of testing instances generated in this computational study is listed as follows:

- Number of machines $m = 5, 20$.
- Number of orders $n = 5, 10, 50$.
- Number of product types $t = 50, 100$.

To simulate the unrelated machine environment, all the machines can process any product type and speed v_{ik}'s are randomly generated from the uniform distribution [40, 60]. Furthermore, order weight W_j's are randomly generated from the normal distribution $N(\mu = 5, \sigma = 1)$ and take the absolute values.

To evaluate the impact of workload variability on the heuristic performances, the following two scenarios are considered:

- **Relatively Uniform Workload (RUW)**: The workload of each product type in every order is randomly generated from the uniform distribution $U[400, 600]$. The coefficient of variation (CV, the standard deviation divided by the mean) equals approximately 0.1 under such circumstances, and this setting represents the case when workloads of all orders are relatively uniform.
- **Highly Variant Workload (HVW)**: The workload of each product type in every order is randomly generated from the uniform distribution $U[1, 1000]$. CV under such circumstances equals approximately 0.6, which, as stated in [6], indicates that there exists high variability in workloads. This setting represents the cases when workloads are highly variant among different orders.

There are $|m| \times |n| \times |t| = 2 \times 3 \times 2 = 12$ cases tested under each of the two scenarios listed above, and 5 independent replicates are randomly generated for each individual

case. Therefore, a total of $2 \times 3 \times 2 \times 2 \times 5 = 120$ replicates need to be constructed in the entire computational experiment.

The performance of the proposed heuristic algorithms is evaluated using the following two criteria:

-Performance gap

Since the problem $Rm|O|\sum W_jC_j$ is NP-complete, it could be computationally challenging to obtain the optimal solution for large scale problem instances. However, since the calculation of the proposed LB mainly requires solving a linear programming, LB can be obtained within polynomial time and used to evaluate the performance of each heuristic algorithm. Therefore, the heuristic performance is gauged with reference to the lower bound as follows:

$$\text{Performance Gap } (\%) = \frac{OBJ(\pi^H) - LB}{LB} \times 100\%,$$

where $OBJ(\pi^H)$ denotes the objective function value of heuristic.

-Computational Time

The running time of heuristic is recorded in CPU seconds (sec) to measure computational efficiency.

The simulation is coded using Matlab, and runs on a desktop computer with 3.40GHz CPU and 8G memory.

4.2 Experiment Results and Analysis

4.2.1 Analysis of Lower Bound Performance

As lower bound is involved in the calculation of performance gaps of heuristics, in order to provide convincing perspective, it is helpful to elaborate the efficiency of the proposed lower bound. The efficiency is measured by the gap between lower bound and optimal solution, which is shown in Table 1.

Table 1 The performance gap of the lower bound

Problem Setting			Reference	Gap (%)			
				RUW		*HVW*	
t	m	n		Mean	Max	Mean	Max
5	5	5	OPT	2.11	4.61	6.69	8.48
		10	OPT	2.23	3.89	5.57	6.11
	10	5	OPT	1.34	1.99	6.01	9.74
		10	OPT	2.28	3.41	9.39	10.17
20	5	5	OPT	0.65	1.34	4.52	6.49
		10	OPT	1.71	2.20	3.72	5.03
	10	5	OPT	0.58	0.81	1.83	2.84
		10	OPT	0.73	1.12	5.12	7.53

It is straightforward to conclude from Table 1 that the proposed lower bound is highly efficient in all testing instances, especially in cases with relatively uniform workload, where all mean gaps are bounded in 2.3%. Even in cases with highly variant

workload, the mean gaps are still bounded in 10%. This phenomenon also demonstrates the slight influence that workload variance causes on lower bound performance.

Problem scale can also affect the efficiency of the lower bound. Given that the numbers of machines (m) and orders (n) are fixed, the mean gaps decrease as the number of product types (t) grows under all the testing scenarios. For example, for the case $m = 5, n = 5$ with relatively uniform workload, the mean gap plunges from around 2.11% to no more than 0.65%, when the product type number (t) increases from 5 to 20. This phenomenon can be heuristically interpreted as the "balance effect" of product types, which means a large number of product types will erode the heterogeneity of machines. This characteristic inspires more confidence in taking the lower bound as reference to evaluate the two heuristics when the target problem involves a large number of product types.

The maximum gaps shown in Table 1 provide an additional insight. In each problem scale, consider the 5 replications of every testing scenario as a group. It can be observed that the absolute difference between the mean and maximum gaps is bounded in 3.73%. This observation suggests the performance of the lower bound is stable within group. Furthermore, under the same problem scale, the absolute difference between the mean and maximum gaps has no significant variance across two different scenarios (RUW and HVW). This shows that the performance of the proposed lower bound is robust over all scenarios considered.

4.2.2 Analysis of Heuristic Performance

The efficiency of heuristic is measured by both of the performance gap with lower bound and the computational time. The proposed heuristics are investigated in 2 testing scenarios under 12 problem settings. Results of two scenarios are reported in Table 2 and Table 3, respectively. The best dispatching method between heuristics $H1$ and $H2$ has also been demonstrated in column "$\min\{H1, H2\}$" of Table 2 and Table 3.

Table 2 The performances of two algorithms under scenario *RUW*

Problem Setting			Performance Gap (%)						Computational Time (sec)	
			$H1$		$H2$		$\min\{H1, H2\}$		$H1$	$H2$
t	m	n	Mean	Max	Mean	Max	Mean	Max	Mean	Mean
50	5	5	0.39	1.02	3.08	3.83	0.39	1.02	1.25	0.01
		10	0.81	1.17	2.69	3.63	0.81	1.17	2.92	0.01
		50	1.17	1.79	1.89	2.25	1.15	1.70	11.63	0.03
	20	5	0.39	0.67	11.12	13.41	0.39	0.67	1.29	0.02
		10	0.68	1.22	7.99	9.36	0.68	1.22	3.44	0.02
		50	0.92	1.14	3.22	3.65	0.92	1.14	15.82	0.11
100	5	5	0.55	1.31	2.16	2.65	0.55	1.31	1.26	0.01
		10	0.46	0.86	1.59	2.24	0.46	0.86	3.51	0.01
		50	0.75	0.91	1.38	1.63	0.75	0.91	18.39	0.06
	20	5	0.43	0.81	6.77	7.89	0.43	0.81	1.88	0.02
		10	0.53	0.95	4.16	4.46	0.53	0.95	5.02	0.04
		50	0.67	0.70	2.11	2.33	0.67	0.70	20.89	0.20

As can be observed in Table 2 and Table 3, heuristic $H1$ outperforms $H2$ in terms of the mean gap in all testing cases. For the cases with relatively uniform workload (RUW), heuristic $H1$ provides solutions with no more than 1% performance gaps for

Table 3 The performances of two algorithms under scenario HVW

Problem Setting			Performance Gap (%)						Computational Time (sec)	
			$H1$		$H2$		$\min\{H1, H2\}$		$H1$	$H2$
t	m	n	Mean	Max	Mean	Max	Mean	Max	Mean	Mean
50	5	5	1.61	5.21	6.20	10.76	1.61	5.21	1.32	0.01
		10	3.73	6.66	6.99	10.62	3.73	6.66	2.95	0.01
		50	3.82	5.28	5.94	7.37	3.82	5.28	11.74	0.03
	20	5	2.79	4.79	24.14	27.13	2.79	4.79	1.32	0.01
		10	4.46	5.42	16.56	19.52	4.46	5.42	3.46	0.02
		50	4.48	5.25	9.06	9.65	4.48	5.25	16.01	0.11
100	5	5	1.74	3.21	3.85	5.76	1.74	3.21	1.26	0.01
		10	2.35	3.74	4.07	5.90	2.35	3.74	3.57	0.01
		50	3.13	3.59	4.44	4.71	3.13	3.59	18.42	0.06
	20	5	1.88	3.51	12.12	14.61	1.88	3.51	1.94	0.02
		10	2.41	3.34	8.60	9.66	2.41	3.34	5.15	0.04
		50	3.22	4.15	5.50	6.36	3.22	4.15	30.13	0.21

nearly all cases. For the cases with highly variant workload (HVW), heuristic $H1$ is also capable of delivering solutions with mean performance gaps that are less than 4.5% compared with the lower bound. On the other hand, however, heuristic $H1$ can be challenged when it comes to computational efficiency. For solving the same problem, the computational time required by $H1$ is approximately 100 times more than that of $H2$. The reason lies in the fact that heuristic $H1$ needs to solve n subproblems $Rm|O|C_{\max}$, which is equivalent to solving a linear programming.

Heuristic $H2$ has been shown to be less efficient than $H1$ in terms of mean performance gap. However, mean gaps of no more than 10% are also observed in nearly all the testing cases of relatively uniform workload setting. Even in the cases with highly variant workload, the mean gaps of heuristic $H2$ are still be bounded in 25%. It is worth noting that $H2$ is the only proposed heuristic that does not require workload split. In some applications where significant setup time is incurred between type switch (such as die change or paint switch), $H2$ may become a better alternative between the two proposed heuristics.

The efficiency of heuristic is also affected by the problem scale. The mean gaps of all the proposed heuristics tend to decrease as the number of product type (t) grows. It accounts for the fact that the lower bound can provide a better approximation to the optimal value when product type number becomes large. Another contributing factor to this phenomenon is that large product type number dilutes the impact of the order differences and therefore provides a slight remedy for undesirable variability in machine capability.

The maximum gaps demonstrated in Table 2 and Table 3 provide an additional insight into the stability and robustness of proposed heuristics. It can be observed that, within each testing group (5 replications of the same problem scale and scenario), the maximum gap does not drift too far from the mean gap. This finding confirms the stability of intra-group performance gaps provided by all proposed heuristics. Moreover, under the same testing scenario, the relative differences between the maximum and mean gaps do not vary too much across all problem scales tested. It can therefore be concluded that all proposed heuristics are robust over all scenarios and problem scales considered.

5 Conclusion

This study addresses a customer order scheduling problem in unrelated parallel machine environment. The objective is to minimize the total weighted completion time of all customer orders. In this study, several important optimality properties of the studied problem have been derived. Based on these properties, a computable lower bound of the objective function has been established. Numerical studies suggest that this lower bound provides a good approximation to the optimal value and performs even better as the number of product types grows.

Two heuristics are also proposed to solve the customer order scheduling problem. Numerical studies provide additional insights into the heuristic performance under various scenarios and problem settings. Both heuristic $H1$ and $H2$ can be implemented quite easily. Heuristic $H1$ behaves quite well and outperforms heuristic $H2$ in terms of the performance gap in most cases. Heuristic $H2$ shows its advantage in applications where significant penalty is incurred between type switch. In addition, the performance of both heuristics improves as the number of product type grows.

Further research on this topic may involve some other important criteria to evaluate the quick responsiveness of industries. Another possible generalization, which is of interest during recent years, would be taking setup time between product type exchange and resource constraints into consideration. The problem considered in this study can also be extended to more complex production environments, such as flow-shop, job-shop and so forth.

6 APPENDIX

6.1 Proof of Theorem 2

Proof It is obvious that

$$\max\{C^O_{\max} - \sum_{k=j+1}^{n} C^{[k]}_{\max}, C^{[j]}_{\max}\} \geqslant C^{[j]}_{\max},$$

$\forall j \in \{1, 2, \ldots, n-1\}$, and $C^O_{\max} \geqslant C^{[n]}_{\max}$. Thus, a lower bound for LB can be obtained that

$$LB \geqslant \sum_{j=1}^{n} W^{[n-j+1]} C^{[j]}_{\max}. \tag{8}$$

Consider the $1||\sum W_j C_j$ problem where the job size $p_j = C^{[j]}_{\max}, j \in \{1, 2, \ldots, n\}$. It is known from [13] that applying the WSPT rule can obtain the optimal solution to $1||\sum W_j C_j$ problem. Therefore, among all possible sequences of $p_j = C^{[j]}_{\max}, j \in \{1, 2, \ldots, n\}$, the summation of weighted completion time of the sequence according to WSPT rule is minimum.

Consider the following n sequences of $C_{\max}^{[j]}$'s:

$$\begin{aligned}
&Sequence\#1 : C_{\max}^{[1]} \to C_{\max}^{[2]} \to \cdots \to C_{\max}^{[n-1]} \to C_{\max}^{[n]};\\
&Sequence\#2 : C_{\max}^{[2]} \to C_{\max}^{[3]} \to \cdots \to C_{\max}^{[n]} \to C_{\max}^{[1]};\\
&\quad\vdots\\
&Sequence\#n : C_{\max}^{[n]} \to C_{\max}^{[1]} \to \cdots \to C_{\max}^{[n-2]} \to C_{\max}^{[n-1]}.
\end{aligned}$$

Let $V^{[j]}$ denote the weight value corresponding to the job size $p_j = C_{\max}^{[j]}$. Consider the summation of weighted completion time of the Sequence #1, the WSPT rule guarantees that:

$$OBJ(\pi^{H1}) \leqslant \sum_{j=1}^{n}(V^{[j]} \times \sum_{i=1}^{j} C_{\max}^{[i]}).$$

From Lemma 4, an upper bound for the summation of weighted completion time of the Sequence #1 can be obtained as:

$$\sum_{j=1}^{n}(V^{[j]} \times \sum_{i=1}^{j} C_{\max}^{[i]}) \leqslant \sum_{j=1}^{n}(W^{[j]} \times \sum_{i=1}^{j} C_{\max}^{[i]}).$$

Therefore,

$$OBJ(\pi^{H1}) \leqslant \sum_{j=1}^{n}(W^{[j]} \times \sum_{i=1}^{j} C_{\max}^{[i]}).$$

In order to express other sequences expediently, it is necessary to extend the definition of $C_{\max}^{[i]}$ such that

$$C_{\max}^{[i]} = C_{\max}^{[i-n]}, \forall i \in \{n+1, n+2, \ldots, n+n-1\}.$$

In the same way, n inequalities can be obtained:

$$OBJ(\pi^{H1}) \leqslant \sum_{j=1}^{n}(W^{[j]} \times \sum_{i=1}^{j} C_{\max}^{[i]});$$

$$OBJ(\pi^{H1}) \leqslant \sum_{j=1}^{n}(W^{[j]} \times \sum_{i=2}^{j+1} C_{\max}^{[i]});$$

$$\vdots$$

$$OBJ(\pi^{H1}) \leqslant \sum_{j=1}^{n}(W^{[j]} \times \sum_{i=n}^{j+n-1} C_{\max}^{[i]}).$$

Summing the above inequalities, one can obtain that:

$$n \times OBJ(\pi^{H1}) \leqslant (\sum_{j=1}^{n} jW^{[j]}) \times (\sum_{j=1}^{n} C_{\max}^{[j]}).$$

Therefore,

$$OBJ(\pi^{H1}) \leqslant \frac{1}{n}(\sum_{j=1}^{n} jW^{[j]}) \times (\sum_{j=1}^{n} C_{\max}^{[j]}).$$

The worst-case performance bound for heuristic $H1$ can be obtained that

$$\begin{aligned}\frac{OBJ(\pi^{H1})}{OBJ(\pi^{opt})} &\leqslant \frac{(\sum_{j=1}^{n} jW^{[j]}) \times (\sum_{j=1}^{n} C_{\max}^{[j]})}{n \times LB} \\ &= \frac{(\sum_{j=1}^{n} jW^{[j]}) \times (\sum_{j=1}^{n} C_{\max}^{[j]})}{n \times \sum_{j=1}^{n} W^{[n-j+1]} C_{\max}^{[j]}}.\end{aligned}$$

6.2 Proof of Theorem 3

Proof Let C_j^i denote the completion time of the first j orders on machine i. Moreover, let

$$C_j^{\min} = \min_{i \in \mathbb{M}} \{C_j^i\}, j \in \{1, 2, \ldots, n\}.$$

The following proof proceeds in two steps.

First, it is shown that

$$C_j(\pi^{H2}) \leqslant C_{j-1}^{\min} + T_j^{\max}, j \in \{1, 2, \ldots, n\}.$$

According to heuristic $H2$, the starting time of the j-th order equals to $C_{j-1}^{\min}$. Suppose the machine i^* is the one with minimum completion time of the first $(j-1)$ orders. Machine i_1 is one of machines assigned with at least one type of the j-th order. In Figure 1, time spot A is the completion time of the first j orders on machine i^*. Time spot B_1 is the completion time of the first $(j-1)$ orders on machine i_1. Then,

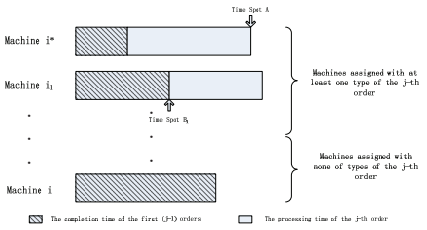

Fig. 1 Heuristic $H2$

time spot A must be later than time spot B_1. If not, machine i_1 is assigned with one type of the j-th order for the first time, but at this time, current completion time of machine i_1 is greater than that of machine i^*. A contradiction.

Let $\mathbb{M}'$ denote the set of all machines assigned with at least one type of the j-th order. Let T_{jk}^i denote the processing time of the entire type k in the j-th order processed by machine i and $T_{jk}^{\max} = \max_{i \in \mathbb{M}} \{T_{jk}^i\}$. Replace each $T_{jk}^i, \forall i \in \mathbb{M}, k \in \mathbb{T}$ with corresponding $T_{jk}^{\max}$ (Figure 2). Then, remove the new processing time of the j-th order on machine $i_1, \forall i_1 \in \mathbb{M}'$ to machine i^* (Figure 3). Since time spot A is later

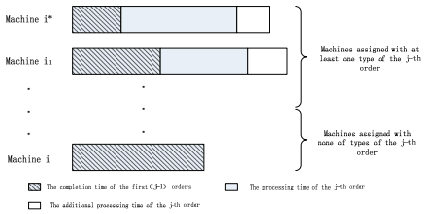

Fig. 2 Replace each T_{jk}^{i} with corresponding $T_{jk}^{\max}$

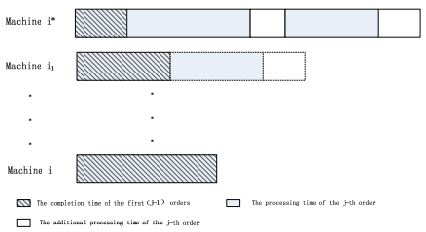

Fig. 3 Remove the new processing time of the j-th order

than time spot B_1, it is trivial to show that the new completion time of the first j orders on machine i^* is greater than $C_j^{i_1}, \forall i_1 \in \mathbb{M}'$. Since $C_j(\pi^{H2}) = \max_{i \in \mathbb{M}'}\{C_j^i\}$, the new completion time of the first j orders on machine i^* is greater than $C_j(\pi^{H2})$.

Therefore,

$$\begin{aligned} C_j(\pi^{H2}) &\leqslant C_{j-1}^{\min} + \sum_{k=1}^{t} T_{jk}^{\max} \\ &= C_{j-1}^{\min} + T_j^{\max}, j \in \{1, 2, \ldots, n\}. \end{aligned}$$

Second, it is shown that

$$C_j^{\min} \leqslant \frac{1}{m} \sum_{k=1}^{j} T_k^{\max}, j \in \{1, 2, \ldots, n\}.$$

Since $C_j^{\min} = \min_{i \in \mathbb{M}}\{C_j^i\}$, it is obvious that

$$m \times C_j^{\min} \leqslant \sum_{i=1}^{m} C_j^i.$$

Moreover,

$$\sum_{i=1}^{m} C_j^i \leqslant \sum_{n=1}^{j} \sum_{k=1}^{t} T_{nk}^{\max} = \sum_{k=1}^{j} T_k^{\max}.$$

Therefore,

$$C_j^{\min} \leqslant \frac{1}{m} \sum_{k=1}^{j} T_k^{\max}, j \in \{1, 2, \ldots, n\}.$$

From the above two steps, one can obtain the following upper bound for $C_j(\pi^{H2})$:

$$C_j(\pi^{H2}) \leqslant (\frac{1}{m}\sum_{k=1}^{j-1} T_k^{\max}) + T_j^{\max}, j \in \{1, 2, \ldots, n\}.$$

Therefore,

$$OBJ(\pi^{H2}) \leqslant \sum_{j=1}^{n} W_j(\pi^{H2}) \times [(\frac{1}{m}\sum_{k=1}^{j-1} T_k^{\max}) + T_j^{\max}].$$

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Exploration of Logic-Based Benders Decomposition Approach for Mapping Applications on Heterogeneous Multi-Core Platforms

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Abstract The proper mapping of an application on a multi-core platform and the scheduling of its tasks is a key element to achieve the maximum performance. To obtain optimal mapping solutions, the logic-based Benders decomposition principle is employed for applications described by Directed Acyclic Graphs (DAGs). The approach combines integer linear programming (ILP) and constraint programming (CP) for the assignment of the tasks to the cores of the platform and their scheduling per core, respectively. Its performance mainly relies on enriching the assignment sub-problem with parts of the scheduling problem in order to identify infeasible solutions. The purpose of this work is to study and experimentally evaluate through computational results the effect of different aspects of the method to its overall performance in terms of solution time. The introduced approach is compared with a pure ILP model achieving speedups of orders of magnitude. In addition, it is employed as a cut generation scheme for the pure ILP model in a hybrid solution method. The latter optimally solves problems that cannot be solved by any of the integrated methods alone, while the overall solution time is also decreased.

1 Introduction

The mapping of an application on a multi-core platform refers to finding an assignment of its tasks to the cores of the platform and their scheduling per core in order to optimize one or more metrics, such as performance, power dissipation, and system cost. In its general form, this is a well-known NP-complete problem. Moreover, the design of modern multi-core systems leans towards the use of heterogeneous cores, whose features can be exploited to satisfy the diverse functionality of the applications. However, when the platform consists of heterogeneous cores, the complexity of the problem increases since the execution time of each task is not the same for all cores.

In the case of multi-core computing, the applications can be considered as a set of many tasks that need to be distributed on multiple cores and can be represented as a Directed Acyclic Graph (DAG). The nodes of the DAG correspond to the tasks and the edges represent data dependencies between them. When the mapping is static, that is the application's characteristics are known in advance and the DAG does not change during its execution, reasonable design effort and time can be spent to obtain an optimal or near-optimal solution. Moreover, in the context of auto parallelization when high level description languages are used as input, intermediate representations generate extremely complex DAGs that need to be mapped and scheduled in heterogeneous architectures[1][2].

The above problem has been studied extensively in the past [3] and a detailed survey is provided in [4]. Due to its increased complexity, the vast majority of the adopted methods that target the mapping problem are based on heuristic approaches, such as list scheduling [5, 6], or stochastic search algorithms, such as genetic algorithms [7]. These methods have low computational complexity and are able to produce a good solution in reduced time, without guaranteeing that it is the optimal one.

On the other hand, methods that always produce an optimal solution are based on Integer Linear Programming (ILP) [8, 9] or Constraint Programming (CP) models [10, 11]. These methods always provide an optimal solution, but they suffer from large computational complexity; thus their solution time may be prohibitive even for relatively small-scale problems. In this direction, some approaches have been proposed trying to speed up the solution process of the above methods.

One approach that has been proven very effective in solving complex optimization problems integrates ILP and CP models reducing significantly the solution time. This approach is based on the Benders decomposition principle and has been employed in many kinds of scheduling problems, achieving significant speedups in terms of the solution time (orders of magnitude in many cases) [12, 13].

In this paper, a hybrid approach based on integrating the Logic-Based Benders Decomposition (LBBD) principle [14] with the pure ILP-based approach to map static applications represented as DAGs on heterogeneous multi-core platforms is discussed. The LBBD model employs two complementary optimization techniques (ILP and CP) to iteratively solve the assignment and scheduling problems, respectively. The master problem is enriched with various relaxations of the scheduling problem to exclude in advance infeasible solutions, while the sub-problems communicate through Benders cuts that are strengthened by a refinement procedure. The effect of each aspect of the method is evaluated through computational results.

The rest of the paper is organized as follows. In Section 2 the related work concerning the applications of logic-based Benders decomposition is discussed. In Section 3 the target mapping problem is defined and the corresponding ILP model is introduced. In Sections 4 and 5, the logic-based Benders decomposition approach as well as the Hybrid approach are explained. The experimental results and the corresponding discussion are presented in Section 6. Finally, Section 7 concludes the paper.

2 Related Work

Many methods based on the logic-based Benders decomposition principle have been proposed and applied in different kinds of scheduling problems, where jobs have to be assigned to multiple processing elements so that a specific metric is optimized. In [15], the approach is studied in the general case, combining ILP and CP in order to map a set of unrelated non-preemptive jobs given release times and deadlines to a set of homogeneous facilities with a certain capacity. The problem is considered with respect to three different optimization criterions, namely the total cost, the makespan, and the total tardiness. The highlights of the method are illustrated while regarding different cost functions and focusing on the cuts generation scheme.

In [16] a set of independent tasks is mapped on a set of machines having different processing time on each machine and sequence-dependent setup time aiming at optimizing the total execution time. The problem is decomposed into the assignment of jobs to machines and their sequencing, which are performed through an ILP model and a specialized solver for the traveling salesman problem, respectively. In [17], the problem of mapping an application represented by a DAG to a homogeneous multi-core platform is considered so that their deadlines of each task are met and it complies to a real-time system. The authors follow a two-stage decomposition approach based on CP models and focus on finding strong cuts by providing efficient explanations for the infeasibility of the solution after each iteration.

In [18], a homogeneous multi-core platform is assumed and the goal is the minimization of the makespan considering the communication delay between tasks on different cores, and the corresponding memory requirements. The problem is also decomposed into the assignment of the tasks to the cores by minimizing the data on the communication resources followed by the model that performes the final scheduling. This work is extended in [19], where a three-stage approach is proposed by further decomposing the allocation stage into the assignment of the tasks and the communication memory. The scheduling problem was formulated by a CP model while the others by ILP ones. A set of novel methods for the Benders cuts generation along with novel search and filtering methods for the CP model are introduced.

Most of the above works consider homogeneous facilities, which simplifies the solution process compared to the heterogeneous case. Specifically, the produced Benders cuts are much stronger since they can exclude many equivalent solutions at once by applying a symmetry breaking procedure [19]. Moreover, the complexity of the assignment sub-problem is small, since it considers only one processing time for each task and has to assess fewer assignment combinations. Finally, in [16] that targets a heterogeneous environment as well as in [15] for the homogeneous case, precedence relations are not considered between the jobs.

In our previous work an approach targeting the problem of mapping applications on heterogeneous multi-core platforms was presented [20] based on the Benders decomposition principle This approach was extended and combined with a pure ILP approach creating a hybrid solution method [21]. It is the only prior works that addresses the heterogeneous case with exact methods that always find the optimal solution and exhibits significant speedups of the proposed approach compared to an ILP model. This work augments the model by introducing additional relaxation constraints in the master problem that helps to exclude infeasible solutions in advance. The effectiveness of the introduced relaxations as well as the generated Benders cuts are studied through experimental results and the different aspects of the solution method are highlighted.

3 Problem Definition and ILP Model

The problem considered in this work is the allocation of the tasks of an application to a set of heterogeneous cores $P = \{1, 2, \ldots, m\}$ and the determination of their execution sequence. An application is usually described by a Directed Acyclic Graph (DAG), $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is the set of nodes and $E = \{e_1, e_2, \ldots, e_{nE}\}$ the set of directed edges. Each node of the DAG corresponds to a task of the application and each edge, $e = (v_i, v_j)$, represents

a data dependency from task v_i to v_j. Due to the heterogeinity of the platform, the execution time, D_{ip}, of task v_i on different cores is not the same.

It is assumed that a task starts its execution when all its predecessor tasks have finished theirs and completes it without preemption. In addition, the communication between the cores is performed asynchronously via a rich and low-latency interconnection network; thus the communication overhead is ignored. The goal is to find an assignment of the tasks to the cores and their execution scheduling that minimizes the total execution time (makespan) of the DAG.

The above problem can be formulated by the ILP model (1)-(5), where ILP solvers can find an optimal solution given plenty of time. In the following formulation, t_{sink} is the start time of a virtual sink node with zero execution time to which all tasks with zero out-degree connect. The variable t_i denotes the start time of task v_i, while x_{ip} is a binary decision variable that equals to 1 when task v_i is assigned to core p, otherwise $x_{ip} = 0$. The set $ON(v_i)$ contains the tasks that are independent of task v_i.

$$\min t_{sink} \tag{1}$$

$$\forall v_i \in V \ \sum_{p \in P} x_{ip} = 1 \tag{2}$$

$$\forall e=(v_i, v_j) \in E \ \ t_i + \sum_{p \in P} D_{ip} x_{ip} \leq t_j \tag{3}$$

$$\forall p \in P, \ \forall v_j \in ON(v_i) \ \ t_i + \sum_{p \in P} D_{ip} x_{ip} \leq t_j + \left(3 - x_{ip} - x_{jp} - a_{ij}\right) M \tag{4}$$

$$\forall p \in P, \ \forall v_j \in ON(v_i) \ \ t_j + \sum_{p \in P} D_{jp} x_{ip} \leq t_i + \left(2 - x_{ip} - x_{jp} - a_{ij}\right) M \tag{5}$$

Equation (2) ensures that each task is assigned to exactly one core, while (3) enforces that for an edge $e = (v_i, v_j)$, task v_j starts its execution after the execution of task v_i. A non-overlapped in time execution sequence between two independent tasks v_i, v_j is imposed by (4) and (5), when they are assigned to the same core. Specifically, α_{ij} is a binary decision variable that equals to 1 when v_j is executed after v_i, and to 0 in the opposite case; M is a large integer constant.

Because of the existence of (4) and (5), it is often difficult for an ILP solver to find the optimal solution in reasonable time. Specifically, the big-M constraints in (4), (5) lead to large integrality gaps between the integer solution and the relaxed linear solution so that branch-and-bound may require an enormous amount of enumeration, making it impractical even on a very fast computer [22].

4 Logic-based Benders Decomposition Approach

The Logic-Based Benders Decomposition (LBBD) approach is an iterative process that decomposes the initial problem in two (or more) loosely connected sub-problems and solves them sequentially [14]. Each sub-problem is solved to optimality and fixes a subset of the problem's variables, which are used by the subsequent one. After each iteration, extra constraints are derived by inference from the solution of the final sub-problem. These constraints, called Benders cuts, are added to the first solved sub-problem, cutting the solution space.

Assume an optimization problem (Primal Problem – PP) in the form of (6).

$$\begin{aligned} \min \ & z = f(x, y) \\ s.t. \ & C(x, y) \end{aligned} \tag{6}$$

where x and y are the variables, $z = f(x, y)$ is the cost function and $C(x, y)$ is the set of the constraints. As mentioned, the principle of LBBD is to decompose the PP into two (or more) sub-problems that are solved sequentially. The first sub-problem, which is called the Master Problem (MP), is a relaxation of the PP in the form of (7). It contains only the variables, x, a subset $C_1(x)$ of the constraints, and a relaxed cost function $w = g(x)$.

$$\begin{aligned} \min \ & w = g(x) \\ s.t. \ & C_1(x) \end{aligned} \tag{7}$$

After the solution of MP, the decision variables have specific values $x = x^*$ and are fed to the Sub-Problem (SP), which has the form of (8). In addition, the cost value of the MP,

$w^* = g(x^*)$, serves as a lower bound of the global solution, since it is produced by a relaxed version of the initial cost function.

$$\begin{aligned} \min\ & z = f(x^*, y) \\ s.t.\ & C(x^*, y) \end{aligned} \tag{8}$$

The SP may be feasible, producing a new better solution for the whole problem, or it may be infeasible. In either case, Benders cuts are generated by inference and inserted to the MP. Their purpose is to guide the MP to produce a solution in the next iteration that will render the SP feasible so that the global cost value is improved. The iterative process terminates when the MP becomes infeasible or the cost value of the SP, $z^*=f(x^*, y^*)$, becomes equal to its lower bound $w^* = g(x^*)$.

The efficiency of the above method is based on the concept that the sub-problems are significantly simpler and easier to solve. Moreover, the modeling and solution method is independent for every sub-problem, allowing to exploit different paradigms depending on each specific sub-problem. However, its performance strongly relies on the efficiency of the generated cuts in every iteration to drastically prune the remaining search space. In addition, due to the loose interaction between the sub-problems, the MP is usually enriched with a relaxation of the SP in order to have an insight of the remaining problem and produce more suitable solutions.

4.1 Decomposition of the target mapping problem

The problem under consideration is to find an assignment of each task to a core and an execution sequence so that the completion time is minimized. Given a specific assignment, the problem of finding an execution sequence per core (task scheduling) for dependent and independent tasks is much easier. Therefore, the MP should provide the assignment according to which the SP find a feasible execution sequence, that is a schedule that improves the best-found cost value (makespan).

Based on the above decomposition, the purpose of the MP is to find an assignment of the tasks to the cores according to some optimization criterion. The chosen criterion is a relaxed version of the makespan, meaning that the MP should minimize the start time of the sink node by satisfying only the tasks' dependencies. Hence, it has to optimize (9) subject to(10), and (11) . As the constraints of the MP are simple linear constraints without big-*M*s, it is modeled through the same ILP formulation.

On the other hand, the SP should find a feasible execution schedule by deciding about the sequencing of independent tasks assigned on the same core and respecting the data dependencies between the tasks so that the makespan is minimized. The SP is modeled through the following Constraint Programming (CP) formulation, which replaces the big-*M* constraints ((4), (5)) of the initial ILP formulation.

$$\min t_{sink} \tag{9}$$

$$\forall e = (v_i, v_j) \in E \;\; t_i + D_i \le t_j \tag{10}$$

$$\forall v_j \in ON(v_i) \,|\, X_i = X_j \;\; t_i + D_i \le t_j \vee t_j + D_j \le t_i \tag{11}$$

In the above formulation, t_{sink}, t_i are in agreement with the initial ILP model. X_i and D_i are positive integers that denote the assignment and execution time of task v_i, respectively, whose values are derived by the solution of MP. The objective function and the precedence constraints are the same as in the MP. In more details, (10) corresponds to(3), while (11) replaces (4) and (5), and states a disjunctive relation on independent tasks that are assigned on the same processor.

According to the above decomposition, the MP totally ignores the sequencing of independent tasks that may be assigned on the same core. Thus, a relaxed version of the SP should be included in the MP in order to exclude assignment combinations that will definitely

render the SP infeasible [15, 23]. Finally, an important point of the decomposition approach is the generation of the initial solution, since there may be a great gap between the solutions of the sub-problems especially during the first iterations. To overcome this limitation, a heuristically-generated initial solution was used employing the HEFT heuristic [5].

4.2 Enriching Master with Sub-Problem Relaxation

The relaxation of the sequencing problem should be able to identify assignment combinations of independent tasks that will definitely result in an infeasible SP or, in our case, in a worse makespan value. Instead of performing a simple overload cheking, a more sophisticated procedure has been proposed in [24, 25] as a pre-processing algorithm. The introduced procedure is based on examining the time windows in which several tasks can be executed. Then, it builds constraints for the tasks that can be assigned on the same core so that their bounds are satisfied when they are executed sequentially. The time window for the execution of each task is defined by its release (*RL*) time and its deadline (*DL*). This algorithm produces knapsack constraints in the form of (12), where *K* is the set of tasks that can be executed inside the time window [DL_K, RL_K]. Moreover, it consists of an additional stage that produces cover constraints on tasks that where not examined during the computation of knapsacks.

$$\forall p \in P \quad \sum_{i \in K} D_{ip} x_{ip} \leq DL_K - RL_K \tag{12}$$

In order to apply the preprocessing algorithm of [24, 25] to the current problem, the release time (*RL*) and the deadline (*DL*) of each task are considered as the earliest start and latest finish time, respectively. These values can be derived directly from the DAG, since each task can start its execution when all its predecessors have finished theirs, whereas it must complete it so that enough time remains for its successors to complete theirs without increasing the makespan. Thus, the earliest start time of every task is the defined by the maximum value of the earliest completion time among its predecessors, while the latest completion time or deadline is defined by the latest start time of its successors. Hence, they can be computed by simply traversing the DAG.

The deadline for the sink node is set to $z_{best} - 1$, where z_{best} is the best-found solution until that point of the iterative process, stating that the desired solution should be better than the best-found one. The deadline of the other tasks is also computed as a function of the best-found solution, meaning that it can be defined by a value d_i that should be subtracted from z_{best} so that $DL_i = z_{best} - d_i$, where $d_{sink} = 1$.

However, the core with the minimum execution time may be the same for two or more predecessor tasks or the number of predecessor tasks may be larger than the number of the available cores. Both these situations are not taken into account when computing the execution window of each task by traversing the DAG, whereas they are captured by the following ILP model, whose purpose is to compute the minimum completion time of a set of independent tasks.

$$\min z \tag{13}$$

$$\forall v_j \in V^i \quad \sum_{p \in P} x_{jp} = 1 \tag{14}$$

$$\forall s \in S, \forall p \in P \quad r_s + \sum_{j \in s} D_{jp} x_{jp} \leq z \tag{15}$$

In the above formulation, *z* is the makespan of the set of independent tasks, V^i. This set corresponds either to the set of direct predecessors or the successors of a task if it refers to the computation of the release time or the deadline, respectively. Set *S* contains all the subsets of tasks that are created based on the different release values of the tasks. In detail, for every distinct value, the tasks whose release time is greater or equal than this value are contained in a subset. Finally, r_s is the earliest start time of the subset *s*. The same formulation can be used to compute the value d_i that should be subtracted from z_{best} in order to compute the deadline for every task. Since the earliest start and latest finish time of every task consider the sequencing of the

predecessors and successors, respectively, the following constraints are also included as a sub-problem relaxation in the MP.

$$\forall v_i \in V \; t_i \geq RL_i \tag{16}$$

$$\forall v_i \in V \; t_i + \sum_{p \in P} D_{ip} x_{ip} \leq DL_i \tag{17}$$

The pre-processing algorithm of [24] generates constraints only on a single core assuming that every task can start its execution on its release time. However, the earliest start time depends on the end time of all its predecessors. Thus, when they are assigned on the same core they will be executed sequentially, increasing the start time of their successor. Moreover, if those nodes have a common predecessor, their start time depends on its end time.

The situation described above is captured by the following constraint, where v_i is a fork node whose dependent tasks contained in the set K are joined on node v_j. Combined with the constraints (16) and (17), it corresponds to a knapsack constraint on the time window defined by $[RL_i, DL_j]$ that consider the assignment of tasks v_i, v_j on different cores than the tasks between them.

$$\forall p \in P, \forall f = (v_i, v_j) \in E \; t_i + \sum_{p \in P} D_{ip} x_{ip} + \sum_{k \in K} D_{kp} x_{kp} \leq t_j \tag{18}$$

Makespan Speculation.

Both relaxations described above introduce a part of the sequencing problem in the MP, while their purpose is to exclude assignment combinations that will definitely lead to worse makespan values. Especially during the first iterations, the best-found solution may be far from the optimal one. Consequently, many inefficient solutions are produced by the MP until a better solution is found and the relaxation is tightened.

To avoid this situation, the relaxation can be tightened artificially by providing a speculated solution that is smaller than the actual best-found one. The speculated makespan value, z_s, ranges between the lower bound of the global makespan and the best-found solution, z_{best}.

The algorithm that follows describes the employed procedure [20]. Initially, the solution of the MP (w^*) serves as a lower bound, but it is updated when the MP becomes infeasible. This happens because it becomes overconstrained due to the tightening of the relaxation by the speculated value. Then, the lower bound of the global solution is updated to the speculated value that causes the infeasibility of the MP. The upper bound is initialized to the solution of the HEFT heuristic, z_{HEFT}.

In the case that the MP is feasible, the SP is solved, and if it provides a better solution than the best-found one, the upper bound is updated, while the speculated value is relaxed. If however the returned makespan is worse, then a tighter speculated value is computed and employed in the next iteration.

```
lb = w*, ub = z_HEFT , z_s = ⌈(lb + ub)/2⌉
while lb ≠ ub do
  solve MP to optimality
  if infeasible then
    lb = z_s
    z_s = ⌈(lb + ub)/2⌉
  else
    solve SP to optimality
    if z_SP < ub then
      ub = z_SP
      z_s = ⌈(lb + ub)/2⌉
    else
      z_s = ⌈(lb + z_s)/2⌉
    end if
  end if
end while
```

The speculated value that causes the infeasibility of the MP is larger than the solution that is normally returned by the MP, since it is based on a relaxation of the sub-problem. Therefore, this procedure also provides better information about the bounds where the optimal solution might lie. Finally, every time when the speculated value is relaxed, the cover constraints of the SP relaxation that where added earlier are removed, whereas the right hand side of the knapsack constraints is updated.

4.3 Benders Cuts

After each iteration where the sub-problems have been sequentially executed, extra constraints are generated and added in the master problem for the subsequent iteration. The purpose of these constraints, called Benders cuts, is exclude the previous solution so that another one is generated. This can be achieved with a simple constraint of the form of (19) that excludes only the current assignment.

$$\sum_{i \in V} x_{ip^*} \leq n-1, \quad x_{ip^*} = 1,\ p^* \in P \tag{19}$$

However, the above constraint is very weak since it excludes only one combination of x_{ip} variables. If the cardinality of the assignment constraint is reduced, that is it contains less decision variables, it becomes significantly stronger. This can be done by gradually adding variables in a set and checking the feasibility of the SP considering only the variables of the set until a failure is detected. The order according to which the variables are inserted in the set is defined by a chosen criterion. In our case, the tasks are sorted in a non-decreasing order according to their slack, which is defined by the difference between their latest and earliest start time.

Even though the cardinality of the conflict set is reduced, its computation imposes a significant time overhead. In order to reduce it, the conflict set C is initialized with the tasks whose slack equals zero. The others are sorted and gradually added to the set until the infeasibility is detected according to the following procedure.

```
feed SP with C and solve to feasibility
while feasible do
  add next task to C
  feed SP with C and solve to feasibility
end while
```

The resulting conflict set can be further reduced employing the algorithm provided in [26]. However, this algorithm also imposes significant time overhead, since it should be used in every iteration. Instead, the following procedure was employed, which considers the task that caused the infeasibility an important one. Then, it adds it to the refined conflict set R, which is initialized with the tasks whose slack equals zero. Afterwards, the remaining tasks are added gradually until the infeasibility is detected. The procedure terminates if less than two tasks have been added in the conflict set until reaching the infeasibility, so the set cannot be further reduced.

```
while |R|<|C|-2 do
  add last task of C to R, set C = R
  feed SP with C and solve to feasibility
  while feasible do
    add next task to C
    feed SP with C and solve to feasibility
  end while
end while
```

The main core of the previous algorithms is the solution of the SP considering different sets of tasks until it becomes infeasible. In order to further decrease the overhead that is caused by this procedure, it was performed in parallel launching multiple CP models and collecting the results. This way, the demanded conflict set was smallest among the infeasible models.

5 Hybrid Approach

The logic-based Benders decomposition approach, when applied to the problem of mapping and scheduling of DAGs on heterogeneous cores, has achieved remarkable speedups in terms of solution time and has managed to find the optimal solutions in problems where other methods have failed to do so [20]. However, this method has some limitations that arise due to the decomposition of the original problem. As the solution procedure progresses and a good or even the optimal solution has been found, many assignment combinations are left to be evaluated that cannot be excluded by the relaxation of the sub-problem that is included in the MP.

In that above case, the method relies only to the Benders cuts in order to prune the remaining search space. As these cuts may be not so strong to drastically prune the solution space, the MP produces many equivalent solutions that do not lead to better makespan values. In this case, the process terminates after all the remaining assignment combinations are enumerated. On the other hand, the pure ILP approach suffers from poor LP relaxations and large integrality gaps when big-M constraints are used. Consequently, without being able to generate efficient cutting planes, the branch-and-bound tree becomes enormous and the problem intractable. Nevertheless, if the search space is reduced by an external source, this method can generate strong cutting planes and can be very efficient in improving the solution or proving the optimality of the best-found one.

To combine the strengths of the solution methods, the LBBD approach presented in the previous section is employed as a cut generation scheme. The iterative process is stopped at some point and the pure ILP model is launched while containing the extra constraints that were generated by the LBBD method. Then, it runs with a time limit so that it is stopped when it cannot quickly close the gap and reach the optimality. Even though the ILP model may not end the solution process, it may produce a feasible solution, that is a solution that improves the best-found makespan up to that point. In this case, a cover cut is derived from this solution using the algorithms of the previous section. The LBBD method is launched again trying to produce more cuts or find the optimal solution [21].

6 Experimental Results and Discussion

The introduced approaches were evaluated on randomly generated DAGs by an in-house software tool. The tool takes as input the number of nodes, minimum and maximum execution times per node, minimum and maximum depth between start and sink nodes, the minimum and maximum number of edges and the distribution policy of edges between nodes and the number of DAG to generate. All models were developed using the FICO Xpress Optimization suite [27]. The experiments were run on an i7 6-core PC operating at 3.2 GHz with 16 GB installed memory and a Time Limit (TL) equal to two hours was set.

The experimental results are shown in Table 1 for the ILP, the LBBD, and the Hybrid approaches. The initial solution provided by the HEFT heuristic was also used as an upper bound on the solution in the ILP model, which is referred to as HILP in Table 1. Each row of the table represents 10 DAG instances, resulting in the total amount of 60 instances. Table 1 shows the number of instances that were solved optimally (*OPT*) and the ones that reached the time limit (*TL*) without finding the optimal solution, as well as their average solution time for each group defined by the number of tasks and cores. For the computation of the average time, the solution time of the instances that were not solved within the time limit was set equal to the limit.

By the results of Table 1, it can be observed that the pure ILP model is not able to find the optimal solution for the majority of cases, while it fails even in most of the small-scale problems.

On the other hand, the LBBD approach is very efficient when compared to the ILP, since it solves optimally the vast majority of instances while it is 9× faster. Finally, the Hybrid method achieves to solve all the instances while being also twice as fast as the LBBD one.

Table 1. Experimental Results

	OPT/TL			*Average Time (sec.)*		
Tasks / Cores	*HILP*	*LBBD*	*Hybrid*	*HILP*	*LBBD*	*Hybrid*
20/2	4/6	**10/0**	**10/0**	4353.1	5.2	**4.3**
20/4	4/6	**10/0**	**10/0**	1137.1	0.4	**0.4**
30/4	0/10	6/4	**10/0**	-	2514.8	**1580.3**
30/6	4/6	**10/0**	**10/0**	4594.4	63.6	**7.7**
50/8	4/6	8/2	**10/0**	5301.8	49.8	**30.6**
50/10	1/9	7/3	**10/0**	6647.6	752.3	**54.4**
Overall	17/43	56/4	**60/0**	5287.4	564.3	**279.6**

An important part of the LBBD method is pruning of the remaining solution space by generating strong Benders cuts. In order to evaluate the efficiency of the refinement procedure (Section 4.3), Table 2 shows the results with and without it for the LBBD and the Hybrid approaches. As it can be observed, the generated cover cuts without the refinement procedure are strong enough, since the corresponding cases solve the same number of instances. The refinement of the cuts reduces the number of performed iterations of the LBBD approach, slightly reducing the solution time. Nevertheless, the Hybrid approach can benefit even from the weaker cuts, whereas the overhead for the computation of stronger cuts causes the slight deterioration of its solution time.

Table 2. Experimental Results Without and With Refined Cuts

	OPT/TL				*Average Time (sec.)*			
	LBBD		*Hybrid*		*LBBD*		*Hybrid*	
Tasks / Cores	*w/o Ref.*	*w. Ref.*	*w/o Ref.*	*w. Ref.*	*w/o Ref.*	*w. Ref.*	*w/o Ref.*	*w. Ref.*
20/2	**10/0**	**10/0**	**10/0**	**10/0**	**3.3**	5.2	**4.3**	**4.3**
20/4	**10/0**	**10/0**	**10/0**	**10/0**	0.5	**0.4**	**0.4**	**0.4**
30/4	7/3	7/3	**10/0**	**10/0**	3228.2	**2514.8**	**1147.4**	1580.3
30/6	**10/0**	**10/0**	**10/0**	**10/0**	79.3	**63.6**	10.3	**7.7**
50/8	**10/0**	**10/0**	**10/0**	**10/0**	58.4	**49.8**	64.4	**30.6**
50/10	9/1	9/1	**10/0**	**10/0**	786.3	**752.3**	**19.1**	54.4
Overall	56/4	56/4	**60/0**	**60/0**	692.7	564.3	**207.7**	279.6

The effectiveness of the presented iterative approaches relies mostly on the relaxations of the sub-problem that are inserted in the master problem, since each achieves in its own way the exclusion of infeasible assignment combinations. However, the complexity of the master after the inclusion of relaxed versions of the sub-problem should remain small. Table 3 shows the experimental results for the LBBD approach when different combinations of the presented relaxations are considered in order to explore their contribution. In all cases, the refinement cut procedure is included.

The first relaxation refers to the Independent tasks Sequencing (*IS*) that is captured by the knapsack constraints generated by the pre-processing procedure of [24]. These constraints (Section 4.2, Eq. (12)) are generated for every core for tasks based on their execution time window, which is defined by their release time and deadline. The second relaxation is the Dependent tasks Sequencing (*DS*) that captures the sequential execution of tasks that are dependent on the same predecessor and have a common successor (Section 4.2, Eq. (18)). This relaxation is considered with conjunction with the previous one in Table 3 (*IDS*). Finally, the makespan speculation procedure is applied to both relaxations and are shown in Table 3 as *ISS* and *IDSS*, respectively.

With the application of only the *IS* relaxation, 12 instances remain unsolved while the solution time is about 30 min. After applying the makespan speculation procedure, 3 more instances are solved while the solution time also decreases, since the relaxation is tightened in

the initial stage of the solution procedure and many unnecessary iterations are avoided. The addition of the second relaxation even without the speculation procedure is very effective and solves more instances than the previous one with smaller solution time.

The effectiveness of the *DS* relaxation is owed to the fact that it more active as the solution process progresses. Specifically, instead of the greedy assignment of the tasks by the master during the first iterations, it is forced by the Benders cuts and the knapsack constraints to assign tasks to cores where their execution time is larger. This way, the start time of the dependent tasks may be larger than their release time, whereas their sequential execution may also increase the makespan. Finally, when this relaxation is combined with the speculation procedure, one more instance is solved while the time also decreases.

Table 3. Evaluation of Relaxations in LBBD Approach

	OPT/TL				*Average Time (sec.)*			
Tasks / Cores	***IS***	***ISS***	***IDS***	***IDSS***	***IS***	***ISS***	***IDS***	***IDSS***
20/2	**10/0**	**10/0**	**10/0**	**10/0**	22.3	6.9	12.9	5.2
20/4	**10/0**	**10/0**	**10/0**	**10/0**	12.9	0.5	10.6	0.4
30/4	5/5	6/4	6/4	7/3	4550.7	3311.6	3256.7	2514.8
30/6	10/0	**10/0**	**10/0**	**10/0**	723.5	399.8	115.6	63.6
50/8	6/4	8/2	**10/0**	**10/0**	3215.1	1584.3	1319.4	49.8
50/10	7/3	7/3	9/1	9/1	2613.8	2619.1	1003.3	752.3
Overall	48/12	51/9	55/5	56/4	1856.4	1320.4	953.1	564.3

The effect of the included relaxations is also important for the performance of the Hybrid approach. As mentioned, it is based on the speculation procedure, which provides better estimation of the lower bound of the solution; thus, this procedure is included in the results of Table 4. With the addition of the *DS* relaxation the solution time is about 4.5 min. while all instances are solved, whereas containing only the *IS* relaxation, one instance remains unsolved with twice as much solution time.

Table 4. Evaluation of Relaxations in Hybrid Approach

	OPT/TL		*Average Time (sec.)*	
Tasks / Cores	***ISS***	***IDSS***	***ISS***	***IDSS***
20/2	**10/0**	**10/0**	6.4	**4.3**
20/4	**10/0**	**10/0**	0.5	**0.4**
30/4	**9/1**	**10/0**	2224.4	**1580.3**
30/6	**10/0**	**10/0**	12.4	**7.7**
50/8	**10/0**	**10/0**	32.7	**30.6**
50/10	**10/0**	**10/0**	**32.5**	54.4
Overall	**59/1**	**60/0**	575.5	279.6

7 Conclusion

In this paper, an iterative solution approach based on logic-based Benders decomposition was presented. The effectiveness of the method relies on the relaxed versions of the scheduling problem that are included to the assignment problem and help to exclude in advance many infeasible solutions. The effect of each relaxation as well as the Benders cuts was computationally evaluated through experimental results. By augmenting the model all the evaluated instances were optimally solved within a time limit of two hours, while the overall solution time was also significantly decreased. As a future work, we intend to extend the above hybrid model to capture the data transfer delay between the tasks as well as the memory requirements for their communication.

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A Weight Assignment Approach for Multicriteria Global Path Planning of an Unmanned Combat Vehicle

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Abstract In path planning of an unmanned combat vehicle, we aim to find solution paths that visit all checkpoints while minimizing the accumulated cost factors. In this paper, we consider the global path planning problem of an unmanned combat vehicle which has been modeled into a multiple criteria traveling salesman path problem on a grid network. An algorithm using weight assignment approach is proposed to find a solution path. To demonstrate the proposed algorithm, computational experiments are conducted on simulated maps. Initial experimentation demonstrates that the solution path generated for different problem instances are affected by the weight assignment of different attributes as well as the tabulation of the terrain and the positioning of the checkpoints.

1 Introduction

An unmanned combat vehicle (UCV) is an armed robotic vehicle used to substitute human participation in high risk military operations. A UCV is controlled via wireless communication signal from an external control station. The path planning and navigation process of a UCV is controlled by operators through user interface in the control center [8]. The navigation control of a UCV is divided into local path planning (LPP) and global path planning (GPP). LPP is a real-time process that maintains the stability and safety of UCV throughout the journey. On the other hand, GPP is a deliberate process of searching for feasible travel paths based on user requirements. Modern UCVs are equipped with LPP navigation systems, thus no human intervention is needed to ensure the stability of the UCV throughout the mission. Hence, in this

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study, we assume that the UCV is equipped with LPP navigation system, where it can perform obstacle avoidance, stability maintenance and direction changes by itself.

To perform the GPP process, the terrain map needs to be represented in a form where a path searching algorithm can be implemented. There are three main representation methods used: cell decomposition, roadmap, and potential field. Among these three methods, cell decomposition method is the most frequently used due to its simplicity and flexibility [3]. By using cell decomposition method, the map of area of interest (AOI) is inscribed into a grid made of uniformly arranged cells. The terrain information of the region inscribed in the particular cell is associated to the corresponding cell. Then, the path searching algorithm is conducted based on user input and the data associated to the grid. If a solution path is found, the algorithm will generate the path and project it onto the map. Otherwise it will report failure.

In general, the GPP of a UCV has the objective of finding a travel path that minimizes resource consumption. Note that the resources are not limited to physical attributes such as traveling distance, fuel consumption or ammunition consumption. It can represent non-physical attributes such as potential threat level, terrain sloppiness, or signal strength. In the grid, the traveling paths are described as a sequence of cells that the UCV needs to pass through. The resource consumption for the UCV throughout the journey can be estimated by the cumulative costs of various attributes along the traveling path. GPP problems with single attribute are known as single criteria problem, while GPP problems that involve multiple attributes are known as multicriteria problems.

The GPP problems with only two checkpoints (origin and destination points) are often modeled as shortest path problems [4], [7]. Also, GPP problems with multiple (more than two) checkpoints are mostly modeled as traveling salesman problem (TSP) or one of its variants [1], [5], [11], [12]. Single criteria GPP problems that modeled as TSPs has been solved using Christofides algorithm [2], convex hull method [6], and various heuristics as reported by [9] in his review paper. On the other hand, multicriteria GPPs that modeled as TSP has been discussed by [1], [11].

In this paper, we consider the GPP problem with multiple checkpoints on a grid map. The objective is to find a path that travels all checkpoints with unique starting and destination point, that minimizes the cumulative cost of the attributes. Our problem is modeled as a traveling salesman path problem (TSPP) with 2 unique endpoints in a grid network. The TSPP with 2 unique endpoints is a variation of the TSP, which search for least cost Hamilton path with 2 unique endpoints, instead of searching for least cost Hamilton cycle as in TSP.

We make the following assumptions:

1. Each attribute is assigned a weight by the user that represent its importance relative to one another.
2. The UCV is in good condition before it starts its journey, so there is no need to return to base for servicing during the mission.
3. The grid is eight-connected, meaning that the UCV can move freely to any one of the eight adjacent cells in the grid if available.
4. The cells in the grid can be traversed more than once if necessary.

Figure 1 shows an example map for a GPP problem, where a series of checkpoints (depicted as flags) are required to be visited, where s and t are starting and ending locations respectively. Throughout the journey, the UCV has to avoid enemy territory and enemy detection (depicted as rifles and radars respectively).

Fig. 1: An example of a satellite terrain map with checkpoints and attributes.

2 Problem Description

In this study, the area of interest is inscribed into an $N \times N$ grid, where each cell in the grid stores the cost value of traverse time (time required to pass through a terrain), risk level (likelihood of an UCV to face a threat), and signal interference level (level of signal interruption encountered by UCV) of the corresponding region. For simplicity, we call these attributes as attribute 1, 2, and 3 respectively. The grid is modeled into an undirected graph G. The terminology, notation, and definitions related to graph G are listed as follows:

G	Undirected graph $G = (V, E, c)$
V	Set of $N \times N$ vertices representing the cells in the grid
E	Set of edges representing the link between two adjacent cells, i.e. $E = \{(i, j)\|i, j \in V\}$
c	Cost function $c : V \to \mathbb{R}^3$ that maps vertex i to cost vector $c(i) = (c_i^1, c_i^2, c_i^3)$, where c_i^k represent the cost of attributes $k = 1, 2, 3$ for vertex i
Λ	Set of $n + 2$ checkpoints need to be visited, i.e. $\Lambda = \{s, v_1, v_2, \ldots, v_n, t\} \subsetneq V$
s, t	Unique starting and ending points
$v_1, v_2, \ldots, v_n$	Intermediate points
$\pi_{i,j}$	Path from vertex i to j, written as a sequence of vertices $\pi_{i,j} =< i, \ldots, j >$
$C_{i,j}$	Cost of path $\pi_{i,j}$, given by the addition of cost vectors of all traversed vertices, i.e. $C_{i,j} = \sum_{k \in \pi_{i,j}} c(k)$
$\oplus$	Path concatenation operator: a concatenation of path $\pi_{i,j}$ and $\pi_{j,k}$, denoted by $\pi_{i,j} \oplus \pi_{j,k}$, gives path $\pi_{i,k} =< i, \ldots, j, \ldots, k >$
$Suc(i)$	Set of successors of vertex i, i.e. $Suc(i) = \{j\|(i, j) \in E\}$
$Pre(i)$	Set or predecessors of vertex i, i.e. $Pre(i) = \{j\|(j, i) \in E\}$

$\pi_{i,j}, i, j \in \Lambda$	Partial path: a path between two vertices in Λ
$\pi^*_{s,t}$	Complete path: a path that joins s to t and pass through all the intermediate points in Λ. i.e. $\pi^*_{s,t} =< s, \ldots, v_{i_1}, \ldots, v_{i_2}, \ldots, v_{i_n}, \ldots, t >$, where $v_{i_1}, v_{i_2}, \ldots, v_{i_n}$ are all the intermediate points

Note that a complete path $\pi^*_{s,t} =< s, \ldots, v_{i_1}, \ldots, v_{i_2}, \ldots, v_{i_{n-1}}, \ldots, v_{i_n}, \ldots, t >$ that contains $|\Lambda| = n + 2$ checkpoints is a concatenation of $n + 1$ partial paths $\pi_{s,v_{i_1}} \oplus \pi_{v_{i_1},v_{i_2}} \oplus \ldots \oplus \pi_{i_{n-1},v_{i_n}} \oplus \pi_{v_{i_n},t}$, where $v_{i_1}, v_{i_2}, \ldots, v_{i_{n-1}}, v_{i_n}$ are all the intermediate points in Λ.

Let x_i be defined by

$$x_i = \begin{cases} 1, & \text{if vertex } i \text{ was traversed in the path,} \\ 0, & \text{otherwise.} \end{cases}$$

Our problem is formulated as follows:

$$\text{Minimize} \quad \sum_{i \in V} x_i c_i^1 \tag{1}$$

$$\text{Minimize} \quad \sum_{i \in V} x_i c_i^2 \tag{2}$$

$$\text{Minimize} \quad \sum_{i \in V} x_i c_i^3 \tag{3}$$

subject to

$$\sum_{j \in Suc(i)} x_j \geq 1, \quad \forall\, i \in \Lambda \backslash \{t\} \tag{4}$$

$$\sum_{j \in Suc(t)} x_j \geq 0, \tag{5}$$

$$\sum_{j \in Pre(i)} x_j \geq 1, \quad \forall\, i \in \Lambda \backslash \{s\} \tag{6}$$

$$\sum_{j \in Pre(s)} x_j \geq 0, \tag{7}$$

$$\sum_{j \in Suc(i)} x_j + \sum_{j \in Pre(i)} x_j = 2k, \quad k \geq 0, \forall\, i \in V \backslash \{s, t\} \tag{8}$$

$$\sum_{j \in Suc(i)} x_j + \sum_{j \in Pre(i)} x_j = 2k + 1, \quad k \geq 0,\, i = s, t \tag{9}$$

$$\sum_{i \in S} \sum_{j \in Suc(i) \cap S'} x_j \geq 1, \quad \forall\, S \subsetneq V, S \neq \emptyset \tag{10}$$

$$x_i \in \{0, 1\}, \quad \forall i \in V \tag{11}$$

Objective functions (1) to (3) represent the three cost factors (traverse time, risk level, signal interference level) to be minimized. Constraints (4) and (5) ensure that the path leaves every checkpoint in Λ at least once (except vertex t, which will only be traversed if necessary). Constraints (6) and (7) ensure that the path enters every checkpoint in Λ at least once (except vertex s, which will only be traversed if necessary). Constraints (8) and (9) preserve flow conservation throughout the path. Constraints (10) prevents the solution path from forming subtours.

3 Solution Approach

In Section 2, the GPP problem was formulated as a multicriteria optimization problem. We propose a heuristic based on additive weighting method to reduce our multicriteria optimization problem into a single criteria optimization problem. We solve our problem using an extended version f the two phase heuristic method proposed in [10]. The extension exists in the form of a preprocessing stage where the multiple attributes are reduced to a single attribute using user defined weights.

3.1 Preprocessing stage

In a terrain map, the costs of different attributes may be assigned in different scales, depending on the map source. To ensure that all attributes have an equal level of significance, the cost of each attribute is normalized into a common range of $[0, 1]$. Here, we implement min-max normalization method to transform original cost vectors into the normalized cost vector for all vertices in the graph. The normalized cost vector correspond to vertex $i \in V$ is defined as $\tilde{c}(i) = (\tilde{c}_i^1, \tilde{c}_i^2, \tilde{c}_i^3)$, such that

$$\tilde{c}_i^k = \frac{c_i^k - c_{min}^k}{c_{max}^k - c_{min}^k},$$

where c_{min}^k and c_{max}^k represent the minimum and maximum cost value for attribute $k = 1, 2, 3$.

Let the user defined weight vector be $\mathbf{w} = (w_1, w_2, w_3)$, where w_1, w_2, and w_3 represent the weights for attribute $k = 1, 2, 3$ respectively, such that $\sum_{k=1}^{3} w_k = 1$ for $w_1, w_2, w_3 \geq 0$. By using additive weighting method on the graph G with normalized cost vector, we construct the graph $G' = (V', E', c')$ in the following manner

1. $V' = V, E' = E$.
2. $c' : V \rightarrow \mathbb{R}$ is a additive weighing cost function such that for $i \in V'$, $c'(i) = \mathbf{w}[\tilde{c}(i)]^T = \sum_{k=1}^{3} w_k \tilde{c}_i^k$.
3. The cost of path between vertex i and j in G' is $C'_{i,j} = \sum_{k \in \pi_{i,j}} c'(k)$.
4. The definition for path, path concatenation, successor set, predecessor set in G' are similarly defined as in G.

Based on the graph G', our problem can be reformulated as single criteria optimization problem that minimizes the objective function

$$w_1 \sum_{i \in V} x_i \tilde{c}_i^1 + w_2 \sum_{i \in V} x_i \tilde{c}_i^2 + w_3 \sum_{i \in V} x_i \tilde{c}_i^3$$

subject to the constraints set in Section 2.

The reduced GPP problem formulation above is an s-t traveling salesman path problem (stTSPP), which is a variation of traveling salesman problem (TSP). In this study, we relax the Hamiltonian property restriction on the solution generated in as we have already made the assumption that the vertices may be visited more than once. In stTSPP, a complete undirected weighted graph is provided and the objective is to find a path that starts and ends at uniquely given endpoints and passes through all other points, such that the total travel cost is minimized. The two phase heuristic used to solve the problem was proposed in [10]. We present the detailed explanation for the procedure in the following subsections.

3.2 Phase 1: Generating the complete graph G_K

In Phase 1, we find the shortest partial path between all pairs of checkpoints $i, j \in \Lambda$ on G'. As G' is undirected, $C'_{i,j} = C'_{j,i}$ for all pairs of checkpoints. In total, there are $\binom{n+2}{2}$ non-directed paths connecting all pairs of checkpoints. We implement Dijkstra's algorithm to find all these shortest paths. Note that in typical shortest path problem, the cost is associated on the edges of the graph and the path cost is calculated by summing up the cost of traversed edges. However, in G' the cost value is associated on vertices of the graph. Hence, while applying the Dijkstra's algorithm, the path cost is calculated by adding the cost associated to the vertices instead of edges.

Let Λ_R be defined as $\Lambda_R = \{p_1, p_2, \ldots, p_{n+2}\}$ such that $p_1 = s, p_k = v_{k-1}$ for $k = 2, 3, \ldots, n+1$ and $p_{n+2} = t$. The details of Dijkstra's algorithm is shown in Algorithm 1. For each pair of checkpoints $p_i, p_j \in \Lambda_R$, the minimum cost of path between p_i and p_j is C'_{p_i,p_j}, and its path π_{p_i,p_j} can be found by using procedure `Generate Path` described in Algorithm 2.

Algorithm 1 `Dijkstra's algorithm`

Require: $G' = (V', E', c'), \Lambda_R = \{p_1, \ldots, p_{n+2}\}$
for all $p_i, p_j \in \Lambda_R$, $(i \neq j, i > j)$ **do**
  $S^* \leftarrow \emptyset$ ▷ S^*: Set of vertices k of which minimum $C'_{p_i,k}$ is obtained
  $S \leftarrow V'$ ▷ S: Set of vertices k of which minimum $C'_{p_i,k}$ is not obtained yet
  $C'_{p_i,p_i} \leftarrow c(p_i)$
  $C'_{p_i,k} \leftarrow \infty$ for all $k \in V' \backslash \{p_i\}$
  while $p_j \notin S^*$ **do**
    $i^* \leftarrow \arg\min_{k \in S}\{C'_{p_i,k}\}$ ▷ i^*: Vertex $k \in S$ with minimum $C'_{p_i,k}$
    $S \leftarrow S - \{i^*\}, S^* \leftarrow S^* \cup \{i^*\}$
    for all $v \in Suc(i^*)$ **do**
      if $C'_{p_i,v} > C'_{p_i,i^*} + c(v)$ **then**
        $C'_{p_i,v} \leftarrow C'_{p_i,i^*} + c(v)$
        $Prev(v) \leftarrow i^*$
      end if
    end for
  end while
  perform `Generate Path`
end for

Algorithm 2 `Generate Path`

$\pi_{p_i,p_j} \leftarrow$ empty stack
$u \leftarrow p_j$ ▷ u: Variable used to store the predecessor vertices
while $Prev(u)$ exist **do**
  insert u into π_{p_i,p_j}
  $u \leftarrow Prev(u)$
end while
insert u into π_{p_i,p_j}

3.3 Phase 2: Search for the best complete path

In Phase 1, the shortest path between all pairs of checkpoints in G' is found using Dijkstra's algorithm. We use all the paths to construct a complete graph G_K. The construction of complete graph $G_K = (V_K, E_K, c_K)$ is defined as follows:

1. The vertex set, $V_K = \Lambda$.
2. The edge set, $E_K = \{(i,j) | i,j \in \Lambda\}$, where $|E_K| = \binom{n+2}{2}$.
3. The edge cost function, $c_K : E_K \to \mathbb{R}$ is defined as $c_K(i,j) = C'_{i,j} - c'(i) - c'(j)$.
4. For $i,j \in V_K$, the path from i to j, $\Pi_{i,j}$ is an alternating series of vertex and edges, such that $\Pi_{i,j} =< i, (i,u_1), u_1, (u_1,u_2), u_2, \ldots, u_x, (u_x,j), j >$, where $u_1, u_2, \ldots, u_x \in V_K$. For simplicity, we write $\Pi_{i,j} =< i, u_1, u_2, \ldots, u_x, j >$.
5. The cost of the path $\Pi_{i,j}$, denoted by $C^K_{i,j}$, is given by $C^K_{i,j} = \sum_{(u,v)\in\Pi_{i,j}} c_K(u,v) + \sum_{k\in\Pi_{i,j}} c'(k)$.
6. A cycle that starts at vertex i, denoted by $Cyc(i)$, is a path that begins and ends with vertex i. The cost of $Cyc(i)$ is given by $C^K_{Cyc(i)} = \sum_{(u,v)\in Cyc(i)} c_K(u,v) + \sum_{k\in Cyc(i)} c'(k)$.
7. The concatenation of two paths $\Pi_{i,j} =< i, (i,u_1), u_1, \ldots, u_x, (u_x,j), j >$ and $\Pi_{j,k} =< j, (j,u_{x+1}), u_{x+1}, \ldots, u_y, (u_y,k), k >$, denoted by $\Pi_{i,j} \oplus \Pi_{j,k}$, is given by $\Pi_{i,j} \oplus \Pi_{j,k} =< i, (i,u_1), u_1, \ldots, u_x, (u_x,j), j, (j,u_{x+1}), u_{x+1}, \ldots, u_y, (u_y,k), k >$.

The stTSPP is equivalent to the standard TSP in which the edge cost connecting s and t, is replaced by $-M$, where M is a sufficiently large value such that $M > \max_{(i,j)\neq(s,t)}\{c_K(i,j)\}$. Hence, solution for standard TSP on $G_K = (V_K, E_K, c_K)$ with $c_K(s,t) = -M$ corresponds to the solution path for stTSPP on $G_K = (V_K, E_K, c_K)$ with $c_K(i,j)$ remain unchanged for all $i,j \in V_K$.

We solve our TSP using the nearest neighbor algorithm (NNA). The NNA starts by randomly selects a vertex i as a starting point and visits the immediate best (least cost) vertex until all the vertices are visited and finally returns to the initial vertex. This forms a least cost cycle that passes through all vertices. However, as NNA is a greedy algorithm, the least cost edges are always selected first, which leaves high cost edges to be chosen in later. Hence, NNA does not guarantee an optimal solution is found. The procedure of NNA for TSP on $G_K = (V_K, E_K, c_K)$ with $c_K(s,t) = -M$ is given in Algorithm 3.

Algorithm 3 `Nearest Neighbor Algorithm (NNA)`

Require: $G_K = (V_K, E_K, c_K)$ with $c_K(s,t) = -M$, $\Lambda_R = \{p_1, \ldots, p_{n+2}\}$
$S \leftarrow \Lambda_R$ ▷ S: Set of vertices not selected yet
Select a random $p_i \in S$
$S \leftarrow \Lambda_R - \{p_i\}$
$Cyc(p_i) \leftarrow < p_i >, C^K_{Cyc(p_i)} \leftarrow c'(p_i)$
while $S \neq \emptyset$ **do**
  $l \leftarrow$ last vertex in $Cyc(p_i)$
  $i^* \leftarrow \arg\min_{k\in S}\{c_K(l,k)\}$ ▷ i^*: Vertex $k \in S$ with least $c_K(l,k)$
  $S \leftarrow S - \{i^*\}$
  $Cyc(p_i) \leftarrow Cyc(p_i) \oplus \{(l,i^*)\}$
  $C^K_{Cyc(p_i)} \leftarrow C^K_{Cyc(p_i)} + c_K(l,i^*) + c'(i^*)$
end while
$l \leftarrow$ last vertex in $Cyc(p_i)$
$Cyc(p_i) \leftarrow Cyc(p_i) \oplus \{l, p_i\}$
$C^K_{Cyc(p_i)} \leftarrow C^K_{Cyc(p_i)} + c_K(l,p_i)$

To increase the likelihood in obtaining a better solution, an improvement technique known as repetitive nearest neighbor algorithm (RNNA) is performed. RNNA conducts NNA $n+2$ times by starting at each checkpoint in Λ once. This gives $n+2$ least cost cycles $Cyc(i)$ that pass through all checkpoints. Among these $n+2$ cycles, the cycle with least cost is chosen. Then, the edge (s,t) is removed from the selected cycle. This forms the least cost path $\Pi_{s,t}$ that connects s to t and passes through all checkpoints, which is the solution for stTSPP in G_K. The algorithm for RNNA is described in Algorithm 4.

Algorithm 4 `Repetitive Nearest Neighbor Algorithm (RNNA)`

Require: $G_K = (V_K, E_K, c_K)$ with $c_K(s,t) = -M$, $\Lambda_R = \{p_1, \ldots, p_{n+2}\}$
1: $Cyc^* \leftarrow$ empty sequence ▷ Cyc^*: Least cost cycle among $Cyc(i), \forall i \in \Lambda_R$.
2: **for all** point $p_i \in \Lambda_R$ **do**
3: Perform `NNA` ▷ Line 2 of `NNA`: select p_i instead of randomly select a vertex in S.
4: **end for**
5: $Cyc^* \leftarrow \arg\min_{p_i \in \Lambda_R}\{C^K_{Cyc(p_i)}\}$
6: Remove edge (s,t) from Cyc^* to form a path $\Pi_{s,t} =< s, \ldots, t >$.
7: $C^K_{s,t} \leftarrow C^K_{Cyc^*} + M$

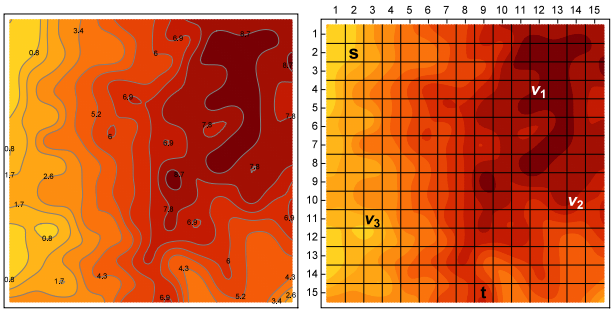
To trace the complete path $\pi^*_{s,t}$ on terrain map, the corresponding edges in $\Pi_{s,t}$ are concatenated, such that $\pi^*_{s,t} = \pi_{s,v_{i_1}} \oplus \pi_{v_{i_1},v_{i_2}} \oplus \ldots \oplus \pi_{v_{i_{n-1}},v_{i_n}} \oplus \pi_{v_{i_n},t}$, where $\pi_{u,v}$ is the shortest path between the vertex u and v as found in Dijkstra's algorithm. The total cost of the complete path in terrain map is given by $C_{s,t} = \sum_{k \in \pi_{s,t}} c(k)$.

4 Results and Discussion

In this section, a GPP problem demonstration and computational experiments are conducted using a simulated terrain. The terrain was designed by using Wolfram *Mathematica*® software and all the algorithms are coded in C++. The details of problem demonstration and computational experiments are explained as follows.

4.1 Algorithm Implementation

We demonstrate our proposed algorithm using the simulated terrain as shown in Figure 2(a). The terrain is inscribed onto a 15×15 grid in Figure 2(b) with the set of checkpoints Λ being marked and their coordinates stated. The distribution of attributes in the terrain are shown in Figure 3. The costs of each cells are assigned with integers ranging from 0 to 9. Then, the grid is modeled into the graph $G = (V, E, c)$ as in Figure 4. Note that the cell (i,j) in the grid corresponds to vertex $15(i-1)+j$ in G for $i, j \in \{1, 2, \ldots, 15\}$. The corresponding vertex number for all the checkpoints are listed in Figure 4. We assign the weight vector to be $\mathbf{w} = (0.7, 0.2, 0.1)$.



(a) Simulated terrain.

(b) 15×15 grid and their coordinates.

Checkpoint	Coordinate
s	$(2, 2)$
v_1	$(4, 12)$
v_2	$(10, 14)$
v_3	$(11, 3)$
t	$(15, 9)$

Fig. 2: Inscribing terrain onto a grid.

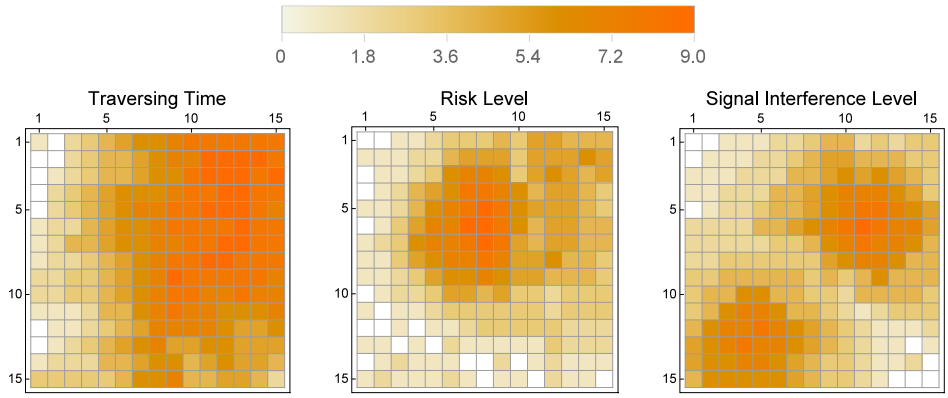
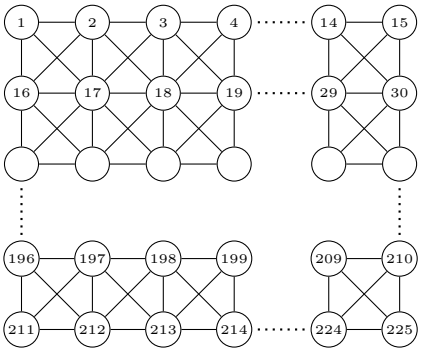


Fig. 3: Terrain attribute distribution.



Checkpoint	Vertex no.
s	17
v_1	57
v_2	149
v_3	153
t	219

Fig. 4: Graph G representing the grid and checkpoints with their corresponding vertices in G.

Next, the cost vector in every vertex i is normalized. For example, the normalized cost vector for vertex 8 is

$$\begin{aligned}\tilde{c}(8) &= \left(\frac{c_8^1 - c_{min}^1}{c_{max}^1 - c_{min}^1}, \frac{c_8^2 - c_{min}^2}{c_{max}^2 - c_{min}^2}, \frac{c_8^3 - c_{min}^3}{c_{max}^3 - c_{min}^3}\right) \\ &= \left(\frac{6-0}{9-0}, \frac{3-0}{9-0}, \frac{3-0}{9-0}\right) \\ &= \left(\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right).\end{aligned}$$

Then, we form a graph $G' = (V, E, c')$ with $c'(i)$ represents the additive weighting cost for vertex i. The $c'(i)$ for vertex i is calculated by $c'(i) = \mathbf{w}[\tilde{c}(i)]^T = \sum_{k=1}^{3} w_k \tilde{c}_i^k$. For example, $c'(8)$ is given by

$$c'(8) = \mathbf{w}[\tilde{c}(8)]^T = \sum_{k=1}^{3} w_k \tilde{c}_8^k = \frac{7}{10}\left(\frac{2}{3}\right) + \frac{2}{10}\left(\frac{1}{3}\right) + \frac{1}{10}\left(\frac{1}{3}\right) = \frac{17}{30}.$$

In Phase 1, $\binom{5}{2} = 10$ shortest paths linking all pairs of checkpoints in Λ are generated based on graph G'. The details of these paths are shown in Table 1. The process is followed by constructing a complete graph G_K (Figure 5) using the paths generated in Table 1.

Table 1: Shortest paths between all pair of checkpoints $i, j \in \Lambda$ in graph G' and corresponding edge cost in G_K.

Vertex pair	Minimum cost path ($\pi'_{i,j}$)	$C'_{i,j}$	$C_K(i,j)$
17, 57	< 17, 18, 4, 5, 21, 22, 8, 9, 25, 41, 57 >	493/90	41/9
17, 149	< 17, 31, 46, 61, 76, 91, 106, 122, 137, 153, 169, 185, 201, 202, 203, 189, 205, 191, 177, 163, 149 >	547/90	97/18
17, 153	< 17, 31, 46, 61, 76, 91, 106, 122, 137, 153 >	46/45	37/45
17, 219	< 17, 31, 46, 61, 76, 91, 106, 122, 137, 153, 169, 185, 201, 202, 218, 219 >	18/5	89/30
57, 149	< 57, 73, 89, 105, 120, 135, 149 >	161/30	172/45
57, 153	< 57, 71, 85, 99, 113, 127, 141, 155, 169, 153 >	31/5	463/90
57, 219	< 57, 71, 85, 99, 113, 127, 141, 155, 171, 187, 203, 219 >	709/90	115/18
149, 153	< 149, 163, 177, 191, 205, 189, 203, 202, 201, 185, 169, 153 >	47/9	22/5
149, 219	< 149, 163, 177, 191, 205, 219 >	29/9	59/30
153, 219	< 153, 169, 185, 201, 202, 218, 219 >	247/90	89/45

In Phase 2, we implement RNNA to find the least cost cycle which begins with checkpoint i for all $i \in \Lambda$. In this example, we set $c_K(s,t) = -7$. This is to ensure that during the RNNA process the edge (s,t) will always be selected, thus making all the cycles generated contain edge (s,t). In the RNNA process, NNA was repeated $|\Lambda| = 5$ times. The list of least cost cycle which begins at different checkpoint is tabulated in Table 2. Also, the cycles generated are shown in Figure 6.

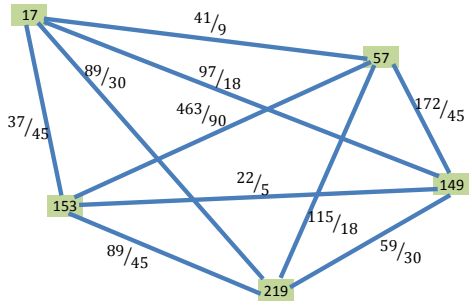
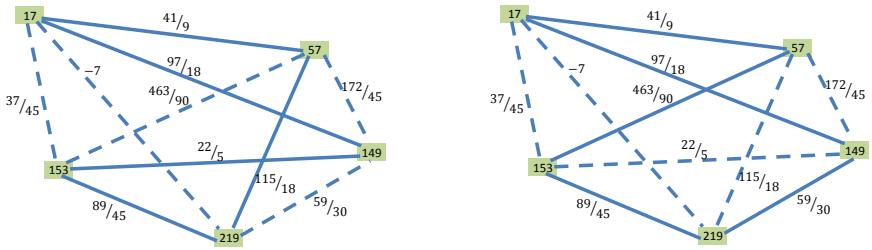


Fig. 5: Complete graph G_K constructed using the vertex pairs in Table 1.

Table 2: Shortest cycles that starts with all checkpoints $i \in \Lambda$ in complete graph G_K and it's corresponding complete path.

Starting point (i)	Minimum cost cycle ($Cyc(i)$)	$C^K_{Cyc(i)}$	Corresponding path ($\Pi_{s,t}$)	$C^K_{s,t}$
17	$< 17, 219, 149, 57, 153, 17 >$	71/10	$< 17, 153, 57, 149, 219 >$	141/10
57	$< 57, 149, 219, 17, 153, 57 >$	71/10	$< 17, 153, 57, 149, 219 >$	141/10
149	$< 149, 219, 17, 153, 57, 149 >$	71/10	$< 17, 153, 57, 149, 219 >$	141/10
153	$< 153, 17, 219, 149, 57, 153 >$	71/10	$< 17, 153, 57, 149, 219 >$	141/10
219	$< 219, 17, 153, 149, 57, 219 >$	97/9	$< 17, 153, 149, 57, 219 >$	160/9



(a) $Cyc(17), Cyc(57), Cyc(149), Cyc(153)$ (b) $Cyc(219)$

Fig. 6: Cycles $Cyc(i)$ in G_K with $c_K(s,t) = -7$ generated using RNNA for $i = 17, 57, 149, 153, 219$ (dashed lines). Note that by removing edge $(s,t) = (17, 219)$ the cycles turn into paths that passes through all the checkpoints with endpoints s and t.

From Table 2, we select the $Cyc(i)$ with least $C^K_{Cyc(i)}$, which is $Cyc(17)$ (Figure 6(a)) with cost $C^K_{s,t} = 141/10$. In cases where there are multiple $Cyc(i)$ with minimum $C^K_{Cyc(i)}$, we randomly select one. Then, the edge $(17, 219)$ was removed to form a path $\Pi_{17,219} =< 17, 153, 57, 149, 219 >$. This path corresponds to the path $\pi_{17,153} \oplus \pi_{153,57} \oplus \pi_{57,149} \oplus \pi_{149,219}$ in graph G. By tracing the path on the simulated terrain, we obtain the complete path as shown in Figures 7 and 8. The total cost for the complete path generated can be calculated by adding up the costs of all the traversed cells, which is $(141, 82, 118)$.

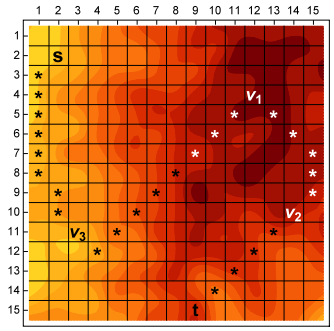


Fig. 7: Solution path with $\mathbf{w} = (0.7, 0.2, 0.1)$ projected onto simulated terrain with checkpoint visiting sequence of $s - v_3 - v_1 - v_2 - t$.

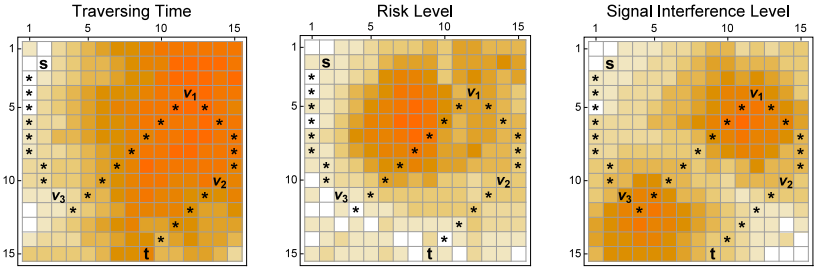
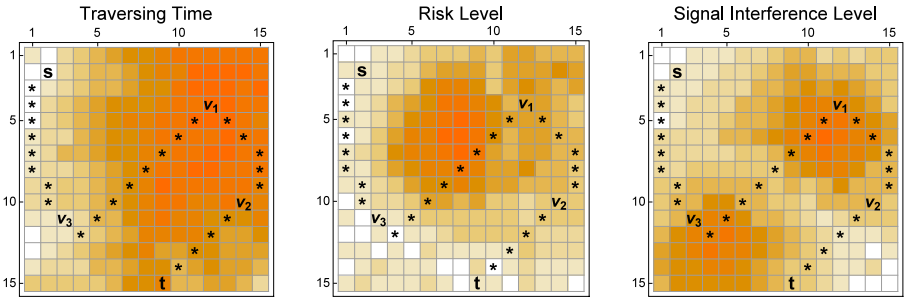


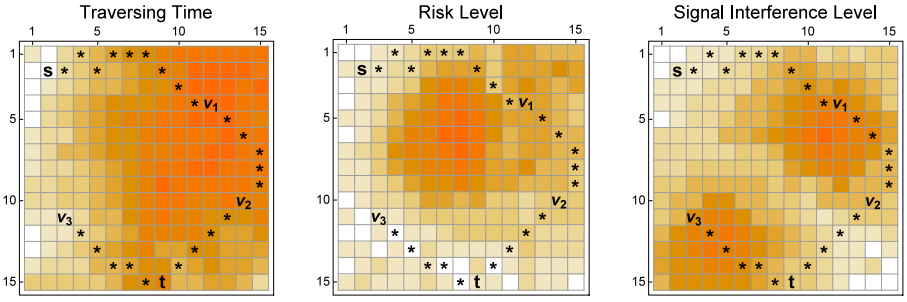
Fig. 8: Solution path for $\mathbf{w} = (0.7, 0.2, 0.1)$ projected onto each attribute grid, with total cost $(141, 82, 118)$.

4.2 Computational Experiment

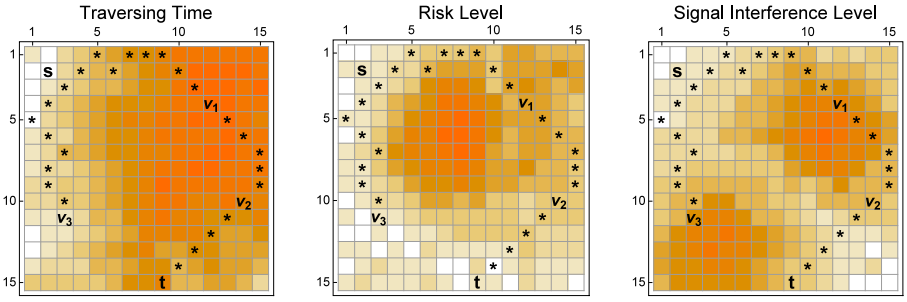
In this section, seven instances of computational experiments are conducted on the same simulated terrain with different weight combinations. The effect of different weight assignments on solution paths that produced are observed. Seven weight combinations are used, which are $(1, 0, 0), (0, 1, 0), (0, 0, 1), (0.33, 0.33, 0.34), (0.5, 0.5, 0), (0.5, 0, 0.5)$, and $(0, 0.5, 0.5)$. The first three instances represent prioritizing a single attribute. The fourth instance represents equal priority for all attributes. Finally, the last three instances represent equally prioritizing two of the three attributes. The complete paths generated using these weight combinations are shown in Figure 9(a)- 8(g).



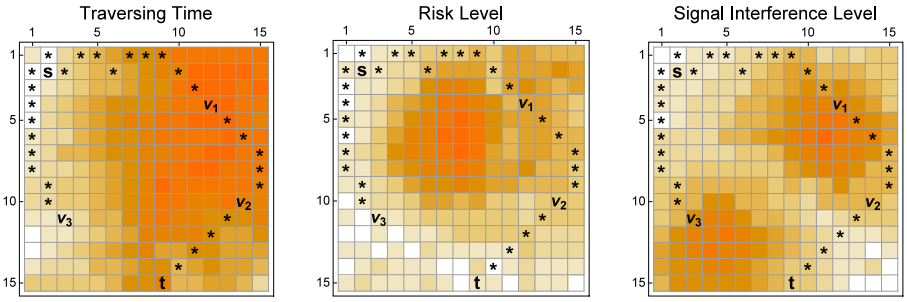
(a) $\mathbf{w} = (1, 0, 0)$, vertex visiting sequence: $s - v_3 - v_1 - v_2 - t$, solution cost $(141, 82, 118)$



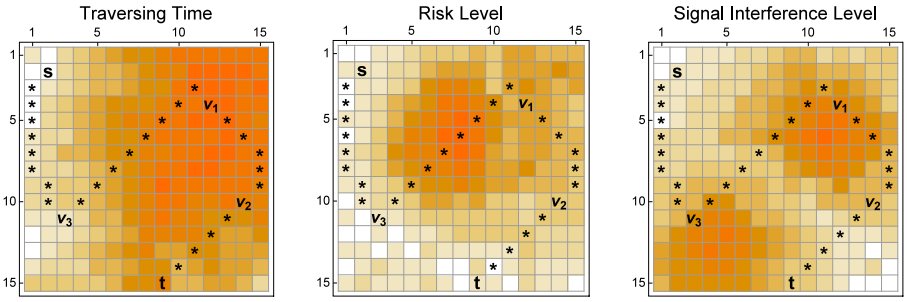
(b) $\mathbf{w} = (0, 1, 0)$, vertex visiting sequence: $s - v_1 - v_2 - v_3 - t$, solution cost $(184, 65, 146)$



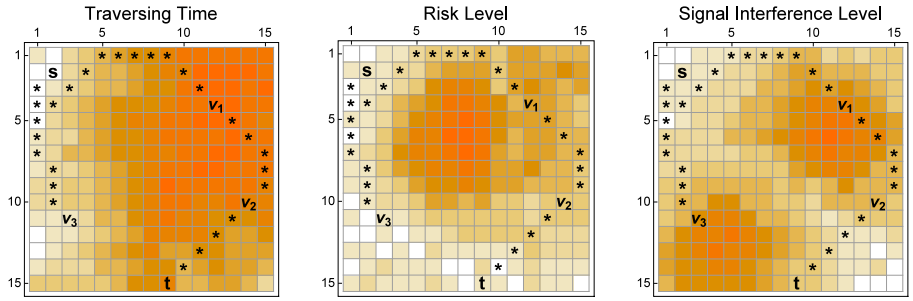
(c) $\mathbf{w} = (0, 0, 1)$, vertex visiting sequence: $s - v_3 - v_1 - v_2 - t$, solution cost $(166, 85, 109)$.



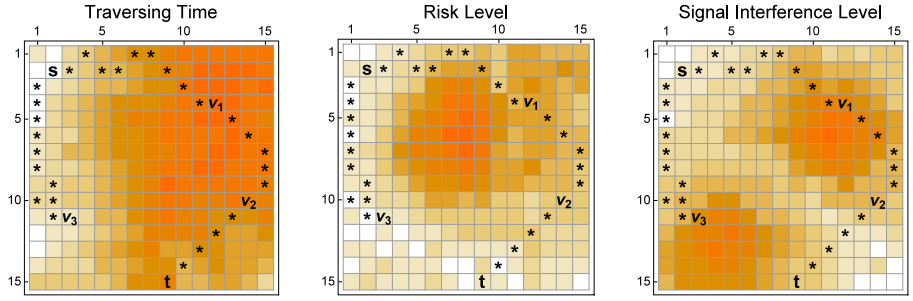
(d) $\mathbf{w} = (0.33, 0.33, 0.34)$, vertex visiting sequence: $s - v_3 - v_1 - v_2 - t$, solution cost $(148, 74, 110)$.



(e) $\mathbf{w} = (0.5, 0, 0.5)$, vertex visiting sequence: $s - v_3 - v_1 - v_2 - t$, solution cost $(143, 92, 111)$



(f) $\mathbf{w} = (0, 0.5, 0.5)$, vertex visiting sequence: $s - v_3 - v_1 - v_2 - t$, solution cost $(153, 74, 109)$.



(g) $\mathbf{w} = (0.5, 0.5, 0)$, vertex visiting sequence: $s - v_3 - v_1 - v_2 - t$, solution cost $(149, 73, 119)$.

Fig. 8: Complete paths for different weight combinations.

In Figure 9(a)- 8(c), the weight combinations are such that only one attribute is prioritized per instance. Thus, the traversing path tends to only avoid entering into high cost region for the prioritized attribute map. Hence, each complete solution path generated has the least total cost for prioritized attribute. For problem setting with $\mathbf{w} = (0.33, 0.33, 0.34)$ in Figure 8(d), the complete solution path is very similar to the solution path generated in problem setting $\mathbf{w} = (0, 0, 1)$ in Figure 8(c), except for the partial paths π_{s,v_3} and π_{v_3,v_1} where the paths traverse the cells at the border of the grid. This is because in $\mathbf{w} = (0.33, 0.33, 0.34)$ the three attributes are given equal priority. Thus the generated solution path tends to avoid high cost regions in all three attribute maps as much as possible.

On the other hand, Figure 8(e)- 8(g) shows the solution paths generated with respect to $\mathbf{w} = (0.5, 0, 0.5), (0, 0.5, 0.5)$ and $(0.5, 0.5, 0)$ respectively, where equal priority are given to two of the three attributes in each instance. We observed that $\mathbf{w} = (0.5, 0, 0.5)$ and $\mathbf{w} = (1, 0, 0)$ gives very similar solution paths. This implies that giving the third attribute equal weight to the first attribute does not affect the solution path by much. Note that the high cost region for attribute 1 and 3 has significant overlap, especially in the upper right corner. We belief that this, along with the positioning of the checkpoints contribute to the observed phenomena. This behavior can also be seen for $\mathbf{w} = (0, 0.5, 0.5)$ and $\mathbf{w} = (0, 0, 1)$.

However, note that in Figure 8(g) the solution path generated from $\mathbf{w} = (0.5, 0.5, 0)$ is not similar to solutions for problem setting $\mathbf{w} = (1, 0, 0)$ as well as $\mathbf{w} = (0, 1, 0)$ which

is unusual. This is possibly due to the position of checkpoint v_1 which is located at high cost region in map of first attribute but in low cost region in map of the second attribute. In such situation, the solution path needs to pass through the region with low costs in both first and second attributes as equal priorities are placed on both first and second attributes.

Also, note that the cost of the solution path generated in problem setting with $\mathbf{w} = (0, 0, 1)$ is dominated by solution path in problem setting $\mathbf{w} = (0, 0.5, 0.5)$. This is due to the arbitrary selection of a partial path when there are multiple partial paths with similar minimum cost exists in the graph in Phase 1 of our proposed algorithm. In single criteria optimization problem, the objective is focused on finding a solution with the minimum cost, rather than finding all possible solutions that have the minimum cost. Thus, when there exist multiple solutions that gives the same least cost, the solution is selected arbitrarily. However, when a multicriteria optimization problem is transformed into a single criteria optimization problem, all possible solutions relative to the minimum single criteria cost must be identified in order to eliminate the non-Pareto optimal solutions. Thus, if a path is arbitrarily chosen when multiple solution paths with similar lowest costs exist, we might end up choosing a path which is non-Pareto optimal. Hence, an efficient way to find Pareto optimal solutions among all possible solution paths with minimum costs exists is currently being studied.

5 Conclusion

In this paper, we studied the multiple criteria GPP problem of UCV where the terrain map was represented on a grid. Our extension to the two phase heuristic algorithm proposed in [10] was based on the additive weighting method. Our multicriteria optimization problem was transformed into a single criteria optimization problem and the GPP problem was modeled as an s-t traveling salesman path problem (stTSPP). We performed 7 instances of computational experiments on the same terrain with different attributes weight combinations.

We found that the positioning of the checkpoints as well as the high cost regions for each attributes affects the solution path obtained. We also observed that our method could generate solutions that were non-Pareto optimal due to solving our multicriteria problem using a method suited to single criteria problems. Improvements such as generating all least cost path and performing dominance checking can be made to the current solution method.

This research can be extended in several ways. For example, we may consider GPP in real time, as environment factors are related to each other and dynamic. In such cases, each cost factor is represented by functions instead of constants. Also, we may consider GPP involving multiple UCVs, where multiple UCVs with similar (or different) capabilities are to be assigned in the path planning process. This is similar to multiple traveling salesman problem. Also, we may consider GPP of UCV in clustered form, where a UCV travels to multiple sites in a region before visiting the next nearest regions. This can be formulated into clustered TSP.

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Optimal Interdiction of Vehicle Routing on a Dynamic Network

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Abstract Network interdiction problems are often defined on a network with instantaneous flow. In vehicle routing applications, however, the flow is time dependent instead of instantaneous. Moreover, the properties of a transportation network are likely to vary over time and, hence, such a network is dynamic. In this work, we consider the extension of a static network interdiction problem to a dynamic network. To solve this problem, we propose an efficient solution by means of complementary slackness constraints, a reformulation of the dynamic network to a static time-expanded network, and finally a linearization of complicating non-convex constrains. The result consists of a mixed integer linear programming formulation. We show the applicability of this method in a computational experiment with two small but significant examples.

1 Introduction

We consider a Stackelberg game on a dynamic network, in which the player moving first tries to reduce capacities on edges to best miminize the obtainable profit of the second moving player, who tries to maximize profit, by solving a min-cost-flow problem. The latter can be seen as model for a company, who can choose to deliver goods to various distribution points up to a certain demand from various sources. The network under consideration is dynamic in the sense that edge traversal times and costs are variant and the reward obtained by delivering commodities is time dependent, such that e.g. late delivery is penalized. It

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should be noted, that the we assume that for each destination there is only one commodity, making it essentially a single commodity problem. In this paper we do not assume an upper-bound on the number of vehicles that would be required to implement the optimal solution obtained, but this extension could be made without too much work. This is an extension of the static network interdiction problem [1] to a dynamic network with time horizon T.

The problem of optimal routing of a fleet of transportation vehicles has been first proposed in an abstract manner by Ford and Fulkerson [2] as the maximal flow through a network. The following approaches for the interdiction of such a flow exist: Wood [3] shows that the problem is NP-complete and proposes integer models for variations of the problem on a static network. Israeli and Wood [4] propose a mixed integer programming formulation to maximize the shortest path from a source to a sink and develop more efficient decomposition algorithms, e.g. an enhanced Benders' decomposition method. Royset and Wood [5] consider a bi-objective version of the problem: minimizing total interdiction cost and minimizing maximum flow. They propose a branch-and-bound method exploiting Lagrangian relaxation for solving the problem. Wood [6] provides an overview of basic theoretical models and solution techniques for bilevel network interdiction. The problem extension of optimal interdiction of a flow on a dynamic network has been studied in the following works. Lunday and Sherali [7] propose a mixed integer non-linear problem formulation for a dynamic network interdiction problem that can be solved directly using a commercial solver or by a proposed heuristic. Szeto and Lo [8] consider time variant network properties and propose models and algorithms applied to not only flow interdiction games but to an entire class of Stackelberg games on networks. Rad and Kakhki [9] consider the problem with variant traversal times for each edge. They present a new formulation based on the concept of Temporally Repeated Flow to interrupt the flow of a single commodity and solve the problem using a Benders' decomposition approach.

In this work, we assume a fleet of vehicles to transport multiple commodities on a network with variant edge traversal times and time dependent delivery rewards. The proposed method for optimal interdiction consists of a mixed integer linear program, which can be solved efficiently using a commercial solver. It is easy to implement as the need for decomposition algorithms is omitted. Furthermore, optimal solutions are found as opposed to heuristic solution methods.

The remainder of this paper is structured as follows. First, a formal description of the problem is given in section 2. The proposed method for optimal flow interdiction on a dynamic network is then presented in section 3. Computational experiments are described in section 4 and, finally, this paper is concluded in section 5.

2 Problem Description

Let $\mathbb{T} = \{1, \ldots, T\}$ be the makespan defined by T. The dynamic network $\mathscr{N} = (V, V_+, V_-, E, w, \tau, d, c, r)$ consists of the directed graph (V, E), capacities $w_{ij} \geq 0, (i,j) \in E$, a transit times $\tau_{ij} \in \mathbb{T}, (i,j) \in E$, demands $d_i \geq 0, i \in V_+$, costs at each time $c_{ij}(t) \geq 0, (i,j) \in E, t \in \mathbb{T}$, and rewards at each time $r_i(t) \geq 0, i \in V_+, t \in \mathbb{T}$. Here $V_- \subset V$ is the set of possible source nodes and $V_+ \subset V$ the set of sink nodes. Intermediate nodes are referred to by $V_* = V \setminus (V_- \cup V_+)$. We aim to find an optimal interdiction plan $q_{ij}(t) \in \mathbb{N}, (i,j) \in E, t \in \mathbb{T}$ that minimizes the maximally obtainable profit, i.e. the difference of the sums of rewards and costs. The latter is the solution to the min-cost-flow problem from sinks to sources, restricted by capacities and demand and subject to costs and rewards. Here, an interdiction plan indicates the amount by which the capacities w_{ij} are reduced on each edge and at each

point in time. To obtain a well-formulated problem with a non-trivial solution, the sum of possible alterations to the capacities is limited by a fraction β of the sum of all capacities.

3 Dynamic Network Interdiction

We propose an efficient method to find an optimal interdiction plan on a dynamic network in this section. First, a model for finding a routing of transportation vehicles that maximizes the cumulative reward for a given fixed interdiction plan q is presented in subsection 3.1. Then, a time expended network is constructed in subsection 3.2, such that the problem of optimal interdiction can be solved by means of the model presented in subsection 3.3. Finally, a linearization of the model is proposed in subsection 3.4, to solve the problem more efficiently.

3.1 Model for Optimal Dynamic Network Flow

The dynamic routing of transportation vehicles is modelled as flow $x_{ij}(t) \in \mathbb{N}_0, (i,j) \in E, t \in \mathbb{T}$. In order to find an optimal flow, we add an artificial super source v_0 and super sink v_1 to the network $\mathcal{N}$, as well as artificial edges $E_{art} = E_- \cup E_+ \cup (v_1, v_0)$. Here, let $E_- = \{(v_0, i) : i \in V_-\}$ with cost $c_{v_0,i}(t) = 0$ and capacity $w_{v_0,i} = \infty$. Furthermore, let $E_+ = \{(i, v_1) : i \in V_+\}$ with cost $c_{i,v_1}(t) = -r_i(t)$ and capacity $w_{v_1,i} = d_i$, which reduces the problem formulation to just costs and capacities. The cost on edge (v_1, v_0) is zero and its capacity is infinite. We formulate the problem of finding a routing of transportation vehicles that minimizes the costs for a given fixed interdiction plan (q_{ij}) as a circulation problem:

$$\min \quad \sum_{t \in \mathbb{T}} \sum_{(i,j) \in E \cup E_{art}} x_{ij}(t) c_{ij}(t) \tag{1}$$

$$\text{s.t.} \quad \sum_{j:(i,j) \in E \cup E_{art}} \sum_{t=0}^{T-\tau_{ij}} x_{ij}(t) - \sum_{j:(j,i) \in E \cup E_{art}} \sum_{t=\tau_{ji}}^{T} x_{ji}(t - \tau_{ji}) = 0 \quad \forall i \in V \tag{2}$$

$$x_{ij}(t) \leq w_{ij} - q_{ij}(t) \quad \forall (i,j) \in E, \forall t \in \mathbb{T} \tag{3}$$

$$\sum_{t \in \mathbb{T}} x_{ij}(t) \leq w_{ij} \quad \forall (i,j) \in E_{art} \tag{4}$$

$$x_{ij}(t) \geq 0 \quad \forall (i,j) \in E \cup E_{art}, \forall t \in \mathbb{T} \tag{5}$$

The objective in (1) is to minimize the cumulative routing costs over time and the set of traveled edges. The constraints in (2) ensure the flow preservation at each time step. The capacity constraints in (3) ensure that the flow does not exceed the interdicted capacity. Finally, the constraints in (4) are introduced to ensure that the cumulative delivered commodities over time do not exceed the demand.

3.2 Optimal Flow on the Time Expanded Network

An established method to tackle an optimal flow problem in a dynamic network is to reduce it to a similar problem on the *time expanded network* [10]. The basic procedure is to generate

replica of each node for all time steps and connect the edges according to E and τ; this is not done for v_0 and v_1. Furthermore, extra care needs to be taken when treating the nodes in V_+. While they are replicated for each timestep as well, a new set of nodes $V_{1+}^{\mathscr{T}}$ needs to be generated, which in essence is a copy of V_+, where we denote the new node corresponding to $i \in V_+$ by $v_{1i} \in V_{1+}^{\mathscr{T}}$. Edges are added between $i_t \in V_+^{\mathscr{T}}$ and $v_{1i} \in V_{1+}^{\mathscr{T}}$ with $c_{i_t,v_{1i}} = -r_i(t)$, $w_{i_t,v_{1i}} = \infty$, and between $v_{1i} \in V_{1+}^{\mathscr{T}}$ and v_1 with $c_{v_{1i},v_1} = 0$, $w_{v_{1i},v_1} = d_i$. This ensures that the demand remains valid across all time steps, while the reward is time dependent. The time expanded network $\mathscr{N}^{\mathscr{T}}$ is then constructed along the lines presented by [10] as follows:

$V_+^{\mathscr{T}} := \{i_t : i \in V_+, t \in \mathbb{T}\}$
$V_-^{\mathscr{T}} := \{i_t : i \in V_-, t \in \mathbb{T}\}$
$V_*^{\mathscr{T}} := \{i_t : i \in V_*, t \in \mathbb{T}\}$
$v_0^{\mathscr{T}} := v_0$
$v_1^{\mathscr{T}} := v_1$
$V_{1+}^{\mathscr{T}} := V_{1+}$
$V^{\mathscr{T}} := V_+^{\mathscr{T}} \cup V_-^{\mathscr{T}} \cup V_*^{\mathscr{T}} \cup v_0^{\mathscr{T}} \cup v_1^{\mathscr{T}} \cup V_{1+}^{\mathscr{T}}$
$w_{ij}^{\mathscr{T}} := w_{ij}$ for $(i,j) \in E$
$c_{ij(t)}^{\mathscr{T}} := c_{ij}(t)$ for $(i,j) \in E$
$q_{ij(t)}^{\mathscr{T}} := q_{ij}(t)$ for $(i,j) \in E$
$d_{ij}^{\mathscr{T}} := d_{ij}$ for $(i,j) \in E_{art}$
$r_{ij(t)}^{\mathscr{T}} := r_{ij}(t)$ for $(i,j) \in E_{art}$
$E^{\mathscr{T}} := \{(i_t, j_{t+\tau_{ij}}), (i,j) \in E, 0 \le t \le T - \tau_{ij}\} \cup \{(i_t, i_{t+1}) : i \in V, 0 \le t < T\}$
$E_-^{\mathscr{T}} := \{(v_0, i_t), \text{ for } i_t \in V_-^{\mathscr{T}}\}$
$E_+^{\mathscr{T}} := \{(i_t, v_{1i}), \text{ for } i_t \in V_+^{\mathscr{T}}\}$
$E_{1+}^{\mathscr{T}} := \{(i_t, v_{1i}), \text{ for } i \in V_+^{\mathscr{T}}\}$
$E_{art}^{\mathscr{T}} := E_-^{\mathscr{T}} \cup E_+^{\mathscr{T}} \cup (v_1, v_0) \cup E_{1+}^{\mathscr{T}}$

The optimal flow circulation problem on the time expanded network is then formulated as:

$$\min \quad \sum_{(i,j) \in E^{\mathscr{T}} \cup E_{art}^{\mathscr{T}}} x_{ij} c_{ij}^{\mathscr{T}} \tag{6}$$

$$\text{s.t.} \quad \sum_{(i,j) \in E \cup E_{art}^{\mathscr{T}}} x_{ij} - \sum_{(j,i) \in E^{\mathscr{T}} \cup E_{art}^{\mathscr{T}}} x_{ji} = 0 \qquad \forall i \in V^{\mathscr{T}} \tag{7}$$

$$x_{ij} \le w_{ij}^{\mathscr{T}} - q_{ij}^{\mathscr{T}} \qquad \forall (i,j) \in E^{\mathscr{T}} \tag{8}$$

$$x_{ij} \le w_{ij}^{\mathscr{T}} \qquad \forall (i,j) \in E_{art}^{\mathscr{T}} \tag{9}$$

$$x_{ij} \ge 0 \qquad \forall (i,j) \in E^{\mathscr{T}} \cup E_{art}^{\mathscr{T}} \tag{10}$$

This problem formulation is very similar to the formulation for the problem on the dynamic network (1)-(5). The single difference is that the time component can be omitted and we yield exactly the formulation as presented by [1] for the problem on a static network. In the next subsection, we drop the invariably of the interdiction plan $q^{\mathscr{T}}$ and present a model that yields an optimal interdiction plan.

3.3 Model for Dynamic Network Interdiction

First, associate dual variables $\lambda = \{\lambda_i : i \in V^{\mathscr{T}}\}$ with the constraints in (7), dual variables $\mu = \{\mu_{ij} : (i,j) \in E^{\mathscr{T}} \cup E_{art}^{\mathscr{T}}\}$ with the constraints in (8) and (9). Furthermore, add slack variables $z_{ij}^{(1)}$ to the left hand side of each constraint in (8) and (9) to bring (6)-(10) in standard form. By formulation of the dual of (6)-(10) as presented by [1] and by *complementary slackness*, the constraints (12)-(17) are the optimality conditions for x, λ and μ. Hence, a feasible solution x, λ and μ that maximizes the profit of the fleet is optimal if these conditions hold. An optimal interdiction plan can then be obtained by solving the following mixed integer nonlinear program (MINLP):

$$\max \quad \sum_{(i,j)\in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art}} x_{ij}c^{\mathscr{T}}_{ij} \tag{11}$$

$$\text{s.t.} \quad \sum_{(i,j)\in E\cup E^{\mathscr{T}}_{art}} x_{ij} - \sum_{(j,i)\in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art}} x_{ji} = 0 \qquad \forall i \in V^{\mathscr{T}} \tag{12}$$

$$x_{ij} + z^{(1)}_{ij} = w^{\mathscr{T}}_{ij} - q_{ij} \qquad \forall (i,j) \in E^{\mathscr{T}} \tag{13}$$

$$x_{ij} + z^{(1)}_{ij} = w^{\mathscr{T}}_{ij} \qquad \forall (i,j) \in E^{\mathscr{T}}_{art} \tag{14}$$

$$\lambda_i - \lambda_j + \mu_{ij} + z^{(2)}_{ij} = c^{\mathscr{T}}_{ij} \qquad \forall (i,j) \in E^{\mathscr{T}} \cup E^{\mathscr{T}}_{art} \tag{15}$$

$$\mu_{ij} z^{(1)}_{ij} = 0 \qquad \forall (i,j) \in E^{\mathscr{T}} \cup E^{\mathscr{T}}_{art} \tag{16}$$

$$x_{ij} z^{(2)}_{ij} = 0 \qquad \forall (i,j) \in E^{\mathscr{T}} \cup E^{\mathscr{T}}_{art} \tag{17}$$

$$\sum_{(i,j)\in E^{\mathscr{T}}} q_{ij} \leq \beta \sum_{(i,j)\in E^{\mathscr{T}}} w^{\mathscr{T}}_{ij} \tag{18}$$

$$x_{ij}, z^{(1)}_{ij}, z^{(2)}_{ij} \geq 0 \qquad \forall (i,j) \in E^{\mathscr{T}} \cup E^{\mathscr{T}}_{art} \tag{19}$$

$$\mu_{ij} \leq 0 \qquad \forall (i,j) \in E^{\mathscr{T}} \cup E^{\mathscr{T}}_{art} \tag{20}$$

$$\lambda_i \in \mathbb{R} \qquad \forall i \in V^{\mathscr{T}} \tag{21}$$

$$q_{ij} \in \mathbb{N}_0 \qquad \forall (i,j) \in E^{\mathscr{T}} \tag{22}$$

Here, the objective (11) is to minimize the profit of the fleet. Constraints (12)-(17) are the optimality conditions that ensure that the flow x is indeed the optimal response of the fleet to the interdiction plan q. Constraint (18) ensures that the total interdiction does not exceed a given fraction β of the cumulative network capacity.

It should be noted that in this formulation $w^{\mathscr{T}}_{ij}$ and q_{ij} have to be interpreted as being checked when entering the edge at the given time, not as being valid at this point in time for the entire flow on the edge. If that were the intention, constraint (14) would need to be replaced by

$$\sum_{s=0}^{\tau_{ij}-1} xij(t-s) + z^{(1)}_{ij} = w^{\mathscr{T}}_{ij} - q_{ij} \quad \forall (i,j) \in E^{\mathscr{T}},$$

and constraint (15) would need to be adjusted accordingly.

3.4 Optimal Dynamic Network Interdiction

A mixed integer non-linear problem is difficult to solve in general. To solve the MINLP in (11)-(22) more efficiently, we propose a new formulation containing only linear constraints. The two sets of constraints in (16) and (17) are the quadratic constraints which will be replaced by a total of six linear sets of constraints. To this end, four binary auxiliary decision variables $a_{ij}, a'_{ij}, a''_{ij}, a'''_{ij} \in \{0,1\}$ are introduced. The derivation of the linear constraints is explained in detail by [1]. The two original quadratic constraints are replaced by the six resulting linear constraints in (28)-(33) in the following mixed integer linear programming (MILP) formulation:

$$\max \quad \sum_{(i,j)\in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art}} x_{ij}c^{\mathscr{T}}_{ij} \tag{23}$$

$$s.t. \quad \sum_{(i,j)\in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art}} x_{ij} - \sum_{(j,i)\in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art}} x_{ji} = 0 \qquad \forall i \in V^{\mathscr{T}} \tag{24}$$

$$x_{ij} + z^{(1)}_{ij} = w^{\mathscr{T}}_{ij} - q_{ij} \qquad \forall (i,j) \in E^{\mathscr{T}} \tag{25}$$

$$x_{ij} + z^{(1)}_{ij} = w^{\mathscr{T}}_{ij} \qquad \forall (i,j) \in E^{\mathscr{T}}_{art} \tag{26}$$

$$\lambda_i - \lambda_j + \mu_{ij} + z^{(2)}_{ij} = c^{\mathscr{T}}_{ij} \qquad \forall (i,j) \in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art} \tag{27}$$

$$-\mu_{ij} - \left(\max_{i'} |r^{\mathscr{T}}_{i'}|\right) a_{ij} \leq 0 \qquad \forall (i,j) \in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art} \tag{28}$$

$$z^{(1)}_{ij} - w^{\mathscr{T}}_{ij} a'_{ij} \leq 0 \qquad \forall (i,j) \in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art} \tag{29}$$

$$a_{ij} + a'_{ij} \leq 1 \qquad \forall (i,j) \in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art} \tag{30}$$

$$x_{ij} - w^{\mathscr{T}}_{ij} a''_{ij} \leq 0 \qquad \forall (i,j) \in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art} \tag{31}$$

$$z^{(2)}_{ij} - \left(3\max_{i'} |r^{\mathscr{T}}_{i'}| + c^{\mathscr{T}}_{ij}\right) a'''_{ij} \leq 0 \qquad \forall (i,j) \in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art} \tag{32}$$

$$a''_{ij} + a'''_{ij} \leq 1 \qquad \forall (i,j) \in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art} \tag{33}$$

$$\sum_{(i,j)\in E^{\mathscr{T}}} q_{ij} \leq \beta \sum_{(i,j)\in E^{\mathscr{T}}} w^{\mathscr{T}}_{ij} \tag{34}$$

$$x_{ij}, z^{(1)}_{ij}, z^{(2)}_{ij} \geq 0 \qquad \forall (i,j) \in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art} \tag{35}$$

$$\mu_{ij} \leq 0 \qquad \forall (i,j) \in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art} \tag{36}$$

$$\lambda_i \in \mathbb{R} \qquad \forall i \in V^{\mathscr{T}} \tag{37}$$

$$q_{ij} \in \mathbb{N}_0 \qquad \forall (i,j) \in E^{\mathscr{T}} \tag{38}$$

$$a_{ij}, a'_{ij}, a''_{ij}, a'''_{ij} \in \{0,1\} \qquad \forall (i,j) \in E^{\mathscr{T}}\cup E^{\mathscr{T}}_{art} \tag{39}$$

Here, the objective (23) is still to minimize the profit of the fleet. The constraints in (24)-(33) are the *linear* optimality conditions that ensure that the flow x is indeed the optimal response of the fleet to the interdiction plan q. Finally, the constraint (34) did not change and remains the budget constraint. Reformulation of the problem in linear terms results in a significant reduction in run time, especially when solving the problem with a commercial solver. The computational experiments in the next section will show the efficiency and applicability of the proposed method.

4 Computational Experiments

The computational experiment was performed on an Intel(R) Core(TM) i7-5600U CPU processor with 2.6 GHz and a usable memory of 7.7 GB. The simulation platform is written in Python, using IBM Cplex with default parameter settings to solve the instances of the proposed MILP. The first instance studied can be found in Figure 1, where each edge (i, j) is labeled with $\tau_{i,j}, w_{i,j}$. The only source node is A, the only sink with a demand of 10 is H. The values of c and r can be found in table 1 and $\beta = 0.1$. The costs follow one of three structures: they are either constant in time, or have two or three phases - like on and off peak - or they are alternating. All of these yield different implications for the exact schedule of the routes. The solution can be found in figure 2, where dashed edges indicate interruption and $5(2-4), 3(0-1)$ should be read as "'a flow of 5 is leaving at every time step from 2 to

4, while the capacity is reduced by 3 for flow starting at time 0 up to time 1"'. In the case of disruptions on an edge at just one point in time, the endpoint is omitted. It can be seen that instead of blocking the same edges for every time step, the two best routes are being blocked at the earliest time, they would be of use, as to delay the flow. Due to the decreasing structure of the rewards, this increases the overall costs. The decay in rewards is not rapid enough though, to make it worth going through the center.

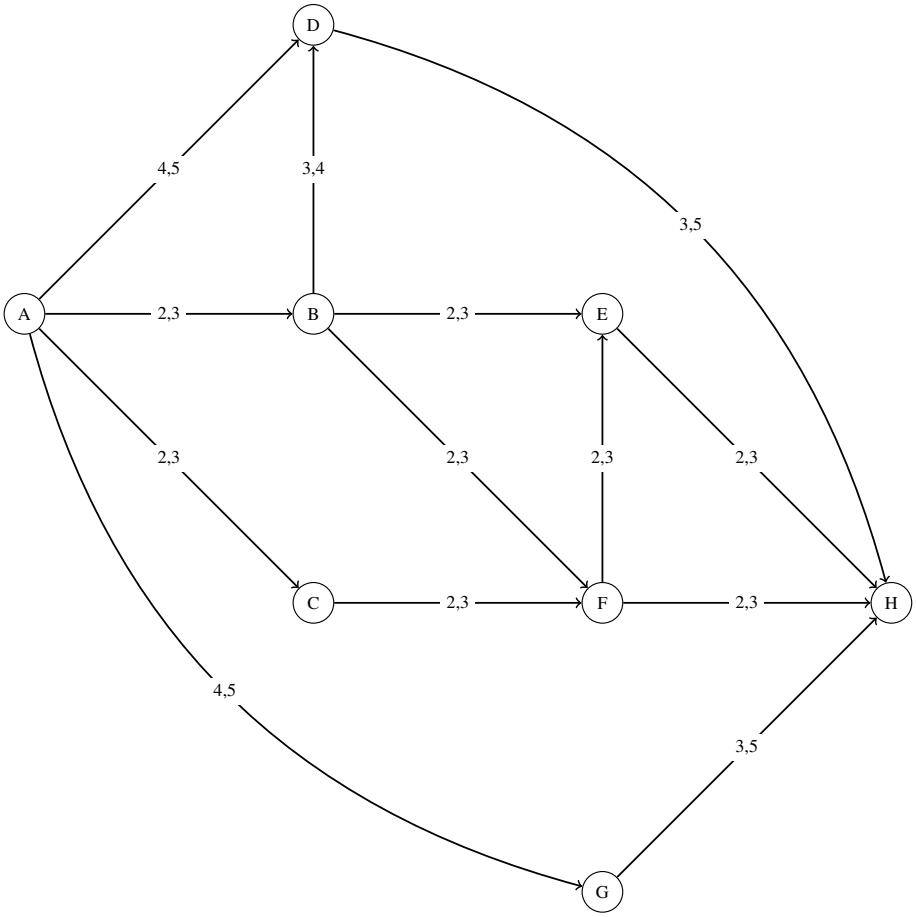

Fig. 1 The network for example 1 with $\tau_{i,j}, w_{i,j}$ on each edge.

The second example chosen can be found in figure 3 and table 2. It is a reduced version of the first example to be able to run it at two different timescales. The longer version runs for 12 time steps and the solution can be found in figure 4. Of particular interest is here, that the edge (E,F) could not be interrupted fully at time 9, which is exploited by the optimal flow. For the shorter version, the problem is run for 8 time steps. The costs and rewards are the ones in table 2 without brackets, $\beta = 0.05$ and the solution can be found in figure 5. It can be seen that in this instance all the flow is routed through the lower half of the network. And that this time the edge (C,F) is blocked as much as possible. The reason to reduce

Table 1 The values of $c_{i,j}(t)$ and $r_H(t)$ for example 1.

t	(A,B)	(A,C)	(A,D)	(A,G)	(B,D)	(B,E)	(B,F)	(C,F)	(D,H)	(E,H)	(F,E)	(F,H)	(G,H)	r_H
1	1	1	4	5	3	1	2	1	4	1	2	2	4	20
2	2	1	4	5	4	1	2	1	4	1	2	2	4	20
3	1	1	4	5	3	1	2	1	4	1	2	2	5	20
4	2	1	4	4	4	1	2	2	5	2	2	1	5	20
5	1	2	4	4	3	1	2	2	5	2	2	1	5	20
6	2	2	4	4	4	1	2	2	5	2	2	1	4	19
7	1	2			3	1	2	1	5	1	2	2	4	18
8	2	2				1	2	1		1	2	2		17
9														16
10														15

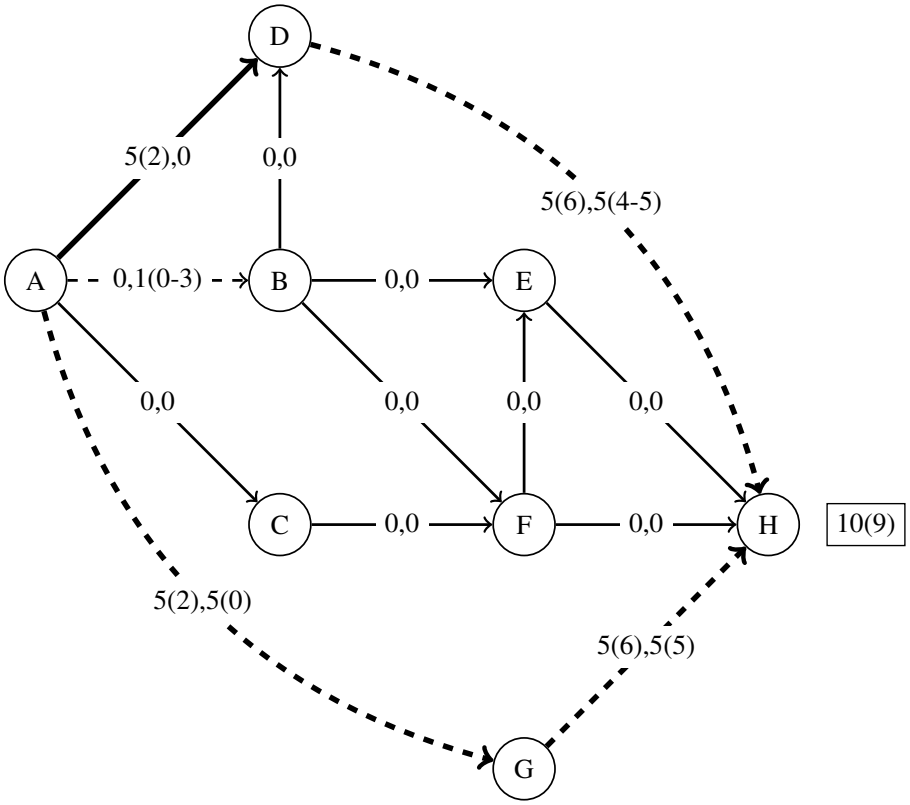

Fig. 2 The solution for example one. Dashed edges indicate interruptions, while thicker errors indicate flow. Further more and edge with 2(3-5),4(1-2) has a flow of 2 starting on each of the time steps 3-5 and has a capacity reduction of 4 at each the beginning of each time step 1,2.

β was that with the original value the interdiction was sufficiently potent to suppress flow all together. More interesting however is the comparison of the run times. While example 1 could be solved in 33.07s the short version of example 2 took just 0.28s. This might be not too surprising, since the latter is smaller and shorter. However, in the long version of just 4 more time steps, the solution time increases to 567.11s, demonstrating the massive increase due to rolling out the network for more time steps.

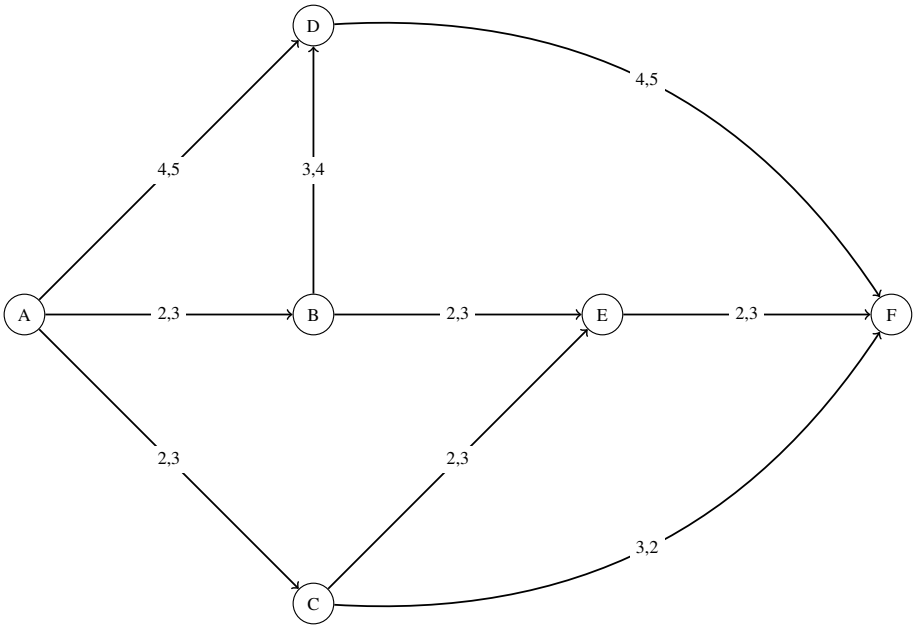

Fig. 3 The network for example 2 with $\tau_{i,j}, w_{i,j}$ on each edge.

Table 2 The values of $c_{i,j}(t)$ and $r_H(t)$ for example 2.

t	(A,B)	(A,C)	(A,D)	(B,D)	(B,E)	(C,E)	(C,F)	(D,F)	(E,F)	r_H
1	1	1	4	3	1	2	2	4	1	15
2	2	1	4	4	1	2	2	4	1	15
3	1	1	4	3	1	2	(2)	(4)	(1)	15
4	2	1	4	4	1	2	3	(4)	2	15
5	1	1	(4)	3	1	2	3	5	2	14
6	2	1	(4)	(4)	1	2	(3)	5	(2)	(14)
7	(1)	2	(4)	(3)	(1)	(2)	(3)	(5)	1	13
8	(2)	2	(4)	(4)	(1)	(2)	2	(5)	1	(13)
9	(1)	(2)		(3)	(1)	(2)	(2)		(1)	12
10	(2)	(2)			(1)	(2)			(1)	(12)
11										11
12										(11)

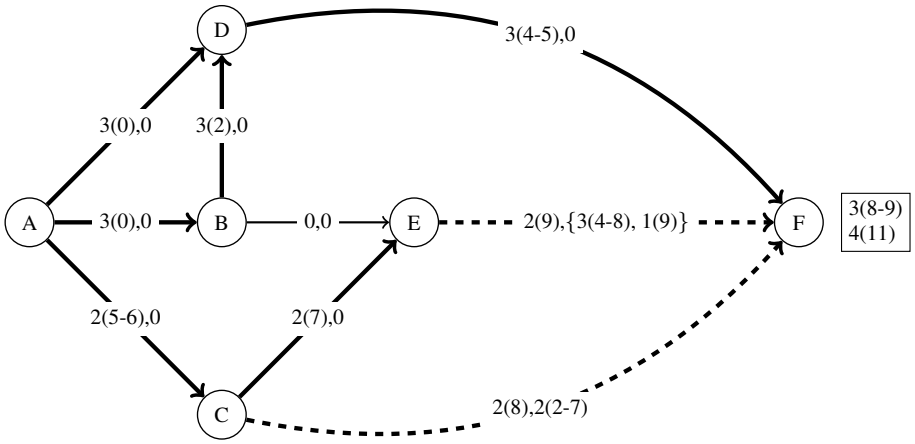

Fig. 4 The solution for the 12 step version of example two. Dashed edges indicate interruptions, while thicker errors indicate flow. Further more and edge with 2(3-5),4(1-2) has a flow of 2 starting on each of the time steps 3-5 and has a capacity reduction of 4 at each the beginning of each time step 1,2.

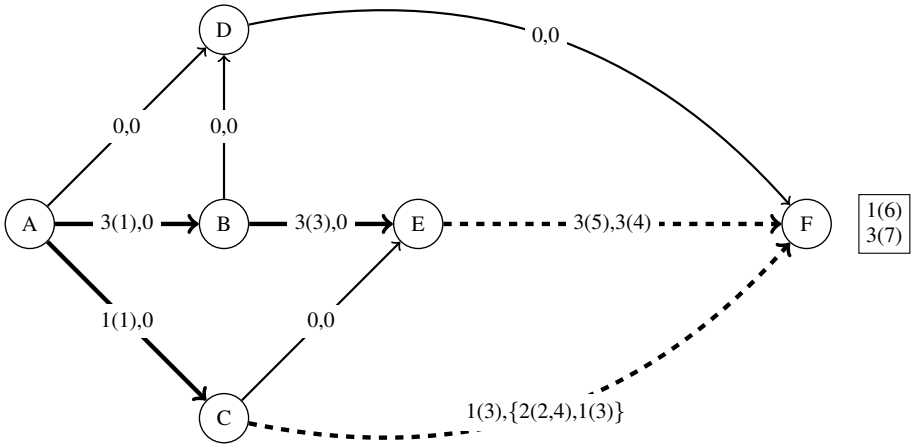

Fig. 5 The solution for the 8 step version of example two. Dashed edges indicate interruptions, while thicker errors indicate flow. Further more and edge with 2(3-5),4(1-2) has a flow of 2 starting on each of the time steps 3-5 and has a capacity reduction of 4 at each the beginning of each time step 1,2.

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5 Conclusion

In this paper we have extended our previous work on network interdiction by making the underlying network dynamic. In order to be able to transfer the approach, the network is being rolled out in time first, to obtain a static network again. The experiments not only show, that this approach works, but also demonstrate that this procedure incurs a rather high computational cost.

Future work should focus on finding a more efficient way to find the solution in problems with larger time horizons to make this approach more relevant to practical applications. This could go several ways. If the considered time frame is for example much longer than the longest time it takes from any source to any sink, then the problem could be decomposed to a set of potentially overlapping episodes of shorter length. Another option would be to try to exploit this kind of symmetry by decomposition methods. Finally, it can be considered to try a completely different approach like dynamic programming or some form of hybrid method.

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Evolving Adaptive Evolutionary Algorithms

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Abstract This paper presents a Grammatical Evolution framework for the automatic design of Adaptive Evolutionary Algorithms. The grammar adopted by this framework can generate a novel adaptive parameter control strategy, aiming to evolve the design of evolutionary algorithms. The Travelling Salesman Problem is used to investigate the potential of the proposed framework to evolve the adaptive evolutionary algorithms. Results show that the proposed framework is capable of not only generating new adaptive evolutionary algorithms but also confirms that automating the design of adaptive evolutionary algorithm can outperform the standard evolutionary algorithm.

1 Introduction

Evolutionary Algorithms (EAs), as there are genetic algorithms, genetic programming and evolution strategy based methods, are general population-based metaheuristics inspired by biological evolution and natural selection [1]. When applying EAs to optimization problems many different parameter configurations have to be set to achieve optimal performance. The choice of different genetic operators and their relative rates is usually based on experience. It has been argued that different parameter values may be optimal at different optimization stages [2-4], which makes the current research more focused on tackling the issue by using adaptive parameter control. Adaptive Parameter Control (APC) is used to tune the parameters in an online manner and during the algorithm execution by considering feedback from an EA run - such as solution quality- to monitor the algorithm performance and adjust the parameter values for future iterations.

Recently, EA parameter control has been widely studied, and many of APC methods have been proposed in the literature [5, 6]. The variation in these methods (regarding its performance and structure) makes the choice of suitable APC methods for a specific problem or instance non-trivial, as there exists no general optimal APC method for EAs. As a consequence, for EAs, the

optimal APC method can considerably vary depending on the problem or instance at hand. Some of the previous work ([7] and [8]) tried to combine different APC methods to improve EA performance. Several alternative combinations of APC methods remain unexplored. In the context of EAs, reference [6] has presented a vital survey of parameter control methods. The authors concluded that more research on the combination of different control mechanisms could be performed and that developing a generic framework for APC could be helpful.

Reference [5] proposes a generic model of the state-of-the-art APC methods based on a comprehensive review of the literature. The authors distinguish between the optimization process and the control process of the **Ad**aptive **E**volutionary **A**lgorithms (AdEA). The control process is further divided into four components, namely, feedback collection, effect assessment, quality attribution, and selection (see section II-A). Each component represents a stage of the parameter control accompanying with several possible adaptive strategies for each stage. Considering this model, the architecture of every AdEA can be mapped into the four components. Thus, a complete design on any AdEA should have at least one strategy for each component, and each strategy can be implemented by using one of many predefined rules or methods. In fact, we can use the model to facilitate the design of any new AdEA, but it is still time-consuming to perform this task manually. Several non-trivial selections or combinations of the most suitable variants of strategies (and so its rules) are needed to optimally specify each component with respect to the problem at hand.

In recent years, several automatic designs of the algorithm were presented to overcome this limitation. Reported in literature, a range of approaches are used to automate the algorithm design. An example of automatic design, Grammatical Evolution (GE) has been used to evolve an algorithm such as presented by [9] for evolving data mining algorithms, and [10] used GE for evolving new local search algorithms for the bin-baking problem. More recently, GE has been used to evolve the design of EAs for solving Royal Road Functions [11], Integration and Test Order problems [12].

In this paper, we propose a Grammatical Evolution framework to evolve the design of AdEA. GE automates the design of AdEA by defining a grammar guiding the selection of an APC strategy for different EA parameters and defining the corresponding implementations. The main aim of the automatic design of AdEA is to surrogate the human design leading to significant performance improvement. We introduced a set of experiments to examine the evolved AdEAs for solving the Traveling Salesman Problem (TSP).

The structure of the paper is as follows: section 2 presents related work of AdEA and the automatic design of algorithms. In section 3 we introduce the **G**rammatical **E**volution framework for **Ad**aptive **E**volutionary **A**lgorithms (GE-AdEA) including APC components description and the grammar definition. Section 4 presents the experimental results and analyses, and finally, the conclusion and future work are presented in section 5.

2 Background

2.1 Adaptive Parameter Control Design Model

The problem of finding optimal parameter configuration for an algorithm is a nontrivial optimization problem. Currently, the study of techniques for automatic parameter tuning is an active research area [5, 6, 13, 14]. One distinguishes parameter tuning and parameter control [14]. The former assigns parameter values before algorithm execution, while the latter decides on optimal parameter values during algorithm execution. According to [14], parameter control can be further classified into deterministic, adaptive and self-adaptive. Deterministic parameter control means the parameter value is assigned based on deterministic rules without further

knowledge about the search progress. Self-adaptive parameter control combines the search of optimal parameter values with the solution search, i.e., encodes the parameter values in the genome to enable them to co-evolve with the solutions. According to [5], APC separates the search for optimal parameter values from the solution search, monitors algorithm properties during the optimization process and adjusts the parameter values accordingly. The set of algorithm parameters that need to be tuned can be formulated as a $P = \{v_1, v_2, .., v_n\}$, where n denotes to the parameter numbers. Each v_i has a set of values $v_i = \{v_{i1}, v_{i2}, .., v_{im}\}$ that could be either a discrete number or intervals continues numbers. Note that m refers to the number of values associated with each parameter v_i. For example, assume that v_i represents a crossover parameter in EA, then the value v_{i1}could be the single point crossover and the value v_{i2} could be the uniform crossover. However, the parameter control aims to find the best next parameter value v_{ij} to optimize algorithm performance.

Over the past two decades, a wide variety of APC has been proposed with several adaptation strategies. A comprehensive study of the research direction of parameter control methods has been shown a significant number of research conducted in the field in the recent years [5, 6, 15, 16]. In the context of EA, more sophisticated literature reviews, such as [5, 17], have focused on studying the design structure of the existing APC methods and tried to decompose the APC process into several elements or components. The components that play a role to generalize the design strategies of the most existing APC methods. Corriveau et al. 2016 [17] divides the adaptive parameter control process in EA into four essential elements, namely, parameters involved (the type and states of the parameters involved), feedback indicators (used to evaluate the impact of the current state of the involved parameters), credit assignment schemes (used to convert feedback information into a suitable reward) and parameter selection rules (used to update parameter states).

Aleti et al. 2015 [5].proposed a more sophisticated and comprehensive conceptual model of APC which also consists of four main components. Fig. 1 illustrates the general algorithmic flow and components involved in the AdEA [5]. The algorithm starts with a population (initial solutions). This population is evolving during algorithm execution until the stopping criterion is reached. The EA parameters, such as population size, genetic operators and the probabilities of performing of the genetic operators, the number of offspring, etc., can be adjusted by the parameter control methods. At each cycle in the optimization process, the generated solutions are evaluated by using the fitness function(s), which provides valuable information about the algorithm performance as a feedback to guide the parameter control method. The feedback information can be used by effect assessment strategy to assess the cause of a change of the properties of the solutions during the run by measuring the effects of each parameter values on the algorithm performance. The vector of all parameter effects can be defined as $\vec{e} = \{e(v_{11}),\ e(v_{12}), \dots, e(v_{1n}), \dots e(v_{mn_n})\}$. The aim is to adapt the vector of probabilities $\vec{p}$ such that the expected value of the cumulative effect $E[\overrightarrow{e}] = \sum_{j=1}^{n} e(v_{ij})$ is maximized. The information is then used by quality attribution components to estimate the quality of each parameter value $q(v_{\mathrm{ij}})$. The vector of quality estimates for all parameter values is denoted as $\vec{q} = \{q(v_{11}), q(v_{12}), \dots, q(v_{1n}), \dots q(v_{mn_n})\}$. However, the selection comment will use $\vec{q}$ for selecting a good parameter value for future iterations.

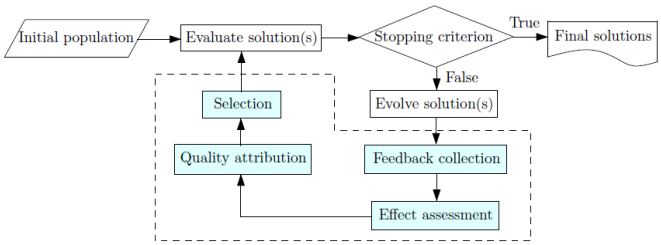

Fig 1 The main steps of optimization with Evolutionary Algorithms using adaptive parameter control [5]

To illustrate the AdEA model, consider the Integrated-Adaptive Genetic Algorithm (IAGA) [7] as an example. The authors adjust genetic operators and their rates. In the feedback collection strategy, they use phenotype feedback by considering the fitness of the solutions. Two different effect assessments have been applied, namely, best solution effect and ancestor solution effect. A reinforcement-based rate adaptation model is applied to measure the quality of the parameter values and can be categorized as learned quality attribution. Finally, the author used the proportional selection of reinforcement values of the operator rates to be used in next iterations. Considering several adaptive parameter control method for EAs in the literature [5].

We can see a degree of variation in AdEA design and a combination of some existing strategies is also possible. However, classifying the existing AdEAs based on decomposing them into several components and strategies can help to develop an optimized variant of AdEAs, mainly when an automatic algorithm designed approach is used; then the task can be much easier.

2.2 Related Work

Recently, several approaches for automating the metaheuristics design were developed, by both researchers and practitioners. Two main approaches for automatic algorithm design have been defined by [18], namely, top-down approach and bottom-up approach. The former one uses a parametrized algorithmic framework to generate an algorithm by starting with a general procedure and integrating different high-level algorithmic components [19]. On the other hand, the bottom-up approach is used by grammar-based genetic programming [9, 10, 18]. In this approach, the design of the algorithm is carried out by defining a context-free grammar, and the design space is represented by a set of production rules. The advantage of this approach is the ability to combine a valid algorithmic component in more fine-grained than top-down approach. Reference [20] used genetic programming to design a hybrid large neighborhood search algorithm for solving vehicle routing problems by applying crossover and mutation of a predefined set of terms derived from the grammar. References [9, 10, 18, 21] used a grammatical evolution [22], a variant kind of genetic programming, to generate a complete algorithm by composing a predefined set of algorithmic components. Grammatical Evolution (GE) [22] is a grammar-based genetic programming capable of generating a variable length code in any language by using a leaner genuine system representation. The GE uses a Backus Naur-form (BNF) specification language to represent a grammar which is used in genotype to phenotype mapping process to generate a complete program from the genotype binary string.

3 The Evolutionary Framework

As aforementioned, the EA parameter control has been studied intensively, and many parameter control approaches have also been proposed recently. The momentum of research in this field is still increasing because developing a new or applying an existing APC strategy for different optimization problems is always in demand [5, 6] as literature witnesses the success of AdEAs in many applications. In fact, developing a robust AdEA leads us to consider several issues such as the nature of a problem, the difficulty of target instances and the decision which APC strategy for a specific parameter in AdEA should be adopted, which is, of course, affecting the performance of the AdEA. All of these issues also make the design of AdEA a difficult task. The difficulty is because several APC strategies exist today and deciding the optimal among many alternatives, for a specific problem or even for a specific problem instance, is infeasible by considering the human design alone. Using an automatic algorithm design is an affordable alternative approach that not only helps to reduce the difficulty of selecting the best among several algorithm components but also can help to generate a novel algorithmic design which is in many cases superior to the standard algorithms.

Recent research work in the literature [11, 12, 23] used the GE framework to evolve the design of EAs. These focus on automating the design and the tuning of EA parameters utilizing from the GE grammar to generate EAs with different architectures. In this work, the idea of using GE to evolve AdEA has been inspired by work presented in [11, 12, 23]. Their results confirm that GE has a capacity of generating a novel design of EAs with better performance comparing to human design (standard EA). Therefore, the grammatical evolution framework GE-AdEA is used to evolve AdEAs by adopting different APC strategies. The framework is composed of two main components: GE Optimizer (GEO) and AdEA Executor (AdEA-EX). The task of GE-O is to evolve individuals that encode effective APC strategies for AdEA, whereas AdEA-EX responsible for executing many possible AdEA architectures based on the newly generated APC strategies (GE individuals) from GEO. The component is necessary for evaluating the generated solutions by assigning fitness to each individual by training it on TSP problem instances.

The GE-O adopts the standard architecture of GE, which consists of a search engine, a mapper function, and a BNF grammar. GE uses a standard genetic algorithm as its *search* engine [22]. The chromosome (genotype) is represented by a variable length binary string, The gene in each chromosome (so-called codon) has 8-bit binary values which are later decoded into an integer (in the range between 0 to 2^8-1). Repeatedly, the integer values produced by a chromosome can be used to convert all terminal to non-terminal symbols via a mapper function. The fitness of each chromosome is then evaluated by executing its corresponding program. The *mapper function* converts the genotype to phenotype by taking a binary string and BNF grammar as input and map it to the corresponding program.

The BNF grammar should be defined according to the four adaptive parameter control components, namely, Feedback Collection (FC), Effect Assessment (EFA), Quality Attribution (QA) and Selection (SE). The four components have been selected based on the conceptual model proposed by [5], and are reflecting the actual design of variant APC strategies of AdEA in the literature. Each of these components has its own set of adaptive parameter control strategies. These are explained as follows:

1. **Feedback collection (FC)**: the feedback collection provides valuable information about the algorithm performance as feedback to guide the parameter control method. This kind of information can be processed in the feedback collection strategy, which can measure and record a specific property of the algorithm performance during the execution, such as phenotype/genotype quality and phenotype/genotype diversity. We have employed several feedback collections methods. The feedback collection that is

used in our framework has been widely used in the literature [5] and is presented in Table 1. Note that in the proposed framework, we use *Phenotype* and *Genotype* feedback strategies to provide information for adaptive crossover and mutation operators effect assessment, while the *Phenotype* and *Genotype* diversity feedback strategies are used to control crossover and mutation rates directly.

Table 1 The feedback collection strategies description used in the proposed framework

Feedback Strategy	**Description**
Phenotype	Refers to the fitness function of solutions. The effect assessment will rely on fitness value differences among solutions, i.e., best, worst average fitness
Genotype	Provides feedback information about the solution components (genome information) of a given solution. We used a Hamming distance to measure the distance between two solutions as follows: $$d_{ij} = \sum_{k=1}^{n} \lvert i_k - j_k \rvert$$
Phenotype diversity	To measure the diversity in the population The population diversity measure by [24]: $$curr\_val = \frac{f_{best} - f_{avg}}{f_{best} - f_{worst}},$$ $$prev\_val = \begin{cases} curr\_val, if curr\_val > prev\_val \\ unchanche \ , \quad else \end{cases}$$ $$w = \left(\frac{curr\_val}{prev\_val}\right)^2,$$ Where $curr\_val$ and $prev\_val$ represent the fitness of the current value and previous value of the best solution, respectively. If the value of w reaches 1.0 that means the solution population is highly diverse.
Genotype diversity	To measure the diversity in the solution components over a set of solutions. The population diversity measured by [26]: $$d_{iX} = \sqrt{\sum_{k=1}^{n} (\lvert i_k - b_k \rvert)^2}$$ $$avg(d_{iX}) = \frac{1}{N_X} \sum d_{iX}$$ N_X is the number of operator applications of type x. i is the current solution and, b is the best solution generated by operator x

2. **Effect Assessment (EFA)**: The parameter control utilizes the effect assessment information to determine which parameter values will potentially perform well in future iterations by using a specific rule. The main difference between the existing effect assessment methods, however, is how they evaluate the success of parameter values. For instance, the effect assessment method presented in [25] calculates the effect of the parameters using quality differences between the generated solutions compared to their parents, while [26] used the overall best solution and [27] used median. A brief description of each APC component and its related strategies, adopted from [5], is presented in Table 2

Table 2 The effect assessment strategies description used in the proposed framework

Effect Assessment Strategy	**Description**
Ancestor	The effect of operator X_i calculated as : $e_{iX}^G = \begin{cases} f_{Parent} - f_{c_j} & , if\ f_{Parent} \geq f_{c_j} \\ 0, & else \end{cases}$ Where f_{Parent} denotes to the parent fitness, f_{c_j} fitness of the child c_j that is generated using operator X_i on generation G.
Ancestor Best	The effect of operator X_i calculated as : $S_{iX}^G = \begin{cases} \frac{f_{Parent} - f_{c_j}}{f_{Parent} - f_{best}}, if\ f_{Parent} \geq f_{c_j} \\ 0, & else \end{cases}$ Where f_{best} is the best parent fitness used by operator X_i, f_{Parent} is the best parent fitness, f_{c_j} is the fitness of the child c_j that is generated using operator X_i on generation G.
Ancestor Median	The effect of operator X_i calculated as : $e_{iX}^G = \begin{cases} f_{median} - f_{ci} & , if\ f_{median} \geq f_{ci} \\ 0, & else \end{cases}$ Where f_{median} is the median parent fitness used by operator X_i, f_{c_j} is the fitness of the child c_j that generated using operator X_i on generation G
Ancestor Worst	The effect of operator X_i calculated as : $e_{iX}^G = \begin{cases} f_{cj} - f_{worst} & , if\ f_{worst} \leq f_{cj} \\ 0, & else \end{cases}$ Where f_{worst} is the worst parent fitness used by operator X_i, f_{c_j} is the fitness of the child c_j that generated using operator X_i on generation G
Current	The effect of operator X_i is calculated by using the current fitness of the children as: $e_{iX}^G = f_{cj}$ Where f_{c_j} is the fitness of the child c_j that is generated using operator X_i on generation G.
Current Best	The effect of operator X_i calculated as : $e_{iX}^G = \begin{cases} f_{best} - f_{ci} & , if\ f_{best} \geq f_{ci} \\ 0, & else \end{cases}$ Where f_{best} is the best child fitness generated by operator X_i, f_{c_j} is the fitness of the child c_j that generated using operator X_i on generation G
Current Median	The effect of operator X_i calculated as: $e_{iX}^G = \begin{cases} f_{median} - f_{ci} & , if\ f_{median} \geq f_{ci} \\ 0, & else \end{cases}$ Where f_{median} is the median child fitness generated by operator X_i, f_{c_j} is the fitness of the child c_j that generated using operator X_i on generation G
Current Worst	The effect of operator X_i calculated as : $e_{iX}^G = \begin{cases} f_{ci} - f_{worst} & , if\ f_{worsti} \leq f_{ci} \\ 0, & else \end{cases}$ Where f_{worst} is the worst child fitness used by operator X_i, f_{c_j} is the fitness of the child c_j that generated using operator X_i on generation G
The average effect of operator $X_i$calculated as: $e_X^G = \frac{\sum e_{iX}^G}{N_x}$	

3. **Quality Attribution (QA)**: this component is built from the rules used in the effect assessment strategy. The parameter quality attribution is useful to estimate the quality of parameter values as it can help to choose the next parameter values. Two quality attribution strategies adopted in our framework, namely immediate and learned quality attribution. In the immediate quality attribution, the quality of a parameter referred directly to the effect value of that parameter, as

$$q(v_i) = e(v_i) \quad (1)$$

However, in the learned quality attribution, the quality of a parameter can be measured using accumulative information, the information from what the algorithm has learned in the previous iteration. So the quality of any parameter can be calculated by computing the ratio of $e(v_{ij})$ with respect to the average effects of all parameters, using the following equation [28]:

$$q(v_i) = \frac{e(v_{ij})}{\sum_{j=1} e(v_{ij})} * P_{Operator} \quad (2)$$

4. **Selection (SE)**: the parameter selection strategy is used to determine which of the parameter values will be used for future iterations. In our framework, we used two different parameter selection strategies. The first is *quality proportionate* which uses parameter quality values to estimate the selection probability of each parameter value for next iterations. The second is *quality proportionate with a minimum probability* which differs from the prior one by assigning a minimum selection probability to each parameter value to avoid missing some parameter values with inferior quality.

$$q(v_i) = \ q(v_i) * (P_{Operator} - i * \delta) - \delta \quad (3)$$

3.1 Grammar Definition

The objective of the proposed framework is to generate an AdEA. Its grammar contains rules that define the adaptive parameter control strategies for EA parameters and EA parameter configuration for parameters that are not selected for adaptation; such grammar is presented in Fig. 1. Every item placed between "<" and ">" is a non-terminal. Items without "<" and ">" represents terminal nodes. Everything coming after "::=" represents an option. "|" presents alternative options that can be assigned to a specific rule. More formally, the grammar can be formulated as a tuple <T, N, S, P>, where T denotes the terminal set (in our case represents a set of values), N denotes the non-terminal set (a set of APC strategies and a set of EA parameters). S is the start symbol (in our case "<Start>"). Finally, P denotes the production rules that map the elements of N to T (in our case, the production rules that used to generate a variant kind of APC strategies).

The rules of the presented grammar consist of all APC components and parameters that can be used to construct a valid AdEA. Each terminal consists of a value of parameters or an implementation of an Adaptive strategy that can be assigned to a specific parameter. For example, "`<Crossover_opr>`" denotes the crossover operator parameter. All options after "::=" might be either a terminal (a value of the parameter) or non-terminal. "`OXCrossover`" or "`0.8`" could be an example for a terminal option which is denoting that the parameter should have a fixed value making the parameter turn to non-adaptive mode. In the other hand, "`<AdaptiveStrategy>`" shown an example of a non-terminal optinon that can be used to extract the adaptive parameter control strategy that makes crossover operators in adaptive mode.

```
<Start>:=<Crossover_opr>;<Crossover_rate>;<Mutation_opr>;<Mutation_rate>

<Crossover_opr>::= <Adaptive Strategy>| OXCrossover | PMXCrossover | CycleCrossover

<Crossover_rate>::= <Diversity control>| 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0

<Mutation_opr>::= <Adaptive Strategy>| Order2OptMutator | OrderSublistMutator | ShiftMutator

<Mutation_rate>::= < Diversity control >| 0.01 | 0.02 | 0.05 | 0.5 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5

<AdaptiveStrategy>::=<Feedback>| <Effects> | <Quality>|<Selection>

<Feedback>::= Phonotype | Genotype

<Effects>::= Ancestor Best | Ancestor Median| Ancestor Worst |

             Current| Current Best | Current Median | Current Worst

<Quality>::=learned Quality | Immediate Quality

<Selection>::=Proportionate | Proportionate_w_m_p

<Diversity control>::= null | Phonotype Diversity | Genotype Diversity
```

Fig 2 GE-AdEA grammar

4 Experiments

In this section, we study the behavior of evolving AdEAs to solve a Travelling Salesman Problem (TSP) using GE and a grammar defined earlier. We conducted a set of empirical evaluations using conventional EA called Simple Generational Elitist (SGE) [29], and the evolved AdEAs by GE-AdEA. Both SGE and AdEA have been implemented based on a classical genetic algorithm that uses a generational schema but ensures any time that the best individual passes to the next generation. The main difference between AdEA and SGE, as shown in Fig 3, is that AdEA uses adaptation process that implements the generated adaptive parameter control strategies.

SGE template	AdEA template

generate initial population;
evaluate individuals;
while termination condition not met **do**
select individuals
apply variation operators
Evaluate (offspring Population);
Replacement
end while
return best individual in the population
	generate initial population;
	evaluate individuals;
	while termination condition not met **do**
	select individuals
	apply variation operators
	Evaluate (offspring Population);
	Replacement
	Adaptation
	end while
	return best individual in the population

Fig 3 Basic template of SGE and AdEA

The evaluation is divided into two main phases 1) training phase and 2) testing phase. In the training phase, we execute GE-AdEA 10 times on *eil76* TSP training instances from the TSPLIB to learn the GE-AdEA to generate ten different AdEAs. The choice of the training instance was adopted from [21]. The authors used three instances (*eil76* with a uniform random distribution, *pr76* with a clustered distribution and *gr96* with a random distribution). However, *eil76* shows the best training instance for learning an ACO architecture because of the probability related to the spatial distribution which is a uniform random distribution. Furthermore, in the training phase and to make fair a comparison between the generated algorithms and SGE, we used GE with different grammars for automatically tuning the SGE parameters. In the testing phase, the performance of the algorithms, the SGE and the 10 evolved AdEAs, is evaluated by conducting several experiments using 10 standard benchmark TSP dataset ranged from 48 to 318 cities. The experiments measure the solution fitness, the fitness that is obtained from 30 runs for each data

set, with 10,000 iterations for each independent run. This number of iterations is required to reach the best fitness.

For all experiments (training and testing), the Adapted EA settings are: Population size *100*; Initial individuals generation: nearest neighborhood heuristic; Crossover operators: *Order Crossover (OX), Partially Matched Crossover (PMX)* and *Cycle Crossover (CX)* with initial rate *0.9*; Mutation operators: *Order 2-opt mutation*, *sub-list mutation* and *shift mutation* with initial rate *0.1*; Tournament selection with tournament size: 2. Similarly, the tuned SGE settings are Population size *100*; Initial individuals generation: nearest neighborhood heuristic; Crossover operators: *Order Crossover (OX)* with rate *0.6*; Mutation operators: *shift mutation* with rate *0.1*; Tournament selection with tournament size: 2. We used Java Class Library for Evolutionary Computation (JCLEC) [29] for implementing both SGE and AdEAs, and GEVA v2.0 [30] for GE.

4.1 Training

In this phase, GE-AdEA parameters were fixed to the values presented in Table 3 [11]. By using these parameters, the GE-AdEA executed ten times on training instances resulting in 10 different AdEAs (here named Alg1 to Alg10). Each algorithm was trained with 1000 iteration budget, and the fitness value provided as feedback to the GE-AdEA corresponds to the best solution so far generated by each AdEA. The limited number of runs was adopted because evaluating an EA is a computationally intensive task. However, the best and the worst performed algorithms (as shown in the next section) are the Alg_2 and Alg_9, respectively. Their APC strategy and parameter settings are presented in Table 5. Due to the limited space available, Table 4 highlights the frequency of appearance of APC strategies in the evolved AdEAs (values are in percentage).

Table 3 Parameters of GE-AdEA

Parameters	**Value**
Population Size	100
Number of GE generations	50
One Point Crossover Probability	0.9
BitFlip Mutation	0.01
Selection Operator	Tournament with size equal 3
Replacement	Steady State
Number of Wraps	3
Number of Runs	10

Table 4 the frequency of evolved adaptive strategies generated from 10 GE-AdEA runs

APC Components	**Strategies**	**Freq.**
Feedback	Phonotype	35 %
	Genotype	25 %
Effect Assessment	Ancestor	5 %
	Ancestor Median	5 %
	Ancestor Worst	10 %
	Current	20 %
	Current Best	15 %
	Current Median	5 %
Quality Attribution	Learned	50 %
Selection	Quality_proportionate with_min_probability	35 %
	Quality_proportionate	15 %
Diversity Control	Phenotype diversity	35 %
	Genotype diversity	1 %

Table 5 Parameters control strategy of the best and worst Evolved algorithm

Parameter	**Best$_{alg_2}$**	**Worst$_{Alg_9}$**

Crossover operator	Adaptive{
	Feedback collection :Genotype;
	Effect Asses. : Current Midian;
	Quality attribution: Learned quality;
	Selection Proportionate with min. prob.;
	}
		Non adaptive {value: OX Crossover}

Crossover rate	Non adaptive {value: 0.4}	Non adaptive {value: 0.9}

Mutation Operator	Adaptive{
	Feedback collection :Phenotype;
	Effect Asses. : Ancestor;
	Quality attribution: Learned quality;
	Selection Proportionate with min. prob.;
	}
		Non adaptive {value: 2opt mutation}

Mutation rate	Non adaptive {value: 0.3}	**Non adaptive {value: 0.09}**

4.2 Testing

To validate the evolved AdEAs in an optimization scenario, this section provides the comparison results between tuned SGE and the evolved AdEAs. The testing is essential to measure the effectiveness of the evolved algorithms in different TSP instances. Ten TSP instances were selected to assess the performance, namely, *att48*, *eil51*, *berlin52*, *kroA100*, *lin105*, *gr137*, *u159*, *d198*, *pr226* and *lin318*.

Table 6 presents the experimental results of the tuned SGE and the evolved algorithms on 10 TSP instances. The result values are expressed by the percentage deviation of the best solutions generated by 10,000 iterations of all algorithms compared to the best-known solutions of the instances. It can be seen that the results of the evolved algorithm are generally better than SGE in most instances. From the table, we can also see an expectable degree of variation in the performance of the evolved algorithm which is confirmed that the GE-AdEA can generate AdEAs with different adaptive control strategies as well as with different optimization performance. Looking at the algorithms ranking presented in Table 6 and calculated by using the ranking method [31] based on the mean of the best fitness for each algorithm, we can see that 2 out of 10 evolved AdEAs had a better performance when they compared with SGE. However, the results obtained reveal that GE-AdEA can generate AdEAs with a novel adaptation strategy and 20% of them have a superior performance comparing to SGE.

To see if there is any statistical difference between the evolved AdEAs and SGE, we used the results of the mean fitness values obtained from the experiments in the testing phase and examines the statistical difference using the pair-wise t-Test with a significance level α=0.05. Table 7 demonstrates the significance of the statistical results of all datasets. A "++" symbol denotes to the significant differences in the mean values with superior performance comparing to SGE, "+" denotes to the significant differences in the mean values but with worst performance comparing to SGE and finally "~" denotes to no significant difference in the mean. Table 7 statistical results show that both *Alg_1* and A*lg_2* have a significant difference with superior performance in the majority of datasets. The analysis also reveals that three of evolved AdEAs namely *Alg_3*, *Alg_4*, and *Alg_6*, have a comparable or equal performance with SGE. However, the rest, which is five evolved AdEAs, have worst performance.

Table 6 Optimization results of evolved AdEA, with 10000 iterations for 30runs

Problem		SGE	Alg_1	Alg_2	Alg_3	Alg_4	Alg_5	Alg_6	Alg_7	Alg_8	Alg_9	Alg_10
att48	B	1.24	0.88	3.10	3.10	1.60	1.11	0.88	0.88	**0.52**	1.47	2.16
	M	4.76	3.07	3.06	5.14	4.51	3.07	3.05	3.05	**3.00**	3.60	3.10
eil51	B	2.58	2.58	3.05	3.05	3.05	2.58	2.35	3.05	**1.41**	3.05	2.58
	M	4.15	4.08	4.23	5.69	4.37	4.08	**3.85**	4.16	4.12	4.81	4.64
berlin52	B	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.30	0.00
	M	2.71	3.33	3.47	4.76	4.16	3.33	2.88	2.25	**1.56**	5.08	3.38
kroA100	B	1.41	**1.37**	1.41	1.41	2.92	1.41	1.41	1.41	1.41	2.18	1.41
	M	4.87	3.70	**3.49**	6.16	4.22	3.70	4.06	4.29	4.62	4.44	4.14
lin105	B	**1.48**	2.60	1.72	3.89	1.97	1.57	2.37	1.89	**1.37**	5.08	1.72
	M	5.30	5.21	**4.48**	7.83	5.90	5.21	5.51	5.40	4.73	6.95	4.89
gr137	B	**7.78**	10.13	8.91	11.99	10.41	10.96	8.47	11.10	9.76	12.03	10.29
	M	**11.87**	13.99	12.57	15.25	12.28	13.99	13.29	15.01	15.43	16.88	16.54
u159	B	10.05	10.30	9.83	10.20	9.83	**8.87**	10.08	9.88	10.49	11.30	10.64
	M	11.10	10.86	**10.50**	11.73	10.99	10.86	11.32	11.55	11.84	12.64	11.86
d198	B	5.71	7.31	5.14	**5.01**	5.28	5.64	5.90	7.00	6.45	6.39	6.91
	M	7.41	7.28	6.50	7.15	**6.17**	7.28	7.51	8.70	9.19	9.47	8.38
pr226	B	4.01	6.12	**3.45**	5.46	3.57	4.18	4.05	7.08	7.16	7.45	6.35
	M	6.07	7.31	**5.54**	7.88	5.86	7.31	7.38	9.26	9.31	8.90	10.13
lin318	B	11.67	10.94	10.85	9.89	11.09	12.08	12.52	12.37	13.08	12.72	11.67
	M	11.04	11.67	10.94	10.85	**9.89**	11.09	12.08	12.52	12.37	13.08	12.72
Algorithm ranking	B	3.6	6.2	**3.0**	6.4	6.0	4.9	5.0	7.4	5.6	9.9	8.0
	M	4.6	3.9	**3.3**	8.4	5.0	4.9	4.9	6.4	6.1	10.1	8.3

Table 7 Results of the pair-wise t-Test at a significance level $\alpha = 0.05$

Problem	SGE-Alg_1	SGE-Alg_2	SGE-Alg_3	SGE-Alg_4	SGE-Alg_5	SGE-Alg_6	SGE-Alg_7	SGE-Alg_8	SGE-Alg_9	SGE-Alg_10
att48	++	++	~	++	++	++	~	++	++	++
eil51	~	+	+	+	++	++	+	~	+	+
berlin52	~	+	+	+	+	~	~	~	+	+
kroA100	~	++	~	++	++	~	++	~	~	~
lin105	++	++	+	+	++	~	+	++	+	++
gr137	+	+	+	+	+	+	+	+	+	+
u159	++	++	~	++	++	~	++	+	+	+
d198	++	++	++	~	++	~	+	+	+	+
pr226	+	**++**	+	++	+	+	+	+	+	+
lin318	++	++	+	~	+	+	+	+	+	+

The structure analysis of the best and worst ranked algorithms (See Table 5) reveals that the best algorithm *Alg*_2 used an adaptive strategy for crossover and mutation operators. By this, the algorithm can select different operator types, i.e., *OX*, *BMX* or *CX* operators for crossover and *2opt*, *sub-list* or *shift* operators for mutation, in each iteration relying on their quality values. However, as shown in Table 5, the difference of structure and parameter setting of the best and worst algorithm is clear regarding their parameter adaptability, we can see that the *Alg_2* structure uses two different parameter control strategies. It applies the genotype as a feedback collection, and the current median as an effect assessment for crossover operator, and phenotype as a feedback and ancestor effect as an assessment for mutation operator. For the quality attribution and selection strategies, the crossover and mutilation operators used the learned quality attribution and propionate with minimum probability selection, respectively. Alg_9 has never used any adaptive parameter control strategy for its parameters. Finally, we believe that GE-AdEA can produce an AdEA with a comparable optimization performance.

4 Conclusion

In this paper, Grammatical Evolution framework for Adaptive Evolutionary Algorithms (GE-AdEA) has been proposed. GE-AdEA uses a grammar with several Adaptive parameter control strategies adopted from an adaptive parameter control model and parameter settings to generate AdEAs. During the evolution, the GE-AdEA executes the generated AdEAs as a training process and produces the best-trained AdEAs based on their fitness's. The best-trained algorithm can be used later to solve the problem. In the training phase, GE-AdEA shows its capacity to generate not only good performance AdEAs but also novel designs of AdEA architectures.

The results of the experiments using several traveling salesman problem instances confirmed that GE-AdEA has a potential to improve the adaptive parameter control strategy for crossover and mutation operators and their rates. The results also revealed that some of the generated AdEAs outperform a tuned SGE. Finally, the work presented in this paper demonstrates that automatic evolution of AdEA is feasible. To test the framework in its generality, in future work, we aim to enrich this approach with more sophisticated adaptive parameter control strategies, and to test it with different optimization problems and with several benchmark algorithms. Also, we will study the influence of using different training instances considering their characteristics (e.g., instance size, structure and degree of difficulty) on the performance of the evolved AdEAs rather than using one training instance; this will lead us to gain more perception on how to design the most effective training environment.

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Fixed jobs scheduling on a single machine with renewable resources

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Abstract This paper deals with scheduling n jobs on a single machine. Job J_i is defined by: a fixed start time s_i, a fixed finish time f_i, an amount of resource requirements $q_{i,j}$ of type $j = 1 \dots k$. The jobs are independent and should be processed without preemption during their time interval $[s_i, f_i]$ (processing time of job J_i is $p_i = f_i - s_i$). The single machine continuously available in $[0, \infty)$ owns Q_j units of renewable resource of type R_j necessary to carry out jobs. A machine can process more than one job at a time, provided the resource consumption does not exceed Q_j for all $j = 1 \dots k$. In this context, the objective is to find an optimal schedule minimizing the number of rejected jobs. We show that this problem is NP-hard. An Integer Linear Program (ILP) is proposed to solve optimally the studied scheduling problem. Three greedy heuristics are also developed. Proposed algorithms are implemented and experimental results are conducted on a set of randomly generated instances. The obtained solutions show the efficiency of the proposed resolution methods.

Keywords : Scheduling; Single machine; Fixed job scheduling; Resources allocation; Complexity; ILP; Greedy heuristics.

1 Introduction

In a classical scheduling problem, a set of independent jobs characterized by processing times should be scheduled on a machine while respecting the constraints to optimize a given criterion. The machine can process at most one job at a time (see Brucker et al.[5] and Blazewicz et al.[3]). In this study, a set of n independent jobs should be scheduled without preemption on a single machine. Additional renewable resources are however necessary to process each job. Several types of such resources are needed, denoted $R_j, j = 1 \dots k$. Hence, at execution time of job i, q_{ij} units of available resource are required. For each job i, the start time s_i and its finished time f_i $(i = 1, \dots, n)$ are fixed

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where its processing time $p_i = f_i - s_i$. Dealing with each type of resources, the machine can process more than one job at a time provided the resource consumption does not exceed a given value Q_j $(j = 1 \dots k)$. This machine is continuously available during time interval $[0, \infty)$. All data are assumed positive integers. The processing times of jobs is formatted in slotted windows. The total time period $[0, T]$ is partitioned into equal length slots (l_0) with $T = max_{i,i=1,\dots,n}(f_i)$. Without lost of generality we suppose that: $s_i < f_i$ and $q_{i,j} \leq Q_j$ for all $i = 1, \dots, n$ and $j = 1 \dots k$. Our objective is to minimize the number of rejected jobs or equivalently calculate the maximum number of jobs that can be scheduled.

Such problems corresponds to some real world situations as introduced in [1]. The authors consider a set of aircraft (jobs) to be parked in an airport for land side operations. The plane stays parked during a fixed interval of time, from the arrival of a flight to the departure of the next one carried by the same aircraft. The parking space layout or number of parking place (additional resource) is such that a same parking lot (machine) may be occupied by either one large aircraft, or more smaller ones. The problem under their study is to verify if it is possible to schedule all planned flights and, if not, which ones must be rejected. In Angelelli et al.[1], authors consider m parallel machines and only one additional resource. They are interested in at least three types of problems: Does a feasible schedule exist for all jobs? Which is a subset of jobs that can be scheduled with the maximum total value? What is the minimum number of machines required to schedule all jobs? For this identified strongly NP-hard problem, they propose a column generation scheme for the exact solution and develop some greedy heuristics.

In our paper, the addressed problem $\mathcal{ISSR}$ (fixed Interval Scheduling under Several Resources requirement) can be met in a data center where the objective is to optimize the use of resources and satisfy the users. Virtual Machines VMs (jobs) submitted by the users should be executed on the same cluster. For example, this cluster owns three limited types of renewable resources CPU, MEMORY and STORAGE with capacities equal to Q_1 CPU, a certain quantity of memory Q_2 and a certain storage capacity Q_3. In this case, to execute VM i, a number of virtual CPUs q_{i1}, virtual memory q_{i2} and hard drives q_{i3} are needed. Note that the feasibility problem has also been addressed in [1] where the authors consider only one additional resource (memory devices). Let's consider the following example.

Example: Eight jobs have to be scheduled on a single cluster with 1000 Ghz CPU, 1000 Go Memory and 1000 Go Storage. For each job i, the starting times, the finishing times and the quantities of each requirement resources $q_{ij;j=1,2,3}$ are given in Table 1.

Jobs	s_j	f_j	CPU	MEMORY	STORAGE
1	0	6	250	500	250
2	1	4	125	600	300
3	3	8	500	300	700
4	3	6	500	700	250
5	4	8	125	250	200
6	5	9	250	500	300
7	0	4	500	300	400
8	1	5	250	800	600

Table 1: Instance with 8 jobs and 3 resources.

For the above instance, the maximal number of jobs to schedule on a single machine is four. An optimal solution is given in Figure 1.

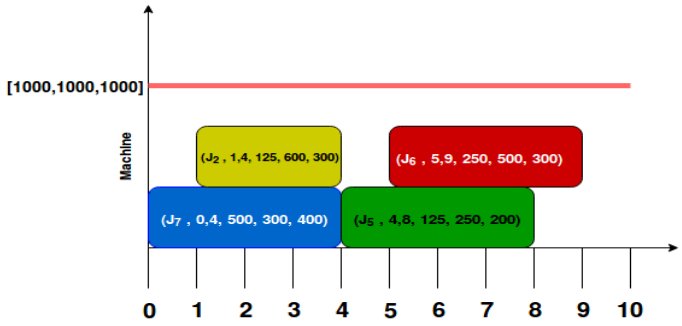

Fig. 1: Optimal Schedule of jobs.

According to Graham et al.[10], the studied problem is denoted by $1|s_i, f_i, q_{ij}|\sum_i RC_i$ where $\sum_i RC_i$ indicates the number of refused jobs that should be minimized.

Several variants of fixed interval scheduling problem have been addressed in dedicated literature. Almost all of them do not consider additional resources to process jobs. We can classify these published results into two categories:

1. **Interval scheduling problems where the number of machines is not fixed and all jobs must be scheduled**. In such problems, the objective is to find a minimum-cost schedule in which all jobs are scheduled. In this case, we can cite the works of Bhatia et al. where the authors proposed 2-approximation algorithm to minimize the number of used machines;
2. **Interval scheduling problems where the number of machines is fixed and some jobs can be rejected**. Maximize the total number of weighted jobs is equivalent to find a min-cost flow and can be polynomially solved (see Arkin et al. [2] and Bouzina et al. [4]). In case each job has unit weight, greedy algorithms maximizing the number of scheduled jobs are proposed (see Faigle et al. [8] and Carlisle et al. [7]). When an availability time interval is associated to each machine, Brucker and Nordmann [6] proposed an $O(n^{m-1})$ algorithm to maximize the number of scheduled jobs.

For more details on existing models and developed algorithms to solve fixed interval scheduling problem and its variants, one can refer to works of both Kovalyov et al. [12] and Kolen et al. [11].

The rest of this paper is organized as follow. In Section 2 we show that the problem $\mathcal{ISSR}$ is NP-hard. In Section 3, we propose an Integer linear programming formulation to solve optimally the studied scheduling problem. In Section 4, three greedy heuristics are proposed and described. All proposed algorithms are implemented and tested on a large set of randomly generated instances and experimental results are presented in Section 5.

2 NP-completeness results

In what follows, we show that the scheduling problem $1|s_i, f_i, q_{ij}|\sum_i RC_i$ with identical time intervals and is NP-hard.

Proposition the problem $1|s_i, f_i, q_{ij}|\sum_i RC_i$ is NP-hard.

Proof The proof is given by reduction from the Partition problem with Equal Size ($\mathcal{PES}$) which is known to be NP-complete [9]. We denote by $\mathcal{ISSRD}$ the decision problem associated to $1|s_i, f_i, q_{ij}|\sum_i RC_i$. $\mathcal{ISSRD}$ is defined by:

Data: A set $\mathcal{N}$ of n jobs, identical time intervals $[s_i, f_i] = [s, f], \forall i, 1 \leq i \leq n$ and an integer value Y.

Question: Is there a machine schedule σ for $\mathcal{N}$ such that $\sum_i RC_i \leq Y$? ($RC_i = 1$ if job J_i is rejected).

We prove that $\mathcal{PES} \propto \mathcal{ISSRD}$.

$\mathcal{PES}$ problem is defined by:

Data: A set $I = \{a_1, \ldots, a_i, \ldots, a_n, \ldots, a_{2n}\}$ of $2n$ integers and an integer value B such that $\sum_{i=1}^{2n} a_i = 2B$

Question: Does there exist a partition of I into two subsets I_1 and I_2 such that $\sum_{i\in I_1} a_i = \sum_{i\in I_2} a_i = B$ and $|I_1| = |I_2|$?

Given an arbitrary instance of $\mathcal{PES}$ problem, we construct an instance of $\mathcal{ISSRD}$ problem as presented in Table 2.

Jobs	s_i	f_i	q_{i1}	q_{i2}	q_{i3}
$j_i, i = 1, \ldots, 2n$	s	f	a_i	$B - a_i$	a_i

Table 2: An instance of $\mathcal{ISSRD}$ problem

Let $Y = n$ and the maximum capacity of the first, second and third resources are $Q_1 = B$, $Q_2 = (n-1)B$ and $Q_3 = B$ respectively.

In the following, we show that there exists a schedule of jobs on one machine, if, and only if, $\mathcal{PES}$ problem has a solution.

- Let us suppose that the answer to $\mathcal{PES}$ problem is 'yes'. Let I_1 and I_2 be the solution. Hence, by considering I_1 a subset of the corresponding jobs that are scheduled, we have $|I_1| = n \leq Y$. This solution is a feasible solution because the quantity of each consumed resource does not exceed the available threshold: $\sum_{i\in I_1} a_i = B = Q_1 = Q_3$ and $\sum_{i\in I_1}(B - a_i) = nB - \sum_{i\in I_1} a_i = (n-1)B = Q_2$. Thus, I_1 is the subset of scheduled jobs for which the answer to $\mathcal{ISSRD}$ problem is 'yes'.
- Now suppose that the answer to $\mathcal{ISSRD}$ problem is 'yes' for sequence σ. We have: $\sum_{i=1}^{n} RC_i \leq Y$. Let I_1 be a subset of scheduled jobs. Then $|I_1| \geq Y = n$ and $\sum_{i\in I_1} a_i \leq B$.
  Suppose that $|I_1| = k > n$. In this case the total quantity of consumed resource of R_2 is then: $kB - \sum_{i\in I_1} a_i > kB - B > (n-1)B$, which is a contradiction. Hence $|I_1| = n$.

Dealing with R_2, we have $\sum_{i\in I_1}(B-a_i) \leq (n-1)B$. Suppose that $\sum_{i\in I_1}(B-a_i) < (n-1)B$. In this case, we have $\sum_{i\in I_1} a_i > B$, which is in contradiction with the maximum available capacities of R_1 and R_3.
Hence, we have $\sum_{i\in I_1} a_i = B$ and $|I_1| = n$ and I_1 is also a solution for $\mathcal{PES}$ problem. Consequently, the answer for the question of the $\mathcal{PES}$ problem is 'yes'.

3 Integer linear programming formulation

We present in this section a time indexed integer linear programming formulation (ILP) for solving the scheduling problem. We define two types of binary decision variables.

1. x_i a binary variable equal to 1 if job J_i is rejected, 0 otherwise.
2. y_{it} a binary variable equal to 1 if job J_i is executed at time t, and 0 otherwise.

The general formulation of the time-indexed ILP is the following.

$$\text{Minimize:} \quad \sum_{i\in\mathcal{N}} x_i$$

$$\text{subject to:} \quad \sum_{t=s_i}^{f_i} y_{it} = (f_i - s_i) * (1 - x_i)\,; \forall i \in \mathcal{N} \qquad (1)$$

$$\sum_{i\in\mathcal{N}} y_{it} * q_{ij} \leq Q_j\,; \forall j \in \mathcal{R}\,; \forall t \in [0,T] \qquad (2)$$

$$x_i \in \{0,1\}, \quad y_{it} \in \{0,1\}\,, \forall i \in \mathcal{N}\,, \forall t \in [0,T].$$

$\mathcal{N}$ is the set of n jobs and $\mathcal{R}$ is the set of type of resource. $[0,T]$ $(T = max_{i,i=1,\ldots,n}(f_i))$ is the time period to schedule all jobs.

The constraints (1) ensure that if job J_i is not rejected then it is scheduled during its time interval. The constraints (2) ensure that no more than Q_j quantities of the required resources are consumed at time t.

4 Greedy heuristics

The scheduling algorithms in this study have many applications, but they were motivated by research into on line system and integrated-services networks. Hence, the resolution method must have low complexity, not just polynomial complexity, and to the extent possible, it should accommodate diverse performance objectives. Furthermore, in many important models that are relevant to the study of integrated-services networks and on line system, the number of jobs to be processed can be extremely large, so low complexity is essential. In this section, we presente low-complexity ($O(n^2)$ or better) greedy algorithm to solve fixed interval jobs scheduling problems. Roughly, this algorithm works as follow: at first, jobs are sorted according to a given priority rule Π. At each new event t (new job is available to be processed), we try to schedule this job with respect to constraints of resources availability at time t. This job is rejected if its assignement causes an unfeasible solution. This procedure is repeated until the last job. Algorithm 1 describes the proposed global greedy heuristic.

4.1 Priority rules

To calculate set L (see Algorithm 1), we describe here three priority rules that were implemented.

1. **Shortest Processing Time (SPT)**: Jobs are sorted in non-decreasing order of their processing times $p_i = f_i - s_i \, \forall i \in \mathcal{N}$, in case of ties, the job with the smallest finished time come first, otherwise lexicographical order is considered. This SPT rule allows resources to be released as soon as possible.
2. **Capacity-makespan ($CC_{\max}$)**: Jobs are sorted in non-decreasing order of their occupied space given by the following formula: $\sum_{j \in R} q_{ij} * (f_i - s_i) \, \forall i \in \mathcal{N}$, in case of ties, the job with the smallest finished time come first, otherwise lexicographical order is considered. The idea of using $CC_{\max}$ rule is to minimize the space occupied by jobs defined by processing time per quantities of consumed resources.
3. **Average Resources Consumed (ARC)**: Jobs are sorted in non-increasing order of resources requirement per unit time during $[s_i, f_i]$ given by the following formula: $\sum_{j \in R} q_{ij}/(f_i - s_i) \, \forall i \in \mathcal{N}$, in case of ties, the job with the smallest finished time come first, otherwise lexicographical order is considered. The idea of using ARC rule is to minimize the average resources consumed by job per unit of time.

Algorithm 1: Global greedy heuristic

- Let L be the set of jobs sorted according to rule Π
- Let S^* be the set of scheduled jobs; Initially, $S^* = \emptyset$;
while $L \neq \emptyset$ **do**
  Let J_k be the next available job
  Let $S \subseteq S^*$ be the subset of overlapping jobs during $[s_k, f_k]$ (i.e their processing intervals have a nonempty intersection)
  if $(\sum_{i \in S} q_{ij} + q_{kj} \leq Q_j \, ; \forall j \in \mathcal{R} \,)$ **then**
    Schedule k
    $S^* = S^* \cup \{k\}$
  end
  1 $L = L \backslash \{k\}$ (J_k is rejected)
end

5 Computational experiments

In this section we present the computational expriments we made in order to analyze the performance of both exact method ILP and proposed heuristics. All heuristics have been implemented in C++. IBM ILOG CPLEX Optimization Studio V12.6.3 is used to solve the ILP formulation. All experiments are conducted on a Pentium i7 computer with 2.80GHz x 8 cores (1 CPU) and 8 GB memory.

The performance evaluation of the algorithms under study has been carried out with 100 instances, with a number of jobs $n \in \{20, 40, 60, \ldots, 200\}$ (10 instances are generated per n). Without lost of generality, we normalize the units of a renewable resource to 1000. Hence, $Q_1 = Q_2 = Q_3 = 1000$ and for each job q_{ij} ($i = 1, \ldots, n$ and $j = 1, 2, 3$) have been generated using a discrete uniform distribution between 1 and 1000. The jobs-starting times s_i have been generated using a discrete uniform distribution between 0 and 1440 mn (one day). Then, the jobs-finishing times have been generated using a discrete uniform distribution between $s_i + 1$ and $1440 - s_i$.

In what follows, experimental results are summarized in Table 3, Figure 2 and Figure 3.

The first result concerns the performance of the exact method ILP. Out of 100 instances, Cplex solves instantaneously all instances (less than 1 second is needed for each instance). We also computed the average computation time in minutes required to obtain an optimal solution for huge size instances. For example, for instances with 1500 jobs the average computation time needed by ILP is 3 minutes, and at maximum 7 minutes for some instances.

In Table 3, the performances of heuristics SPT, CC_{max} and ARC are presented. From this table, the first column gives the size of instances (number of jobs). The second, third and fourth columns indicate the number of optimal solved instances (in percentage), where each line corresponds to 10 randomly generated instances. For example, line with 40 jobs, over 10 instances 70% of them are optimally solved by heuristic CC_{max} where SPT (respectively ARC) finds an optimal solution 6/10 (3/10 respectively) times. Note that out of the 100 instances, SPT, CC_{max} and ARC solve optimally 24, 47 and 12 instances, respectively. ARC is the heuristic that has most difficulty to optimally solve instances of size more than 40 jobs.

n	SPT	CC_{max}	ARC
20	90%	90%	90%
40	60%	70%	30%
60	30%	50%	0%
80	30%	60%	0%
100	10%	40%	0%
120	10%	50%	0%
140	0%	40%	0%
160	0%	30%	0%
180	0%	20%	0%
200	10%	20%	0%

Table 3: Rate of instances giving the optimal solution.

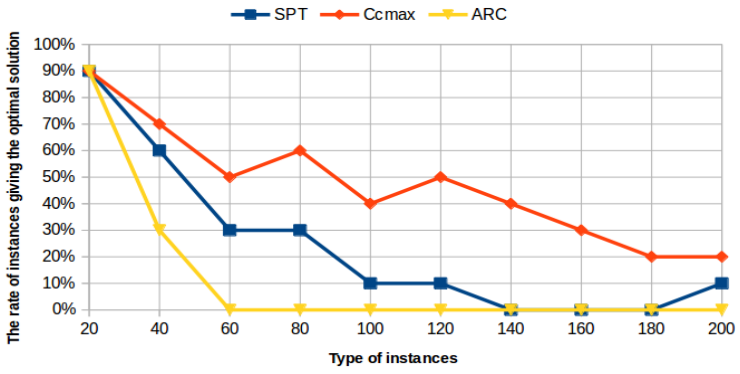

Fig. 2: Comparisons when varying number of jobs.

In Figure 2, we analyze the behavior of the 3 heuristics according to the size of the instances to be solved. We note that when the number of jobs increases, the performance of the heuristics decreases. For instances with 140 jobs, the heuristic CC_{max} presents a rate of 40% (4 instances resolved at the optimum) against 0% for SPT and ARC.

In figure 3 where gaps are averaged with respect to the number of jobs, we observe that SPT and ARC are dominated by CC_{max}, however CC_{max} obtains a clear advantage from the competition. In fact, out of 100 instances, the average gaps compared to optimal solutions are: $CCmax$ 4,41%, SPT 9,40% and ARC 14,96%

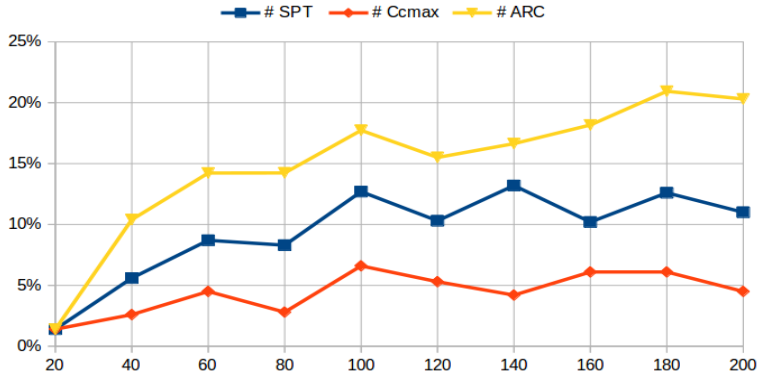

Fig. 3: Gap averaged on number of jobs

6 Conclusion

In this article, we study a new single machine interval scheduling problem. The machine owns k limited types of renewable resources. This problem is motivated by a real application that consists in allocating of a set of virtual machines (VMs) to a cloud computing cluster, for example. The objective is to minimize the number of rejected jobs. We prove that this problem is NP-hard.

However, an efficient exact method (ILP) is proposed. We then propose faster greedy heuristics. All the algorithms have been tested on a large set of randomly generated instances.

This study is still in progress. At first, would there be a pseudo polynomial time dynamic programming algorithm? This question is still open. It will be also interesting to determine other priority rules to improve global greedy heuristic and to consider the weighted jobs scheduling problem. The next step of this study will deal with the case of m parallel machines.

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Improving Ant Colony Optimization algorithm with Levy Flight

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Abstract Ant Colony Optimization (ACO) algorithm is a metaheuristic evolved from the foraging behavior of ants, and the swarm intelligence is realized by the behavior of a single ant. ACO algorithm is widely used to solve combinatorial optimization problems. At the same time, the quality of an ACO algorithm depends on the diversity of solutions and the times spent by the algorithm. Some commonly used ACO algorithms such as the Elite ACO and the Rank-based ACO attempt to reduce the computational time of the algorithm by exploring near the current best solution. Other algorithms such as the Max-Min ACO try to explore the diverse solution spaces by limiting the maximal and minimal pheromone. Levy Flight is a type of random walking behavior based on Levy distribution, which is also widely observed in nature like animal feeding routes, circulation logistics (e.g. banknotes). This article describes the ACO algorithm combining with the Levy Flight mechanism, which can balance the convergence rate and the diversity of solutions. The experiment demonstrates that the proposed Levy ACO algorithm obtains better results than the classical ACO algorithm.

1 Introduction

ACO (Ant Colony Optimization) algorithm is a metaheuristic based on ants foraging behavior, and it relies on the foraging behavior of single ant to embody the foraging intelligence of ant colony. ACO algorithm was first proposed in Dorige's doctoral dissertation in 1992 [1] and more details provided later in 1996[2]. The early version of ACO algorithm was applied to the TSP (Travelling Salesman Problem) problems, and Dorige [2] described how to use ACO algorithm for TSP modeling and the experiments for validations. In his latest paper [3], Dorige surveyed ACO technologies.

Since the birth of the ACO algorithm, there were many researchers carried out in-depth studies, and proposed numerous improved versions, for instance, the Elite Ant Colony algorithm [3], the Rank-based Ant Colony algorithm [4], the Max-Min Ant Colony Optimization algorithm [5]. The core in terms of the efficiency of an ACO algorithm is to balance the diversity of

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exploration (Exploration) and the concentration of exploration (Exploitation). Exploration is to explore new solution spaces as much as possible while exploitation is to search the areas near the current best solution as fast as possible, In the above mentioned algorithms, the Elite Ant Colony algorithm [3] and the Rank-based Ant Colony algorithm [4] focus on the exploitation of current best solution, while the Max-Min ACO algorithm [5] balances the exploration and the exploitation by setting the maximum and minimum pheromone and gets better performance. Furthermore, ACO algorithm is not only applied for solving TSP [6-11,29] but also other optimization problems such as VRP (Vehicle Routing Problem) [12-17], QAP (Quadratic Assignment Problem) [18,19], JSP (Job-shop Problem) [20-22].

The random mechanism is embedded in ACO algorithms, where the probability distribution plays a great role. Some common distributions of continuous distributions include uniform, normal, exponential ones, etc. One class of continuous distribution we are interested has the property called *fat-tail*, which means the tail is thicker than others such as the normal and the exponential ones. Its randomness is weaker, so that a tail value that is rarely selected in normal situation could be chosen with a higher probability. From our aspect (will be discussed below), it increases the diversity of solutions that would contribute to better solutions. Specifically Levy distribution is a kind of typical fat-tailed distribution.。

Levy Flight [23] is a type of random walking patterns that conforms to the Levy distribution, which was named after the French mathematician Paul Lévy, and the step length has the fat-tailed distribution. The walking steps are isotropic random directions when walking in a space with the number of dimensions greater than 1. Many animal's foraging movements also possess the Levy Flight features, e.g. most of the feeding time is spent around known food sources, and occasionally a long-distance flight is needed to find other food sources [24] [26].

2 Integration of Levy Flight and ACO

2.1 Problems background

In this paper, the concrete application for which the Levy ACO will be applied is the Traveling Salesman Problem (TSP). When solving TSP, ACO algorithm utilizes the positive feedback by accumulating the pheromone to focus on possible better solutions and the negative feedback by evaporating pheromone to reduce the history solution effect for exploring more search spaces. The framework of an ACO algorithm is shown in figure 1.

Similar other metaheuristics, ACO not only needs to explore more diverse solution spaces to avoid being trapped at a local optimum, but also attempts to speed up the solution procedure to reduce the time spent. In summary, a good ACO should be able to find higher quality solution within shorter period.

In the ACO algorithm shown in figure 2, the uniform distribution is employed to for each single ant to select the next site from the possible sites (assuming we are solving a TSP) in step 2. At this time, the attractant factor derived from the attraction and pheromone values for each candidate will accumulated to 100%，please refer to formula (1). Each single ant will select the next site according to the random number uniformly distributed between 0 and 1. However, because attractant factors for candidate sites vary, and they are normally or exponentially distributed after sorted, the selection probability of a candidate site will decline quickly and be close to zero as the number of ranks increases. Statistically the candidate sites with small attraction factors then could be two or more standard deviations from the mean and would rarely be selected since their selection probabilities are very low. As the result, the algorithm always focuses on candidate sites with higher attractant and it is hard to achieve diverse solutions.

$$p_{ij}^{k}(t) = \frac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}(t)]^{\beta}}{\sum_{l \in N_i^k}[\tau_{il}(t)]^{\alpha}[\eta_{il}(t)]^{\beta}} \quad if\ j \in N_i^k \qquad (1)$$

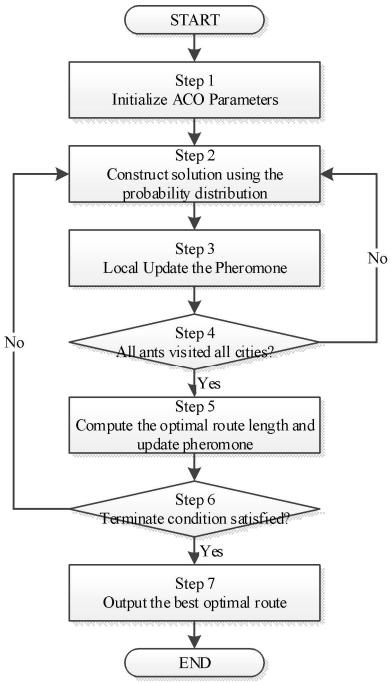

Fig. 1. The framework of ACO algorithm.

The Max-Min ACO algorithm explores more solution spaces and performs better among those improved versions of ACO algorithm. Nevertheless, there is still some room for the improvement in the Max-Min ACO algorithm based on our study. In this paper, the Levy Flight mechanism is employed to improve the original Max-Min ACO algorithm while the original Max-Min ACO is used for benchmarking.

2.2 Modification of Levy Flight for ACO

The Levy Flight mechanism is defined upon the Levy distribution. The Normal, the Cauchy, and the Levy distributions are shown in Figure 2. Unlike the Normal and the Cauchy distributions, the Levy distribution is a fat-tailed one, which means the points in the tail part have higher probability than the one with same value in other two distributions. Upon solving TSP, we are going to utilize Levy distribution (Levy flight) to enhance ACO, which shall be able to explore more solution spaces within a reasonable computational time.

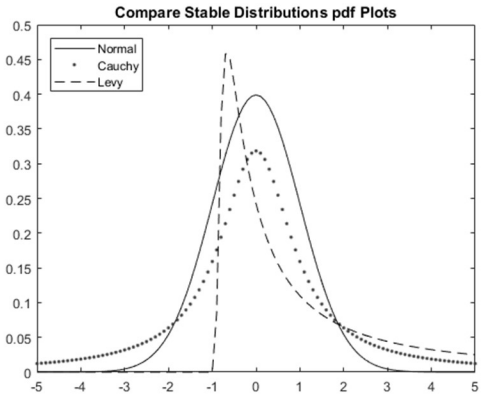

Fig. 2. Different Distributions.

Figure 3 depicts the Brownian motion upon the Normal distribution and Levy flight based on the Levy distribution [26]. It is obvious the area covered by Levy flight is much larger than the one of Brownian motion within the same 1000 steps. Part b of figure 3 illustrates the detailed trajectory of Brownian motion, and it indicates the movements of the Brownian motion mainly concentrate around the current point with step length of 1. The parameters of Levy flight can be adjusted to balance effectively the diversification and intensification. It's a Brownian motion if step length equals to 1, and others are so called Levy flight if step length is great than 1. The fly distance is defined as the step length in our paper.

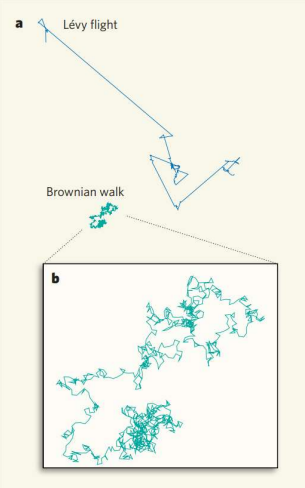

Fig. 3 Levy Flight vs Brownian Walk

The formulas of the standard Levy distribution are represented by:

$$L(s) = |S|^{-1-\beta} \tag{2}$$

$$S = \frac{\mu}{|v|^{\frac{1}{\beta}}} \tag{3}$$

$$\mu \sim N(0, \sigma_\mu^2), \nu \sim N(0, \sigma_\nu^2) \tag{4}$$

The solution procedure for the Levy distribution use formulas (2) to (4). L(s) is Levy distribution for step length S, μ and ν follow the normal distribution, and β is the parameter for Levy distribution. Formulas (2) to (4) illustrate how the step length is computed, which is the most important part of the Levy Flight. The step length is a random number following the Levy distribution, and it is associated with the direction of Levy Flight which follows the uniform distribution, maybe in 2 dimension or 3 dimension, depending on the particular application. There is no direction to be considered if the movement of Levy flight is one dimensional.

The calculation of Levy Flight using formulas (2) to (4) is very complicated and time consuming. The running time of an ACO algorithm will increase significantly if the Levy distribution is applied directly. ACO algorithm is an iterative process in which the Levy flight function will be called repeatedly, therefore it is necessary to pick a simple and approximate model for decreasing the computational cost. Furthermore, the standard Levy flight model requires two parameters, i.e., direction and step length, whereas the direction is uniformly distributed and the step length follows the Levy distribution. In our ACO algorithm, only a random number between 0 and 1 is needed for selecting the next site. Therefore, the Levy flight mechanism in this paper needs only to consider the step length whose value ranges from 0 to 1

after the conversion and follows the Levy distribution.

2.3 Integrating Levy Flight with ACO

The structure of the Levy ACO proposed in this paper is the same as the classic ACO except the application of the Levy flight mechanism to determine the probability of selecting next visiting site (step 2 in Figure 1). Figure 4 lists side-by-side the flowcharts of classic ACO and the Levy ACO for better comparison. Some grey steps in the flowchart of Levy ACO are newly added ones. These steps calculate the new selection probability, sort the candidate sites upon the attractants, and then apply the new selection probability.

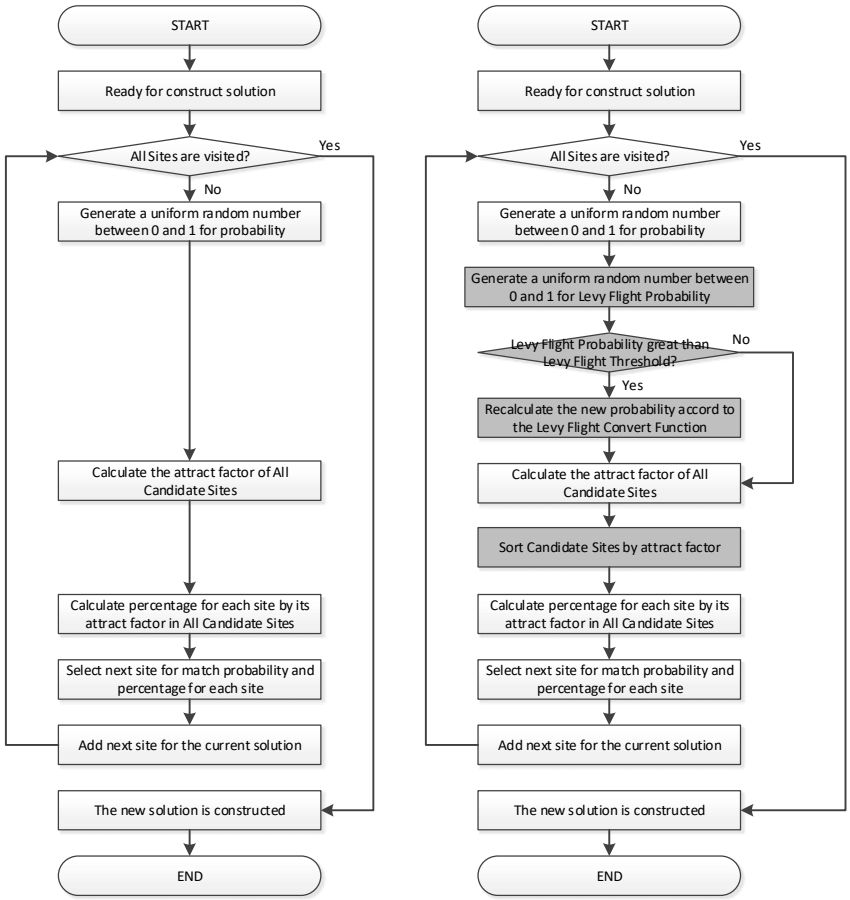

Fig. 4. Comparison of Classic ACO and Levy ACO for Step 2 in ACO Algorithm

The attractive factor η, an exponential function of parameter β in formula (1), makes each candidate site have different attracting value, and thus each candidate site has different and exponentially decreasing probability to be selected after sorted. So very few sites have higher probabilities and will be selected more frequently while most sites have lower probabilities and they are selected seldom. This unfavorable scenario can be improved if Levy distribution is applied instead of using original uniform distribution.

The original uniformly distributed selection probability will be replaced by the step length of Levy Flight after the conversion. Please note that the step length of Levy flight can be any positive number, therefore it needs to be converted so that the result ranging from 0 to 1 as the

probability. Formulas (5) to (7) are designed to convert the step length into the selection probability. In Levy ACO, all candidate sites should be sorted by their attraction factor values (formula (1)) in non-increasing order before applying the converted selection probability. After sorting, obviously the front part contains the candidate sites with higher attractants while the back portion consists of the candidate sites with lower attractants. With the amplification mechanism in our conversion formulas, the new selection probabilities of some candidate sites with lower attractants will be amplified. In this case, these original "unfavorable" sites will more likely be selected and diverse solution spaces can be searched.

Some definitions of the terms used in the conversion formulas:

- P_{new} : New selection probability after Levy flight conversion, it will be used for selecting the next site.
- P_{now} : The original selection probability that is uniformly distributed before the Levy flight conversion.
- P_{levy}: The random number uniformly distributed between 0 and 1 is used for switching Levy flight conversion.
- $P_{threshold}$: Threshold ranging from 0 to 1 for Levy flight conversion, which determines if Levy flight conversion is turned on or not after being compared with P_{levy}.

$$1 - P_{new} = A * \frac{1-P_{levy}}{1-P_{threshold}} * (1 - P_{now}) \quad (5)$$

$$P_{new} = 1 - A * \frac{1-P_{levy}}{1-P_{threshold}} * (1 - P_{now}) \quad (6)$$

$$p_{new} = \begin{cases} 1 - A * \frac{1-p_{threshold}}{1-p_{levy}} * (1 - p_{now}), & if\ p_{levy} > P_{threshold} \\ p_{now}, & if\ p_{levy} \leq P_{threshold} \end{cases} \quad (7)$$

The core steps of Levy flight conversion (formulas (5) to (7)):

- The Levy flight conversion formula (5) is applied to the Levy flight step lengths that are greater than 1 so that they will be mapped to the values between 0 and 1.
- The formula (6) is the transformation of formula (5) for calculating p_{new}.
- Value p_{now} and p_{levy} are generated in uniformly distributed between 0 and 1.
- Levy flight threshold $p_{threshold}$ is set for determining if the selection probability should apply Levy Flight conversion or not.
- New selection probability p_{new} use the value of p_{now} if p_{levy} is less than $p_{threshold}$ (indicating step length is not greater than 1). In this case p_{new} is not converted.
- The new probability p_{new} should be recalculated using formula (6) if p_{levy} is greater than $p_{threshold}$, that is, the Levy flight step length is greater than 1 and Levy flight conversion is turned on.

In the above Levy flight conversion formulas we need two predefined parameters: $p_{threshold}$ (Levy flight threshold) and A (amplification ratio). The new selection probability presented in formula (7) is able to encourage the current ant to choose the candidate sites with original lower selection probabilities to increase the diversity of the solution space with the hope of getting out of local optima.

3 Computational Experiments

3.1 Data and environment setting

In the following computational experiments, we compare the results obtained by the Levy ACO proposed in this paper integrates the original Max-Min ACO algorithm with Levy Flight mechanism and the original Max-Min ACO algorithm without Levy Flight mechanism. The code for the Levy ACO proposed in this paper is implemented based on the code for ACO algorithm [5] available at http://www.aco-metaheuristic.org/aco-code/, The Levy threshold $p_{threshold}$ and amplification ratio A are set to be 0.8 and 1 respectively. Each benchmark runs 100 times considering the probabilistic factor in ACO algorithms.

The platform configuration for benchmarking: Windows 10 x64, CPU 8 cores 2.7GHz, Memory 32GB, the programming language is C with the original Max-Min ACO algorithm [28] and the Levy ACO algorithm in this paper.

3.2 Benchmarks

3.2.1 Benchmark 1 and 2

The experiments are performed using instance link318 and pcb442 in TSPLIB [27], and here are some observations are illustrated in figure 5 and table 1:

- All instances reach the optimal solution 52029 for link318 or 50778 for pcb442, and the average number of iterations to get the best solution for the Levy ACO is lower than the original Max-Min ACO. This result indicates that the Levy ACO can reach the best solution faster.
- For the purpose of analyzing the performance of the algorithm, we conduct the statistical analyses for the benchmarks including the maximal iterations, average iterations, median iterations, and minimal iterations to obtain the best solution.

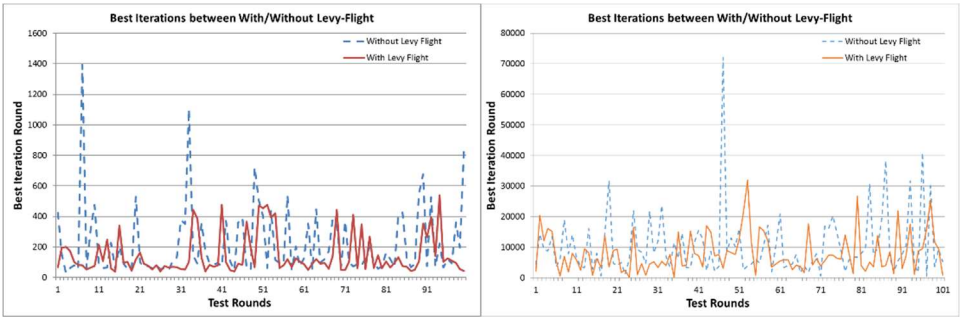

Fig. 5. Benchmark for with/without Levy Flight for link318(left) and pcb442(right)

The statistical results are listed in table 1：

- The maximal number of iterations for link318 is reduced from 1395 (without Levy flight) to 539 (with Levy flight), an improvement of 61.36%. The maximal number of iterations for pcb442 is reduced from 72016 (without Levy flight) to 31956 (with Levy flight) with an improvement of 55.63%.
- The average number of iterations is reduced by 32.13% or 23.53% with Levy ACO.

- The median number of iterations shows the improvement of 9.31% or 20.00% by applying Levy ACO.
- Levy ACO runs more consistently as the variance in the number of iterations is reduced by 43.58% or 38.26%, which means the stability of the algorithm is also improved.
- The minimal number of iterations for Levy ACO increases a bit for link318 and decrease by 85.19% for pcb442. Please note that for link318, the minimal iteration number is already very low (31), and there is very small space to improve. This is reasonable because ACO algorithm is a probabilistic algorithm.

Table 1. Benchmark for with/without Levy Flight for link318/pcb442

	Without Levy Flight		With Levy Flight		Improve Percentage	
	link318	pcb442	link318	pcb442	link318	pcb442
Max round	1395	72016	539	31956	61.36%	55.63%
Average round	216	10403.69	146.59	7956.17	32.13%	23.53%
Mean round	102	7735	92.5	6188	9.31%	20.00%
Min round	31	466	38	69	-22.58%	85.19%
Sample variance	230.79	10289.07	130.22	6352.98	43.58%	38.26%

3.2.2 Benchmark 3 and 4

Instance a280 and pr299 in TSPLIB [27] are selected for the other experiment, and the results are illustrated in figure 6 and table 2:

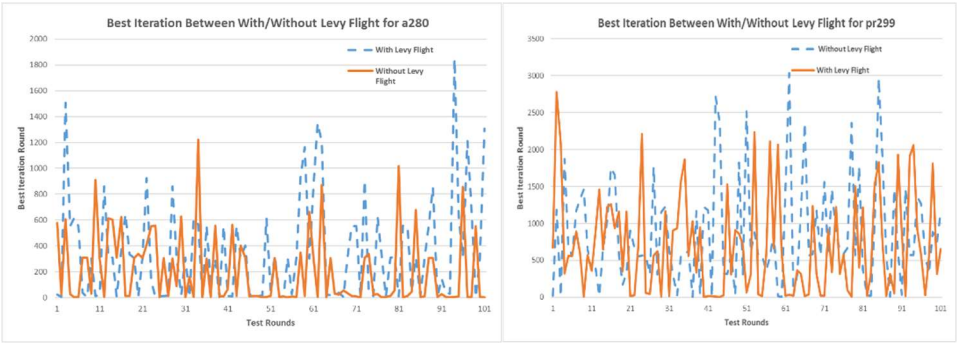

Fig. 6. Benchmark for with/without Levy Flight for a280 (left) and pr299 (right)

Similarly, we can draw the following conclusions

- All instances get the best solution 2579 for a280 and 48191 for pr299, and the average number of iterations to get the best solution of the Levy ACO is lower than the original Max-Min ACO. This indicates that the Levy ACO can reach the best solution faster.
- The number of average iterations shows 41.53% for a280 and 17.85% for pr299 reduction by applying the Levy ACO.
- The median number of iterations of the Levy ACO possesses an improvement 91.50% for a280 and 5.78% for pr299 comparing to the original Max-Min ACO.
- It is obvious that the Levy ACO runs more consistently since it has lower variance in the number of iterations. It indicates the stability of the Levy ACO is improved.
- The minimal number of iterations of the Levy ACO presents same or some improvement as well with a very small value (2 or 3).

Table 2. Benchmark for with/without Levy Flight for a280 and pr299

	Without Levy Flight		With Levy Flight		Improve Percentage	
	a280	pr299	a280	pr299	a280	pr299
Max round	1846	3035	1221	2779	33.86%	8.43%
Average round	346.96	849.25	202.86	697.63	41.53%	17.85%
Mean round	306	606	26	571	91.50%	5.78%
Min round	2	3	2	2	0.00%	33.33%
Sample variance	393.71	701.48	275.90	672.74	29.92%	4.10%

4 Conclusions

The paper proposes the Levy ACO algorithm that is developed based on Levy Flight mechanism and classic ACO algorithm. The experiment demonstrates that the proposed algorithm can achieve the best solution more effectively. The capabilities of exploring diverse solution spaces and avoiding local optima contribute the efficiency of the algorithm as a whole. On the average the proposed algorithm can reach the optimal solution with less (reduced by 32.13%, 23.53%, 41.53% and 17.85%) iterations comparing to the original Max-Min ACO algorithm. The computational results demonstrate the superiority of the Levy ACO proposed in the paper.

In addition to conducting more computational experiments, for example, using the other instances in TSPLIB, we are planning to tune parameters of the Levy ACO with some smarter methods to further improve the performance of the Levy ACO and to study its applicability for other problems such as VRP.

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A Robust Crew Pairing Model for Airline Operations using Crew Swaps and Slack Times

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Abstract The trend in airlines, as more flights are being flown, is that delays have been less from unpredictable events and more from operations. Airlines push to maximize profits by scheduling flights with little to no regard for possible disruptions. Common practices that deal with disruptions, such as purposeful cancellation of flights, and assignment of emergency crew, are usually inefficient. Planning for disruptions in operations, while increasing planned costs, create flight schedules that are capable of handling disruptions. This paper proposes a robust crew pairing model that schedules slack times and crew swaps that can potentially reduce the propagation of delay when a disruption occurs. A mathematical formulation is presented. A small set of flights is presented to show the schedules obtained from the traditional and robust models. The comparison is then made for a set of 1890 flights from a real major airline. The robust model is able to create a solution with higher planned costs but better delay indicators.

1 Background

The time-sensitive nature of service in the airline industry means that several scheduling problems arise during operations. Airline resources work under connectivity and compatibility constraints [1]. Among those resources are the crew members. Scheduling the aircrafts can be extremely costly when being inefficient, and are often inflexible, while airlines have limited control in scheduling passengers. Crew scheduling is both flexible and manageable in cost, and innovations in crew scheduling can lead to improvements in airline operations that are feasible in cost, but still having significant impact.

1.1 Crew pairing

Crew pairing is part of a sequential process done in airline resource management that follows the timetable construction and the assignment of aircrafts. Pairings are constructed by forming sequences of connectable flights within the same fleet that start and end at the same

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crew base [2]. Crew pairing finds sets of specified crew that will operate together for the duration of the pairing. The graphical representation of a pairing is shown in Figure 1.

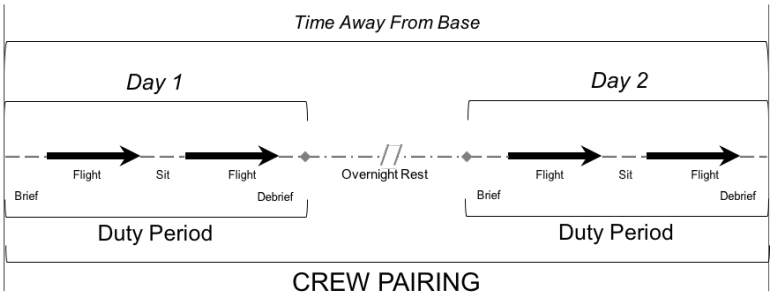

Figure 1: Structure of a crew pairing

A duty period is different from a crew pairing. A duty period is a sequence of flight legs that can be legally flown by a set of crew with only short rest periods. The crew pairing is a sequence of flight legs, and long rest periods if needed, that can be legally flown by a set of crew, and starts and ends at the same home base. A crew pairing is composed of one or more duty periods.

1.2 Delays and delay propagation

The air transport industry neglects shorter delays (departure and arrivals no later than 15 minutes). In the context of air transport, passengers consider small delays as being negligible [3]. The popular 15-minute margin of tolerance is the viewpoint for significance of lateness. A missed connection is what can increase the significance of lateness from the viewpoint of a passenger, which is why connecting flights are scheduled with ample connection time [4].

Delays are what affect punctuality, an attribute that passengers find satisfaction in. Consistent lack in punctuality causes a reduction in the airline's market share. There is therefore a cost to lack of punctuality. Flights that are delayed (or cancelled) to an extent beyond the tolerance limit of its passengers will shift service perception of customers wherein switching airlines of preference is likely [5].

Delays in an airline context usually force the company to solve the problem locally where the disruption occurs. While local resources will allow for lessening the effect of the delay, it is still likely to propagate towards those with connected resources occurs [6]. Shown in Figure 2 is the effect delay propagation from single or multiple delays on a schedule.

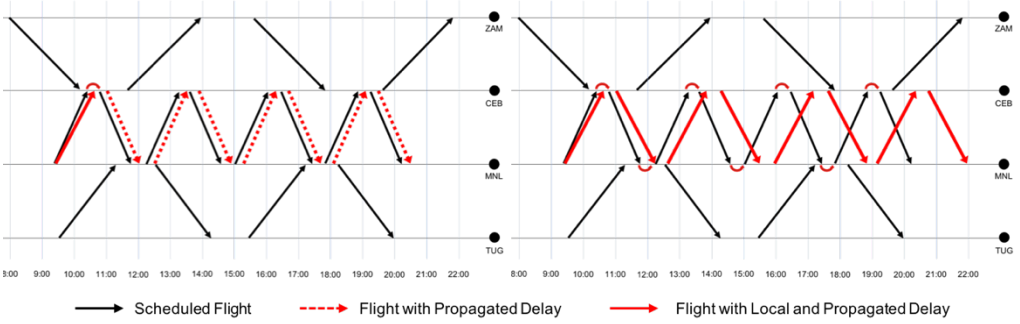

Figure 2: Flight schedule with propagation from a single delay (left), and from multiple delays (right)

A delay on the first flight will translate to subsequent flights. Mitigation of the delay is possible if local resources at each flight will allow for catching up, such as if passengers can be boarded quicker than the standard time. This is unlikely for flights that are high-density, which are usually scheduled with no slack. A single delay can at the beginning of a schedule can cause

the same delay to propagate to further flights. In a realistic schedule, flights will often have delays that were created locally and that were propagated from previous flights Figure 2 follows a timeline network representation, further discussed in chapter 2.2

1.3 Planning for disruptions through slack times and crew swaps

A promising area of research is to construct schedules that perform well under irregularities [7]. Robustness is in the form of stability, plans that as least sensitive as possible to disruptions; or flexibility, which gives options for the plan to remain cost-effective given a perceived set of disruption events [8]. Scheduling by providing ample slack times gives stability, while scheduling for possible crew swaps gives flexibility.

Slack times are purposely idle time in the schedule that can possibly absorb delays. Crew swaps are discussed through an example: suppose that *Crew A* are on standby waiting for their next flight *Flight A's* departure time. At the same time, a flight *Flight B* departing from the same airport is delayed due to *Crew B* still in transit. An example of a crew swap is scheduling *Crew A* to take over duties for *Flight B*, while the in-transit *Crew B* will take over *Flight A* when they arrive. While this might result in *Flight A* being delayed if *Crew B* takes too long to arrive, the delays between *Flight A* and *Flight B* are essentially mitigated and shared between the two.

The prevalent scheduling process in airlines pay insufficient consideration on the impact of uncertainty in operating a complex airline network with multiple interconnected resources [9]. Disruption management as a field has primarily existed as a reactive approach through recovery, which aims to put a schedule back to its original as soon as possible following a disruption. Reactive disruption management uses simple techniques for solving, focuses on having minimal costs during planning, and incurs higher delays when disruptions occur. On the other hand, proactive disruption management in the form of robust schedules incorporates uncertainty of delays into the planning, incurring higher planned costs but often lower delays.

Section 2 discusses on relevant literature, including context on airline resource planning, network representations used in this research, and the traditional modeling approach of the crew pairing problem. Section 3 formulates the robust crew pairing model, an extension of the traditional set partitioning approach. Section 4 discusses on a methodology for solving large instances of the problem. Computational results are shown in Section 5. Emphasis is placed on comparison of the solutions obtained from the robust model and the traditional model. Lastly, conclusions and extensions on the research is discussed in Section 6.

2 Related Literature

2.1 Airline resource planning

The large complexity of scheduling flights, aircrafts, and crew for even just a small set of flights have led to the creation of a sequential process that is now generally applicable for any airline [10].The process, in sequence, is timetable construction, fleet assignment, crew pairing, and crew rostering, illustrated in Figure 3.

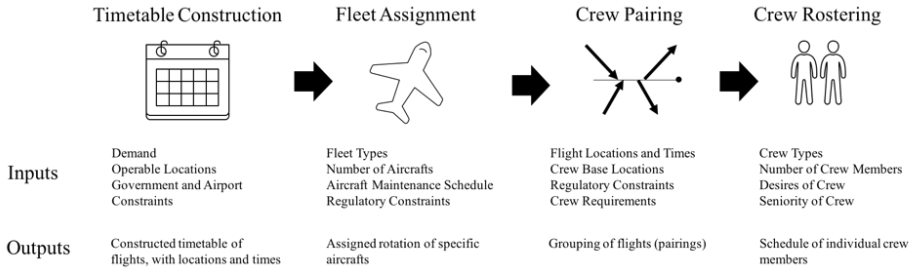

Figure 3: The stages of resource planning in the airline industry

A timetable of flights is created first. Expectations from marketing based on analysis of demand is matched with the available fleet, the available crew, and the time slots available for the airline in the different airports. The timetable consists of flight legs (non-stop flights), each with the locations, dates, and times of departure and arrival. Allocation of aircrafts to the flight legs from the timetable is then done based on the expected revenue, passenger demand, and flight distance of each flight leg. Larger aircrafts are assigned to flights with more demand and longer distances. Crew pairings are then constructed by forming sequences of connectable flights within the same fleet that start and end at the same crew base. Crew pairing finds sets of specified crew that will operate together for the duration of the pairing. The structure of a crew pairing is shown in Figure 3. Crew rostering is then done, which assigns selected pairings from the previous step to the airline's crew. The objective of crew rostering is to cover all pairings, as well as other schedules of the crew including training requirements, vacation days, etc. Work rules and regulations must be satisfied in the rostering [11]. In some airlines, a subset of crew rostering is done known as crew bidding. Crew members bid on their preferred schedules in attempt to satisfy the desires of individual crew members [12].

Disruption management is the monitoring and scheduling of resources close to the day of operations. While disruptions should be resolved with locally available resources, disruptions in the airline industry tend to extend to other flights due to connectivity of resources. Recovering from unexpected events is now a well established part of operations among airlines, and is typically seen as the final stage of airline resource planning.

The timetable construction and the fleet assignment are heavily related to each other, as there is no room for flexibility in the assignment of aircrafts. The goal of constructing timetables and assigning fleets is usually to maximize revenue. For most airlines, this process is repeated semi-annually, accounting for changes in the demand and profitability of each leg [13]. Planning for crew is done much closer to the actual flight legs. A summarization and timeline view of the airline resource planning process is shown in Figure 4.

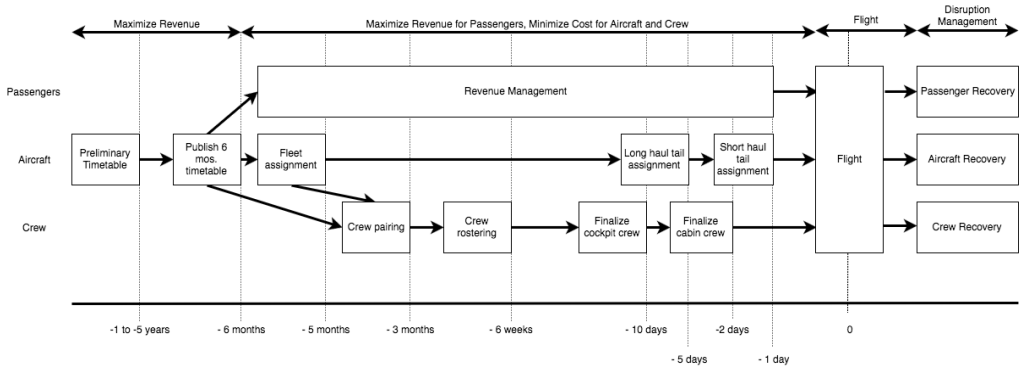

Figure 4: A timeline view of airline resource planning

2.2 Network representation

Mathematical models used to solve airline planning problems or recovery problems use network representations. These figures provide easy interpretation of airline scheduling problems. The three most common network representations in literature are connection networks, time-band networks, and timeline networks [14]. The primary representation used in this paper is the timeline network.

Timeline networks represent schedules in the most natural way possible. A timeline network has arrows that represent each flight. The arrival or departure of any flight is the beginning or end of an arrow. Time-line networks are inherently activity-on-arrow networks. Horizontal lines are set arbitrarily, pertaining to the different airports in the system. Nodes are placed on a specific line corresponding to the airport of the event. The horizontal location of the start and end of an arrow is set based on the time of the flight. An arrow connecting different

horizontal lines pertains to a feasible flight. An arrow connecting nodes found on the same horizontal pertains to a grounded aircraft. All arrows are constructed moving from left to right, indicating chronological flow. Figure 2 uses a network representation based on the data from Table 1. The data in this table, from hypothetical airline 'Hypothetical Phils', connects several airports in the Philippines. Sixteen flights comprise of Hypothetical Phils' schedule, with CEB and MNL acting as high-density ports while ZAM and TUG act as low-density ports. This timetable simulates a small hub-and-spoke structure.

Table 1: Hypothetical Phils' flight schedule

Flight Number	Departure Location	Arrival Location	Departure Time	Arrival Time	Flight Number	Departure Location	Arrival Location	Departure Time	Arrival Time
1	MNL	CEB	0925	1025	**9**	ZAM	CEB	0825	1015
2	CEB	MNL	1050	1150	**10**	CEB	ZAM	1150	1400
3	MNL	CEB	1215	1315	**11**	ZAM	CEB	1540	1750
4	CEB	MNL	1340	1440	**12**	CEB	ZAM	1930	2130
5	MNL	CEB	1505	1605	**13**	TUG	MNL	0935	1120
6	CEB	MNL	1630	1730	**14**	MNL	TUG	1240	1425
7	MNL	CEB	1755	1855	**15**	TUG	MNL	1530	1715
8	CEB	MNL	1920	2020	**16**	MNL	TUG	1820	2005

2.3 Traditional modeling approach

The traditional formulation of the crew pairing problem is the set partitioning formulation, with objective of finding a minimum cost subset of feasible pairings such that every flight is covered by exactly one selected pairing [15]. Pairings are the sequences of flights wherein crew members work together, and are constrained to having to start and end at the same crew base. Let P be the set of all feasible pairings and F the set of all flights, c_p the cost of each pairing, $a_{fp} = 1$ if pairing p covers flight f and 0 otherwise, $x_p = 1$ if pairing p is chosen in the solution and 0 otherwise. The traditional crew pairing formulation is as follows:

$$\text{Min} \sum_{p \in P} c_p x_p \quad (1)$$

$$\text{s.t.} \quad \sum_{p \in P} a_{fp} x_p = 1 \quad \forall f \in F, \; p \in P \quad (2)$$

$$x_p \in \{0,1\}, \quad \forall p \in P \quad (3)$$

This formulation minimizes the total costs of the chosen pairings, ensuring that each flight is covered by one and exactly one pairing only.

2.4 Previous works

An existing research uses a propagation tree to understand how a root delay propagates through aircraft and crew connections. It also evaluates how absorption of this can be done by slack time, and re-allocates slack times to where it's needed the most. [6]. Another research provides a detailed methodology on providing crew swap opportunities as contingencies in delay propagations. Their model prioritizes high delay propagation costs for swap opportunities [16]. There has been work on costing the per delay minute of a flight. Literature suggests that the cost of flight delays follow an 'S' shape curve, in that flight delays are tolerable up to an extent, then

becomes increasingly dissatisfactory until it reaches very long delays and approaches full customer dissatisfaction [4].

3 Robust crew pairing model

The model is formulated as an extension of the set partitioning formulation of the traditional crew pairing problem (CPP). The objective function involves the computation of costs of pairings, as well as the cost of delay propagations for each flight in each pairing. The model involves two phases, wherein the first selects the pairings for the solution, and the second selects the crew swaps for the solution.

3.1 Assumptions

The following are the assumptions taken in the development of the model in this research:

- The schedules of flights can be known with certainty and cannot be changed.
- Minimum ground connection times can be defined for each pair of flights.
- Any crew member can be scheduled to any flight. Crew pairings can be formed around any of the flights within the set of flights being solved in the problem.
- Regulations are known with certainty and do not vary.
- The cost of each crew pairing can be determined as a deterministic value.
- No additional flights need to be added into the timetable.
- There are enough crew members to satisfy the pairings to be formed in the model.
- All crew swaps to be performed have an available aircraft.

3.2 Definition of parameters

The following are defined in the model:

Sets and notations

P	set of all feasible pairings
F	set of all flights
$\rho_p(f)$	preceding flight of *f* at pairing *p* in the same duty period
$Q(f,p)$	set of flights that precede flight *f* and is in the same duty period as *f* in pairing *p*
$R(f,p)$	set of flights that follow flight *f* and is in the same duty period as *f* in pairing *p*
$\alpha(Q)$	the first flight in the set of flights *Q*
$\omega(Q)$	the last flight in the set of flights *Q*
M	a very large number

System and decision variables

ACC_{fp}	accumulated flight duty hours after flying flight *f* in pairing *p*
PND_{fp}	pending flight duty hours after flying flight *f* in pairing *p*
DP_p	delay propagation value for pairing *p*
$s_{fpf'p'}$	binary variable for checking possibility of a swap between two flights from different pairings 1 if a swap is possible between flights *f* and *f'* for pairings *p* and *p'*, 0 otherwise
x_p	binary decision variable for selection of pairings 1 if pairing *p* is selected, 0 otherwise
y_{ip}	binary decision variable for linearization of delay propagation requirement; $i \in \{1,2\}$ 1 if requirement is not violated; 0 otherwise

Input parameters

c_p	cost of pairing	$ap_{ff'}$	binary variable for the location connectivity of two flights 1 if flight *f* arrives at the same airport as flight *f'* departs, 0 otherwise
π_{fp}	binary variable defining flights covered by pairings 1 if flight *f* is covered by pairing *p* 0 otherwise	$hb_{pp'}$	binary variable for considering home bases of two pairings 1 if pairing *p* and *p'* has the same home base, 0 otherwise
g	minimum ground connection time	dp_{min}	minimum delay propagation level to consider a swap that mitigates delay
r_f	arrival time of flight *f*	dp_{max}	maximum delay propagation level to consider a swap that absorbs delay
d_f	departure time of flight *f*		
reg	maximum regulation flight duty period of crew		
b	briefing period time		
b′	debriefing period time		
τ_f	average delay time of flight *f*		

3.3 First phase: Pairing selection

The first phase involves the selection of the pairings into the solution. It has two objective functions that are counteracting, and thus, uses a user-inputted weight for the second objective function.

$$\text{Min} \sum_{p \in P} c_p x_p \quad (4)$$

$$\text{Max} \sum_{p \in P} DP_p x_p \quad (5)$$

Objective function (4) is taken from the traditional crew pairing problem formulation. It aims to minimize the total cost of the pairings chosen to cover the set of flights. Objective function (5) minimizes the total delay propagation value of the chosen pairings. The first expression aims to choose the pairings with the least costs, while the second expression chooses the pairings with least delay propagation. This bi-objective formulation is resolved by using a user-inputted weight to the second objective function.

The following mathematical expressions are the constraints of the model formulated:

$$\sum_{p \in P} \pi_{fp} x_p = 1 \qquad \forall f \in F \quad (6)$$

$$x_p \in \{0,1\} \quad (7)$$

Constraint (6) ensures that all flights must be catered to by exactly one pairing. This constraint is also present in the traditional crew pairing problem formulation. Constraint (7) states that the decision variable x_p is a binary variable. The delay propagation value DP_p of the pairings generated is calculated as the pairings are generated. It is calculated using the average historical delays of flights τ_f, their scheduled arrival times r_f, and their scheduled departure times d_f. The minimum ground connection time g is also needed. An example of how the delay propagation value DP_p is calculated is shown in Figure 5, which takes data from the hypothetical flight schedule in Table 1.

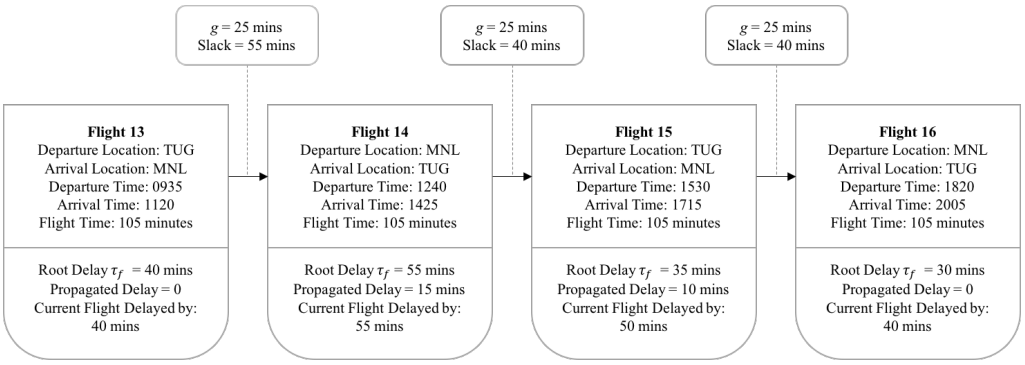

Figure 5: Example of delay propagation value computation

The delay propagation value is the average of the propagated delay for each flight. For the example in Figure 5, the delay propagation value is 6.25 minutes. In the first flight, the root delay (or the historical delay average for that flight) is 40 minutes. The slack between the first and the second flight is 55 minutes. Thus, no delay propagates on to the next flight. So, the first flight only experiences the root delay. The third flight, on the other hand, experiences a propagated 15 minutes of delay from the second flight. This is because the slack times could not absorb all of the root delay, and thus, allows for delay propagation. The delay propagation value DP_p used in the model does not account for how large the root delay is, but for how much the pairing tends to propagate delay. If there is enough slack allocated for a flight that is likely to be delayed, then the delay propagation value is low. The first phase of the model allows for a tradeoff between scheduling with minimal costs, or scheduling with more slack for flights that need the slack. The algorithm for the calculation of the delay propagation value of each pairing is shown in Algorithm 1.

Algorithm 1: Calculation of the delay propagation value of a pairing

Set F as the set of flights that are part of a pairing p
Set f as the first flight
Set DP_f as the delay propagation after flight f
$DP_f = 0$
While ($f \in F$) **do**{
  Calculate the slack time between flight f and the flight after by obtaining the difference in scheduled arrival of the first flight and the scheduled departure of the next flight, and subtracting the minimum ground connection time g
  $DP_f = \max(DP_f + \text{delay of flight } f - \text{slack time}, 0)$
  Set f as the next flight
}
Set DP_p as the average of all DP_f such that $f \in F$

3.4 Second phase: Crew swap contingencies

The second phase involves the selection of the crew swaps into the solution. It gets the outputs from the first phase of pairing selection, and uses these as inputs. These inputs determine the best crew swaps possible for mitigating delay propagation.

$$\text{Min} \sum_{f \in F} \sum_{f' \in F} \sum_{p \in P} \sum_{p' \in P} \varphi(DP_p, DP_{p'}) s_{fpf'p'} \quad (8)$$

Objective function (8) maximizes the number of swaps that can be implemented for the schedule. For a crew swap to be considered in the model, the first pairing should have a high delay propagation value and the second pairing should have a low delay propagation value. The function φ calculates the value of a swap. The value of a swap is higher when the delay propagation of the first pairing is higher and the delay propagation of the second pairing is lower.

This is because when the two pairings to be swapped have a large difference in delay propagation, it is more likely that swapping will allow for better absorption and distribution of delays. The function φ is shown graphically in Figure 6. The value of a swap is zero when the delay propagation for the first pairing is zero when it is below a minimum. The value of a swap also becomes zero when the delay propagation for the second pairing when it is above a maximum.

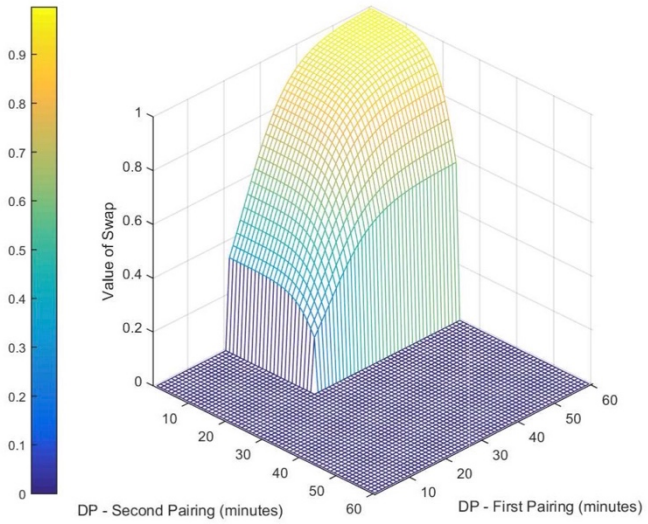

Figure 6: Plot of function φ that calculates for the value of crew swaps

The following mathematical expressions are the constraints of the model formulated:

$$s_{fpf'p'} \leq x_p \qquad \forall f, f' \in F;\ \forall p, p' \in P;\ f \neq f';\ p \neq p' \qquad (9)$$

$$s_{fpf'p'} \leq x_{p'} \qquad \forall f, f' \in F;\ \forall p, p' \in P;\ f \neq f';\ p \neq p' \qquad (10)$$

$$r_{\rho_p(f)} + g \leq d_{f'} + M\left(1 - s_{fpf'p'}\right) \qquad \forall f, f' \in F;\ \forall p, p' \in P;\ f \neq f';\ p \neq p' \qquad (11)$$

$$r_{\rho_p(f)} + g \leq d_f + M\left(1 - s_{fpf'p'}\right) \qquad \forall f, f' \in F;\ \forall p, p' \in P;\ f \neq f';\ p \neq p' \qquad (12)$$

$$s_{fpf'p'} \leq ap_{\rho_p(f)f'} \qquad \forall f, f' \in F;\ \forall p, p' \in P;\ f \neq f';\ p \neq p' \qquad (13)$$

$$ACC_{\rho_p(f)\,p} + PND_{f'p'} \leq reg + M\left(1 - s_{fpf'p'}\right) \qquad \forall f, f' \in F;\ \forall p, p' \in P;\ f \neq f';\ p \neq p' \qquad (14)$$

$$ACC_{\rho_{p'}(f')\,p'} + PND_{fp} \leq reg + M\left(1 - s_{fpf'p'}\right) \qquad \forall f, f' \in F;\ \forall p, p' \in P;\ f \neq f';\ p \neq p' \qquad (15)$$

$$ACC_{fp} = b + r_{\omega(Q(f,p))} - d_{\alpha(Q(f,p))} \qquad \forall f \in F;\ \forall p, \in P \qquad (16)$$

$$PND_{fp} = r_{\omega(R(f,p))} - d_{\alpha(R(f,p))} + b' \qquad \forall f \in F;\ \forall p, \in P \qquad (17)$$

$$s_{fpf'p'} \leq hb_{pp'} \qquad \forall f, f' \in F;\ \forall p, p' \in P;\ f \neq f';\ p \neq p' \qquad (18)$$

$$s_{fpf'p'} \leq y_{1p} \qquad \forall f, f' \in F;\ \forall p, p' \in P;\ f \neq f';\ p \neq p' \qquad (19)$$

$$s_{fpf'p'} \leq y_{2p'} \qquad \forall f, f' \in F;\ \forall p, p' \in P;\ f \neq f';\ p \neq p' \qquad (20)$$

$$DP_p \geq dp_{min} - M(1 - y_{1p}) \qquad \forall p \in P \qquad (21)$$

$$DP_p \leq dp_{max} + M(1 - y_{2p}) \qquad \forall p \in P \qquad (22)$$

$$ACC_{fp} \geq 0, PND_{fp} \geq 0, s_{fpf'p'} \in \{0,1\}, y_{ip} \in \{0,1\} \qquad (23)$$

Constraints (9) and (10) ensure that swaps can only be possible if the pairings of the two flights being swapped are chosen. Constraints (11) and (12) ensure that swaps are possible only if the minimum ground connection times are followed. Constraint (13) ensures that swaps are possible only if the arrival airport of the incoming flight f is the same as the departure airport of the outgoing flight f'. Constraints (14) and (15) ensure that the maximum duty hours of pilots are not violated upon swapping. Constraints (16) and (17) calculate the accumulated and pending hours system variables respectively. These system variables are necessary for determining whether or not the maximum duty hours of the crew are violated upon swapping. Constraint (18) ensures that swaps can only be considered if the pairings have the same home

base. This ensures that the crew can go back to their home base as scheduled. Constraints (19), (20), (21), and (22)restrict swaps such that the first pairing has high delay propagation, and the second pairing with low delay propagation (and consequently high slack). A minimum delay propagation value is set for the first pairing in dp_{min}. A maximum delay propagation is set for the second pairing in dp_{max}. This allows for the swaps to absorb delays and prevent them from propagating forward. Lastly, constraint (23) ensures the bounds for the values of all variables.

4 Methodology for solving large problems

A set partitioning formulation is used for the crew pairing problem. This model formulation chooses pairings from a set of generated pairings that minimize costs while ensuring that all flights are covered by exactly one pairing. However, the generation of the pairings themselves is exponentially difficult depending on the size of the system. Even a small number of flights will have an exponentially large number of pairings. As such, efficient methods must be made for solving realistic problems.

4.1 Flight subsequences

To generate the pairings needed in solving, the subsequences are first obtained for each flight. Subsequences are the flights that can follow a certain flight in a duty period, meeting connection time, duty period length, and location constraints. Establishing the subsequences of each flight allow for faster search and generation of pairings because the computer bypasses searching the entire set of flights for each iteration. Algorithm 2 shows the algorithm used to obtain the subsequences of each flight. In the generation of subsequences, a time window is used to limit the subsequences that are available to flights. A time window specifies a minimum and maximum connection time, the former determined by the legal minimum ground connection time for flight turnarounds, plus a set slack time, and the latter determined by what is deemed as feasible and efficient by the planner. By setting this window, the number of pairings to be generated is exponentially decreased, while likely not affecting the optimal solution. The representation of setting this time window to limit subsequences is shown in Figure 7.

Algorithm 2: Obtaining subsequences of each flight

Set F as the set of flights
Set f_1 as the first flight
While ($f_1 \in F$) **do**{
  Set f_2 as the first flight
  While ($f_2 \in F$) **do**{
    If ($f_1 = f_2$) **then break** and choose next flight for f_2
    Else { **If** f_2 can follow f_1 in a duty period, such that it follows connection time constraints, duty period length constraints, and location constraints
      Then set f_2 as a subsequence of f_1 and choose next flight for f_2
      Else choose next flight for f_2
    }
  }
  Choose next flight for f_1
}
Enumerate all subsequences of each flight

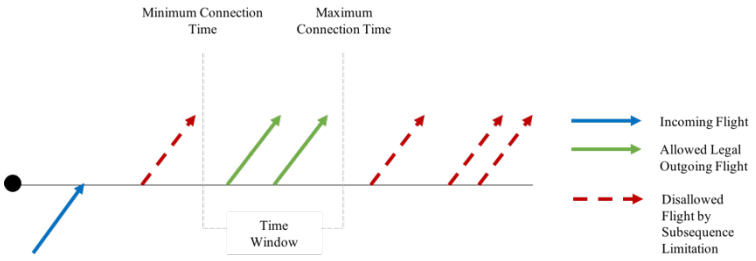

Figure 7: Limiting subsequences of flights by establishing a time window

4.2 Duty tree construction

After generating the subsequences, a duty tree is used to produce the duty periods for generating pairings. The subsequences are used as the building blocks for the duty periods, while the duty periods will be used as the building blocks for the pairings. The duty tree is limited by the maximum number of flights in a duty. Algorithm 3 shows the algorithm used to obtain the valid duty periods of a set of flights.

Duty tree construction can be sped up and limited by specifying the starting and ending locations of the duty periods it generates. A duty period always starts and ends at either a location that is a home base for crew, or at a location that crew stay at for long rests. In a typical airline setting, the home bases are set, and the locations for long rests can be indicated by the planner. For example, a location that is in the rural province that is a 2-hour drive from a major city will never be a starting or ending point of a duty period. It will always be less costly for the airline, and more convenient for the crew, to start and end duty periods at the major city. A major city has more flights connecting through, and thus, any crew that stays here can be more flexibly scheduled. By specifying the home bases and the locations wherein long rests can occur, the number of duty periods can be reduced dramatically. A hub-and-spoke network will have a significantly larger number of spokes than hubs. If the locations for the starting and ending points of duty periods can be limited to the hubs and a few far-off spokes in an airline network, the duty periods that can be generated will be significantly limited while not affecting the optimal solution.

Algorithm 3: Constructing duty tree and valid duty periods

Set F as the set of flights
Set C_i as the set of subsequences for flight i
Set $maxflights$ as the maximum number of flights in a duty
Set f_1 as the first flight
While ($f_1 \in F$) **do**{
  If the series [f_1] satisfies constraints on duty period length **then** series [f_1] is a valid duty period
  Set f_2 as the first flight among subsequences of f_1
  While ($f_2 \in C_1$) **do**{
    If the series [f_1, f_2] satisfies constraints on duty period length **then** series [f_1, f_2] is a valid duty period
    Set f_3 as the first flight among subsequences of f_2
      …
        While ($f_{maxflights} \in C_{maxflights-1}$) **do**{
          If the series [$f_1, f_2, \dots, f_{maxflights-1}, f_{maxflights}$] satisfies constraints on duty period length
          Then series [$f_1, f_2, \dots, f_{maxflights-1}, f_{maxflights}$] is a valid duty period
          Choose next flight for $f_{maxflights}$
        }
      …
    Choose next flight for f_2
  }
  Choose next flight for f_1
}
Establish duty tree based on valid duty periods

4.3 Progressive pairing generation

A concept on column generation can be used to take advantage of the set partitioning model formulation in order to shorten the solution time. Column generation is used when the number of columns in a problem (or the number of possible solutions) is very large. This is typically the case for set partitioning problems. Solution difficulty comes from the large number of possible solutions. If columns were generated iteratively, then the problem can be solved with greater tractability, even though there is no assurance of optimality. The guide for the generation of pairings is the hierarchy of how preferable the types of pairings are. Single-duty pairings are most preferred, then multiple-duty pairings, then pairings that include deadhead. Deadhead is the movement of crew on existing commercial flights of the airline as passengers, but as required by their schedule set by the company and count as working hours with regard to satisfying

regulatory constraints. A progressive process for generation of pairings can be implemented. The steps for progressive pairing generation is shown in Figure 8.

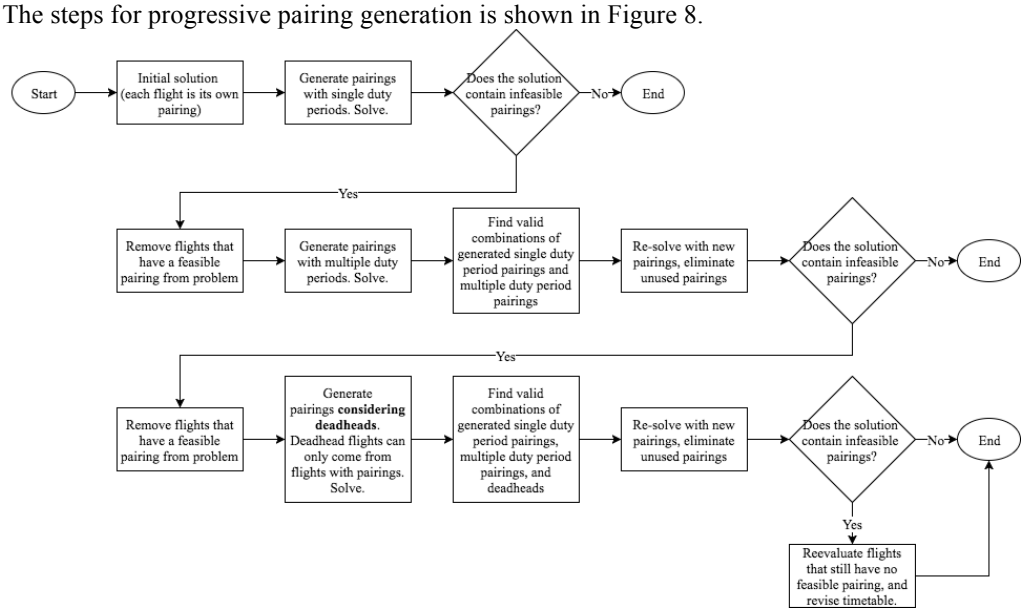

Figure 8: Progressive pairing generation flowchart

The solution methodology starts with an initial solution wherein the flights are each in their own pairing. These are artificial and infeasible pairings to be replaced by feasible ones by the end of the process.

Next is generating pairings that have only one duty period. This means that these pairings will start and end at only a handful of locations that are designated as home bases. The creation of the duty trees becomes much less complex when there are only a handful of possible source locations, realistically less than ten for a major airline, as opposed to considering the hundreds of locations an airline can realistically fly to. The flights covered by a feasible pairing are removed from the problem.

The number of duty periods to consider in the pairing is increased. This continues until the methodology considers pairings with the maximum number of duties possible, as set by the airline. After solving, a set of single duty period pairings and multiple duty period pairings are obtained. The pairings are evaluated as to whether it is possible to combine them to obtain a lower cost. For example, a single duty period pairing can be placed at the beginning of another pairing so that two pairings result into a single longer pairing.

If after considering single and multiple duty period pairings there are still infeasible flights, deadheading pairings are evaluated. Deadheading is only considered on flights that have already been scheduled to a pairing, in order to cater to flights that have yet to be scheduled to a pairing. After generating the pairings, the problem is re-solved. If after this the problem remains infeasible, there is a need to manually alter the timetable of the said flights in order to fit them into a feasible pairing.

5 Results and discussion

5.1 Solution of trivial data

For the validation of the model, a hypothetical set of data is used. Shown in Table 1 are the basic of each flight. The hypothetical flight schedule, referred to as Hypothetical Phils connects four airports in the Philippines: Zamboanga (ZAM), Tuguegarao (TUG), Manila (MNL), and

Cebu (CEB). This data is trivial, containing only sixteen flights with CEB and MNL acting as high-density ports while ZAM and TUG act as low-density ports.

Pairings were generated based on the flight schedule. It is assumed that a crew base can exist in all four airports. Two solutions for Hypothetical Phils are obtained, one using the traditional model, and one using the robust crew pairing model. The traditional model obtains a lower planned cost than the robust crew pairing model, as expected.

A scenario is applied to the schedule for Hypothetical Phils based on the input parameters and locations of the flights, wherein flights coming and going to hubs are more likely to be delayed. The scenario assigns independent local delays for each flight. If connecting resources are present between two flights (same crew with insufficient remaining time to connect), delay will propagate. A minimum ground connection time of 25 minutes is used. If the time between the actual arrival of the previous flight and the scheduled departure of the next flight is less than this, the next flight is delayed.

The number of minutes each flight is delayed for for the two solutions are presented in Figure 9. For many of the flights the deviations in the schedule were often greater for the traditional model solution. Flights 12 was the only flight that was larger in delay for the robust crew pairing model solution as compared to the traditional model solution.

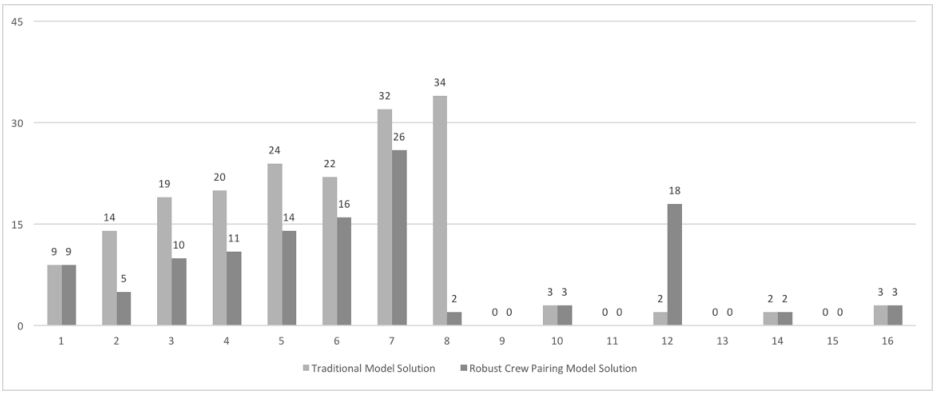

Figure 9: Flight delay (minutes) of each solution obtained for each Hypothetical Phils flight

For both solutions, there was a steady increase in delays from flights 1 through 8. This is because of the short connection time between these flights, and that these flights are between high density hubs. This makes it more likely that these flights become delayed. Swaps taken by the crew pairings in the robust solution were able to lessen the effects of delay propagation. Therefore, despite the additional increase in planned cost, the overall reduction in delay may make it the more desirable solution.

Table 2 shows the comparisons for costs and delays for the two obtained solutions. The cost is obtained using a cost structure based on the duty period lengths (including overtime and fatigue) that simulates crew pay. While planned costs are higher for the robust solution, it has a significantly lower total delay. Even more important is that the traditional model solution has 50% of its flights (8 out of 16) perceived as being late (delayed for more than 15 minutes), and 2 of those flights are perceived being very late (delayed for more than 30 minutes). On the other hand, the crew swap model solution has only 3 of its flight as perceived as late, and none of its flights being excessively late.

This solution serves as a trivial example for showing the effect of mitigation of delay propagation through slack times and crew swaps on an individual flight basis.

Table 2: Cost and delay comparison between the traditional CPP and the crew swap model solutions

	Traditional CPP Solution	*Crew Swap Model Solution*
Planned Cost	17,654	19,830
Total Delay in minutes	184	119
Number of flights delayed for more than 15 minutes	8	3
Number of flights delayed for more than 30 minutes	2	0

5.2 Solution of 1890-flight crew pairing problem

Computational experiments for the solution of realistic data were carried out on a desktop PC with an Intel Core i3 processor with 8GB RAM. The MATLAB Integer Linear Programming solver is used. Each solution of the 1890-flight crew pairing problem from a real airline that spans one week of operations had solution times between 60 to 90 seconds each.

Results for the traditional model in the form of a crew pairing schedule were obtained. To evaluate these crew pairings, they were run through a Monte Carlo Simulation. Data on historical delays of flights were used to obtain the simulated delays for the flights. The pairings constructed determined the propagation of delays. The results for these are shown in Table 3.

Table 3: Monte Carlo simulation results of schedule constructed from the traditional crew pairing model

Planned Cost	695,805.65
Total Delay	48,364.88 minutes
Flights with delays less than or equal to 15 minutes	608 flights
Flights with 15 minutes < delay ≤ 30 minutes	642 flights
Flights with 30 minutes < delay ≤ 45 minutes	381 flights
Flights with 45 minutes < delay ≤ 60 minutes	159 flights
Flights with delays more than 60 minutes	100 flights

The robust model in this research resolves its multiple objective functions (minimizing cost and maximizing robustness) by using a user-inputted weight. By varying this weight and obtaining different solutions, solutions of different robustness levels and planned costs are obtained. The robust crew pairing model obtains a large number of solutions by increasing the weight. The next solution obtained is more robust than the last one, but also more expensive in planned costs. Each solution is a set of crew pairings constructed by the model that caters to all flights.

A function is needed to evaluate the severity of a flight delay that properly reflects the costs of flight delay minutes to the airline. A function proposed for the customer's propensity to switch airlines from a given airline [4] is used as the basis for this function. The graph for this is shown in Figure 10. The 'S' shape curve of this function is based on the Kano model that defines a three-tier approach to customer satisfaction requirements [17]. The 'S' shape curve shows that a small level of delay is insignificant, the significance of the delay increases once the delay moves past what is tolerable to the customer, and plateaus when delay is very high.

The function to evaluate the severity of a flight delay is shown in expression (24). The variable *h* is the hypothetical cost of delay set by the crew planner, and the variable *d* is the delay of the flight in minutes.

$$h\left(1 + e^{4.5-0.07\left(d^{1.12}\right)}\right)^{-1} \quad (24)$$

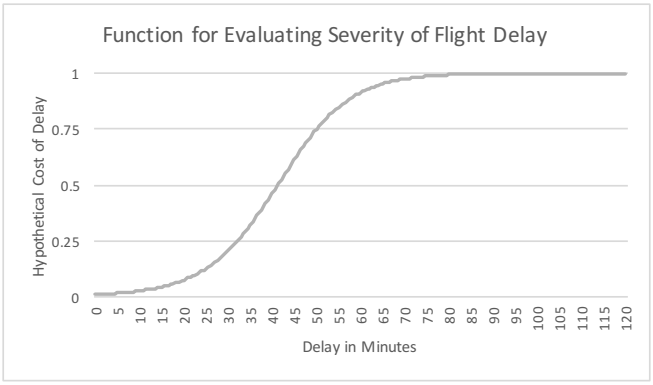

Figure 10: 'S' shape function for evaluating severity of flight delay

After evaluating the flight delays using this function, the delays of each flight in each pairing can be converted to a hypothetical cost of delay. The total cost of each pairing is, the sum of the planned cost of the pairings and their hypothetical costs of delay. Shown in Figure 11 is the graph showing the solutions plotted based on their planned costs and costs of delay. An optimal isocost line is shown to intersect with the optimal solution. This is in comparison to the solution with the minimized planned cost obtained from the traditional model and does not take into account delays.

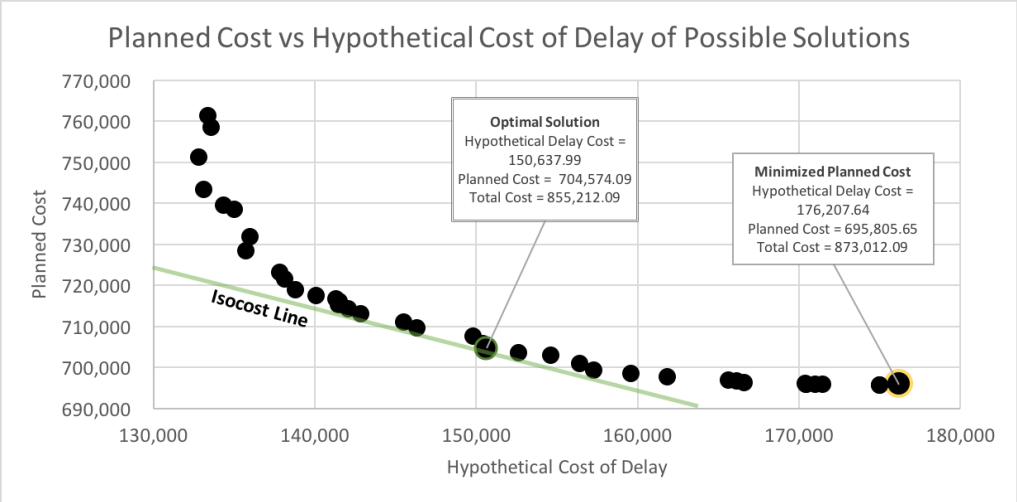

Figure 11: Optimal solution obtained balancing planned cost and cost of delay

The optimal solution from the robust crew pairing model has a higher planned cost, but a lower delay cost. This is compared to the solution for minimizing the planned cost from the traditional model. The planned cost is as minimized as it can be, but the delay cost is higher. Due to not being able to recognize that delays have costs for the airline, most airlines implement the solution with the minimized planned cost. This is from a lack of foresight and initiative to try to minimize delays even before the flight happens. Most airlines simply handle delays after the fact, when the flight has been flown.

Shown in Figure 12 are the results of the Monte Carlo simulations for the optimal robust solution from the robust crew pairing model, and the minimized planned cost solution from the traditional crew pairing model. The number of flights delayed by more than an hour goes from 100 flights to 51 flights when implementing the optimal robust solution. Likewise, the number of flights delayed by 45 minutes to an hour goes from 159 to 133 flights. The flights delayed by 30 to 45 minutes remain relatively unchanged from 381 to 383 flights. The flights delayed by 30 minutes or less increase significantly using the optimal robust solution.

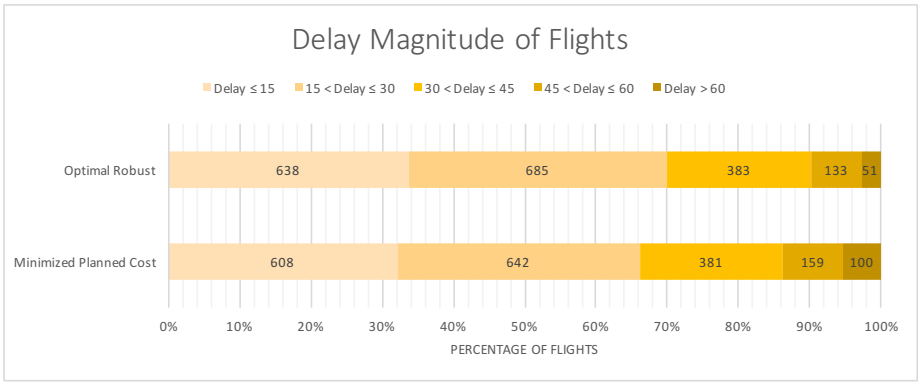

Figure 12: Delay magnitude of flights for optimal robust and minimized planned cost solution

It is clear that the delay performance of the optimal robust solution is better than that of the minimized planned cost solution. Implementing the robust solution results in the planned cost increasing by 1.26%. This increase in planned cost is clearly insignificant compared to the potential improvement in delay performance that can be obtained by using the optimal robust solution.

It should be noted that the cost of delay presented is hypothetical. While literature exists that delays should have a cost based on an 'S' shape curve, the multiplier h in the mathematical function (24) is entirely hypothetical. Research would have to be conducted to determine the proper value for this constant. The value for this constant may vary depending on a number of factors, including the reputation of the airline, the state of competitors of the airline, the geographical location, and the way the airline handles delays. For example, an airline that has prepared meticulous steps for satisfying passengers when delays occur (possibly in the form of discounts or mileage) will have more tangible costs from delay as opposed to just the cost of loss of customer loyalty. An airline that has no competitors in the area will have low costs of delay because customers will have no other airline to choose from, and therefore the loss of loyalty is low. A low-cost carrier that has a reputation for late flights would have lower costs for loss of loyalty because the customers are more particular of obtaining the lowest costs possible, rather than the punctuality of flights. These factors dictate that the proper determination of the cost of delay is important for the robust model presented to be effective. Because the factors for the cost of delay vary greatly depending on the situation, it is important that this cost of delay be established by the airline through research, data analytics of passengers, surveys, analysis of competition, etc., before implementing methods that can deal with robustness.

This also gives greater importance to the use of a Monte Carlo simulation. Because the costs of delay are hypothetical, it is more difficult to evaluate the validity of a numerical cost attributed to the delay of flights. However, being able to conduct simulations and dictate the number of flights delayed by certain magnitudes, as shown in Figure 12, is more tangible and is likely better for airline management to use for making decisions.

6 Conclusions

Delays have been more and more caused, not by natural unforeseen circumstances, but by the inability of airlines to plan for delays caused by the inefficiencies of their own operations. Disruptions that occur locally can propagate delays to connecting flights. The majority of existing literature focuses on recovery from disruptions. Most approaches try to bring a schedule as quick as possible back to its original schedule, which is not cost-effective. Given the data available to modern airlines, it is now possible and even more appropriate to use this data to plan for disruptions as a form or proactive disruption management.

Crew pairing is the most complex among the airline resource planning processes, due to human fatigue constraints and the large amount of crew and flights to consider. Adding a level of complexity by planning for robustness makes the existing formulation of the problem difficult to solve using existing algorithms. An algorithm that takes advantage of the structure of crew pairings was developed that could obtain solutions with no assurance of optimality in tractable time even for large problems. Progressive pairing generation, the solution methodology formulated to solve large crew pairing problems, generates the pairings based on a hierarchy of what is usually easier and more cost effective to implement.

A two-phase robust crew pairing model was developed. The first phase has two objectives – minimization of costs and maximization of robustness – which are resolved by using a user-inputted weight. The second phase finds crew swaps in the model for added flexibility. It was seen that the tradeoff of costs and robustness resulted into solutions that had an asymptotic trend. There is a point wherein the increase in cost when generating a solution is no longer justifiable for its increase in robustness. To resolve the robustness-cost tradeoff, a function is introduced that gives an equivalent monetary cost to a delayed flight. The function follows an 'S' curve, maintaining the characteristic that minor delays are tolerable while major delays are intolerable and costly for the airline. It emphasizes that delay is not linearly related to costs and encourages the model to spread delays, primarily through crew swaps, among different flights rather than allowing it to propagate on one pairing.

The robust model developed is able to create a crew pairing schedule that incorporates robustness such that it performs well under disruptions. The solution of the model contains the flights that belong to the same pairing, the sequencing of the flights, and the crew swaps that could be taken should it be necessary. Progressive pairing generation was used in order to solve the model in tractable time. Each solution, as the user-inputted weight in the model was varied, took within 60 to 90 seconds of computation for a set of 1890 flights. The set of 1890 flights are one week's operations of a major airline.

Moving forward, research on formalizing the cost of flight delays would be important in any robust planning for airline operations. A formalized method on costing of flight delays could be used to provide tangible value to robustness, rather than relying on hypothetical inputs from the crew planner. An integrated approach to airline resource planning with robustness may also be explored. One of the assumptions in this model is that a crew member always has an aircraft available. This is an unrealistic assumption, though it is standard for crew pairing literature. Developing an integrated approach to robust planning of aircraft and crew would remove the need for this assumption.

Another recommendation for future research is to benchmark this methodology using other column generation, or possibly even genetic algorithm techniques. This research benchmarks the methodology proposed for solving big problems against the optimal set partitioning linear programming methodology. It has long been known that this optimal methodology takes too long for large problems. As such, it would be beneficial to benchmark the progressive pairing methodology proposed here to other methodologies in literature that have gained traction in the field.

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Integer programming for home care scheduling with flexible task frequency and controllable processing times

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Abstract Home care scheduling integrates the assignment, sequencing and scheduling of caregivers to household tasks subject to a range of operational constraints and objectives. Due to the increased scale and complexity of client requests, decision support models have become an indispensable tool for management to efficiently deploy available staff. The present paper introduces a time-indexed integer programming formulation for multi-period home care scheduling while considering both flexible task frequency and controllable processing times. Both of these novel characteristics are common in practice, but have never been considered by previous academic models. By using two modeling tools, activity modes and task patterns, these characteristics may be integrated without any assumptions on their cost functions or general structure. Extensive computational experiments are performed to analyze the new formulation's performance on practical problem instances. The results confirm that it is possible to solve realistically-sized instances consisting of 25 caregivers and 100 tasks to optimality within acceptable computation time.

1 Introduction

As home care is steadily becoming the primary source of care for elderly people, there is a high demand which is increasingly difficult for home care organizations to satisfy. Rich decision support models are quickly becoming indispensable for home care organizations in order to effectively manage their available staff. In short, home care scheduling concerns the scheduling and assignment of skilled caregivers to various tasks at different clients' homes. The underlying optimization problem integrates assignment, sequencing and scheduling decisions, subject to a variety of personal and organizational constraints and objectives. Due to its practical relevance and increased impact in recent years, home care scheduling has received considerable attention in the academic literature (Fikar and Hirsch, 2016).

The present paper introduces a new integer programming model for scheduling caregivers in a multi-period setting within which the schedules are constructed for multiple

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consecutive days. The proposed model employs time-indexed decision variables, which are known to result in tight linear programming relaxations for scheduling problems (Van den Akker et al, 2000). Tasks are used as an abstract concept which generalize client care requirements. Each task is associated with skill requirements and must be performed multiple times during the scheduling period. Skill structure is modeled in a general way such that both hierarchical and arbitrary structures may be considered. Various other real-world problem characteristics are included such as availabilities, personal preferences and idle time.

Travel time is an objective commonly considered by academic models for home (health) care scheduling (Akjiratikarl et al, 2007; Liu et al, 2014; Maya Duque et al, 2015). The proposed model, however, does not explicitly minimize travel time for a number of reasons. Firstly, home care tasks, such as housekeeping or accompanying a client to a social activity, are typically very time-consuming. Consequently, caregivers visit a small number of clients per day, thereby limiting the number of times they must travel between clients. Secondly, within real-world problem instances, clients are often clustered in, for example, municipalities. The distance between clients within a cluster is typically relatively small compared to the distance between clients in different clusters. The proposed model thus does not explicitly minimize travel time but instead forbids caregivers to visit clients associated with different clusters on the same day.

Scheduling flexibility is emphasized by considering flexible task frequency (how many times a task is scheduled) and controllable processing times (the duration of each scheduled task). For both of these properties, general cost functions may be defined which model the cost of scheduling a task fewer times than required or scheduling a task for less time than required. When considered independently, task rejection and controllable processing times are known to make polynomially solvable machine scheduling problems NP-hard (Shabtay and Steiner, 2007; Shabtay et al, 2013). While these types of flexibility are common practice in home care organizations, there are no academic models integrating these two properties.

The majority of studies throughout the academic literature address the daily scheduling problem in which only a single period is considered (Rasmussen et al, 2012; Yuan et al, 2015; Braekers et al, 2016). Only a few authors solve the problem in a multi-period setting. However, considering multiple days is essential as home care scheduling concerns human resources. Consequently, not only capacity is important, but also, for example, sequence, frequency, continuity, assignment spreading and variation of assignments. Begur et al (1997) describe an early implementation of a decision support system for a real-world home care scheduling problem. The problem is addressed by heuristically solving a series of daily scheduling problems with varying visiting patterns. Such patterns are also used by Cappanera and Scutellà (2014) in an integer programming model for scheduling visits to palliative patients. Trautsamwieser and Hirsch (2014) present a branch-price-and-cut algorithm for a weekly scheduling problem. Their model includes a number of employee scheduling constraints concerning breaks and rest time which allow for flexible working hours. Finally, Nickel et al (2012) address the planning of home care services at different time horizons. They present a metaheuristic approach based on constraint programming for both mid- and short-term planning. There are no approaches in the state of the art academic literature which consider flexibility both in terms of frequency and duration.

The remainder of this paper is organized as follows. Section 2 presents a formal definition of the considered home care scheduling problem. Section 3 introduces the proposed time-indexed integer programming formulation along with detailed examples

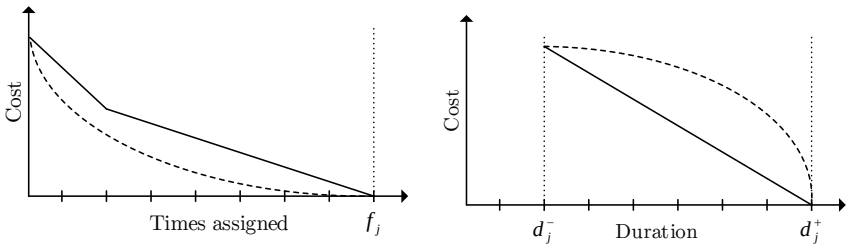
of the novel modeling tools which it includes. Section 4 analyzes a series of computational experiments to gain various insights concerning the model's performance. Finally, Section 5 concludes the paper and identifies areas for future research.

2 Problem definition

Let $D = \{1, ..., |D|\}$ be a set of consecutive days which define the scheduling period and let $E = \{1, ..., |E|\}$ be the set of caregivers. The availability of caregiver $e \in E$ on day $d \in D$ is defined as a hard time window $[b^-_{ed}, b^+_{ed})$ with $0 \leq b^-_{ed} < b^+_{ed}$. The tasks to be scheduled are denoted by $J = \{1, ..., |J|\}$. For each task j, let $E_j \subseteq E$ be the subset of caregivers who meet the skill requirements. The time window in which task j may be scheduled on day d is denoted by $[b^-_{jd}, b^+_{jd})$ with $0 \leq b^-_{jd} < b^+_{jd}$. Let a_{je} be the cost of assigning task j to caregiver e which represents several soft preferences such as pet allergies, gender or language proficiency. Travel restrictions for caregivers are defined by the set $\tilde{J}$, which contains pairs of tasks (j, j') which cannot be assigned to the same caregiver on the same day. Typically, $(j, j') \in \tilde{J}$ if the travel time between the locations of j and j' exceeds some given bound.

Each task $j \in J$ must be scheduled f_j times during the scheduling period. Note that tasks cannot be scheduled more than this frequency. A cost function $p^f_j : [0, f^+_j] \to \mathbb{R}$ defines the cost when task j is scheduled x times. The precise definition of $p^f_j(x)$ depends on the application context where, for example, scheduling a task at least once may be of prime importance, whereas reaching the desired frequency is not critical, or vice versa. Figure 1a illustrates examples of piecewise linear and non-linear cost functions which may be used to model frequency cost. Note that $p^f_j(x)$ is not assumed to be monotonically decreasing but, in practice, often will be.

Each time task $j \in J$ is scheduled, its duration should be between d^-_j and d^+_j, with both bounds considered hard constraints. Similar to the task frequency, a cost function $p^d_j : [d^-_j, d^+_j] \to \mathbb{R}$ defines the cost of scheduling task j with duration x. Figure 1b shows examples of $p^d_j(x)$ whose definition is, again, strongly context-dependent and typically monotonically decreasing.



(a) Frequency cost functions (b) Duration cost functions

Fig. 1: Examples of cost functions for flexible frequency and controllable processing times.

The objective function to be minimized is a weighted sum of five objectives: (i) frequency cost as calculated by $p_j^f(x)$, (ii) duration cost as calculated by $p_j^d(x)$, (iii) the sum of assignment costs a_{je}, (iv) caregiver idle time and (v) task spreading cost. Each of these objectives has a weight $\omega_1, \omega_2, ..., \omega_5$ associated with it.

Task spreading ensures that the days on which a task is scheduled are distributed evenly throughout the scheduling period. This is an important objective in practice, where visiting the same client on two consecutive days is generally undesirable. The cost related to task spreading is calculated as the absolute value of the difference between the ideal task spread and the actual number of days between two scheduled tasks. The ideal spread is defined as the quotient of the length of the scheduling period and the number of times the task is scheduled, rounded down to the nearest integer.

Example 1 Consider a task $j_1 \in J$ which is scheduled three times throughout a one-week scheduling period, as illustrated in Figure 2. The ideal spread is calculated as $\lfloor \frac{7}{3} \rfloor = 2$, which means that ideally there will be exactly two days between each time j_1 is scheduled. Examining the solution in Figure 2 shows that there are two days between the first and second occurrence of j_1 but only one day between the second and third occurrence. The total spreading cost is therefore $|2-2| + |2-1| = 1$.

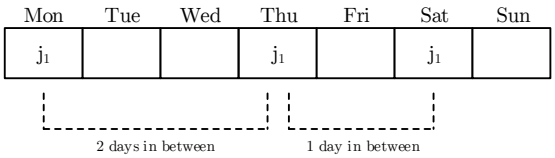


Fig. 2: Example of task spreading

Note that, if j_1 was scheduled, for example, on Monday and Tuesday, the number of days in between these two occurrences would be zero and the task spreading cost would be $| \lfloor \frac{7}{2} \rfloor - 0| = 3$.

3 Time-indexed integer linear programming formulation

A time-indexed integer programming formulation is proposed in which scheduling period D is discretized into time-slots based on some given granularity ϕ. Let $T = \{1, ..., |T|\}$ be the set of time-slots representing the complete scheduling period. The subset of time-slots associated with day d is denoted as T_d. The problem's two novel complex elements, controllable processing times and flexible task frequency, are modeled as *activity modes* and *task patterns*, respectively.

Activity modes are commonly used in resource-constrained project scheduling problems (Wauters et al, 2016). Let $M_j = \{1, ..., |M|\}$ be the set of feasible modes for task j, d_{jm} the duration of task j in mode $m \in M_j$ and $p_{jm} = p_j^d(d_{jm})$ the cost for using mode $m \in M_j$ (Brucker et al, 1999).

Example 2 Consider a task $j_1 \in J$ with $d_{j_1}^- = 60$ mins and $d_{j_1}^+ = 240$ mins. Given a time granularity $\phi = 15$ mins, task j_1 has 13 activity modes in the set $M_{j_1} = \{1, ..., 13\}$. The duration of each mode $m \in M_{j_1}$ is calculated as $d_{j_1 m} = d_{j_1}^- + (m-1)\phi$, such that $d_{j_1 1} = 60$ mins, $d_{j_1 2} = 75$ mins, $d_{j_1 3} = 90$ mins, ..., $d_{j_1 13} = 240$ mins.

Task patterns are binary vectors used to represent the days on which a task is scheduled in the scheduling period. Let $R = \{1, ..., |R|\}$ be the set of all possible patterns with $|R| = 2^{|D|}$. A binary value h_{rd} equals one if day $d \in D$ is an element of pattern r. The number of days in pattern r is denoted by q_r. Note that R also includes an empty pattern which has $q_r = 0$. Let $R_j \subseteq R$ be the (sub)set of patterns which may be selected for task j such that $q_r \leq f_j^+, \forall r \in R_j$. The cost n_{jr} associated with selecting pattern r for task j is calculated as the weighted sum of $p_j^f(q_r)$ and the spread of days in r. Finally, let $k_{rr'}$ be the number of days for which patterns r and r' overlap.

Example 3 For $|D| = 7$, consider the pattern $(1, 0, 0, 1, 1, 0, 0)$ in which a task is scheduled on the first, fourth and fifth day of the scheduling period. When selecting this pattern for task $j_1 \in J$ with $f_{j_1}^+ = 4$ and weights $\omega_1 = 5$ and $\omega_5 = 2$, the unweighted pattern assignment cost is calculated as $n_{j_1 r} = 5(4-3)+2\left((\lfloor \frac{7}{3} \rfloor - 2) + (\lfloor \frac{7}{3} \rfloor - 0)\right) = 9$. The first term calculates the frequency cost, while the second and third terms attribute the task spread to the pattern cost.

The time-indexed formulation uses a number of additional sets. Let S_{jmed} be the set of feasible start time-slots for task j in mode m if assigned to caregiver e on day d, which considers the time windows of both the task and caregiver. Let $S_{jme} = \bigcup_{d \in D} S_{jmed}$ be the union of feasible start time-slots over all days in the scheduling period. Note that $S_{jme} = \emptyset$ if caregiver e is unqualified for task j. Finally, the set O_{jmet} contains all feasible start time-slots for task j in mode m that overlap with time-slot t when assigned to caregiver e.

The proposed formulation uses the following decision variables:

$$x_{jmet} = \begin{cases} 1 & \text{if task } j \text{ is assigned to caregiver } e \text{ in mode } m \text{ starting at time-slot } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_{jr} = \begin{cases} 1 & \text{if pattern } r \text{ is selected for task } j \\ 0 & \text{otherwise} \end{cases}$$

z_{ed}^- = start time of the first task of caregiver e on day d
z_{ed}^+ = end time of the last task of caregiver e on day d
$idle_{ed}$ = total idle time for caregiver e on day d

The objective function (1) is a weighted sum of (i) preference costs, (ii) deviation from preferred task duration, (iii) pattern assignment cost and (iv) idle time. Note that weights ω_1, ω_2 and ω_5 do not explicitly appear in the objective function as they are captured by mode and pattern costs.

$$\text{minimize} \sum_{j \in J} \sum_{m \in M_j} \sum_{e \in E} \sum_{t \in S_{jme}} (\omega_3 a_{je} + p_{jm}) x_{jmet} + \sum_{j \in J} \sum_{r \in R_j} n_{jr} y_{jr} + \omega_4 \sum_{e \in E} \sum_{d \in D} idle_{ed} \tag{1}$$

Constraints (2) ensure that each task is scheduled at most once per day, and that at most one mode and one caregiver are selected. Constraints (3) forbid scheduled tasks to overlap.

$$\sum_{m\in M_j}\sum_{e\in E}\sum_{t\in S_{jmed}} x_{jmet} \leq 1 \qquad \forall j \in J, d \in D \qquad (2)$$

$$\sum_{j\in J}\sum_{m\in M_j}\sum_{t'\in O_{jmet}} x_{jmet'} \leq 1 \qquad \forall e \in E, d \in D, t \in T_d \qquad (3)$$

Constraints (4) update the start time variable z^-_{ed} based on the first scheduled task for each caregiver e on each day d. Constraints (5) update end time variable z^+_{ed} based on the last scheduled task for each caregiver e on each day d. Constraints (6) calculate the total idle time per caregiver per day as the difference between the end time and start time minus the total duration of scheduled tasks.

$$\sum_{m\in M_j}\sum_{t\in S_{jmed}} t\ x_{jmet} \geq z^-_{ed} - b^+_{ed}\left(1 - \sum_{m\in M_j}\sum_{t\in S_{jmed}} x_{jmet}\right) \quad \forall j \in J, e \in E, d \in D \qquad (4)$$

$$\sum_{m\in M_j}\sum_{t\in S_{jmed}} (t + d_{jm}) x_{jmet} \leq z^+_{ed} \qquad \forall j \in J, e \in E, d \in D \qquad (5)$$

$$z^+_{ed} - z^-_{ed} - \sum_{j\in J}\sum_{m\in M_j}\sum_{t\in S_{jmed}} d_{jm} x_{jmet} = idle_{ed} \qquad \forall e \in E, d \in D \qquad (6)$$

Constraints (7) select one pattern per task. Constraints (8) ensure that the number of task start times equals the number of days in the selected pattern. Constraints (9) link the days of the start time variables to the days of the selected pattern.

$$\sum_{r\in R_j} y_{jr} = 1 \qquad \forall j \in J \qquad (7)$$

$$\sum_{e\in E}\sum_{m\in M_j}\sum_{t\in S_{jme}} x_{jmet} = \sum_{r\in R_j} q_r\ y_{jr} \qquad \forall j \in J \qquad (8)$$

$$\sum_{e\in E}\sum_{d\in D}\sum_{m\in M_j}\sum_{t\in S_{jmed}} h_{rd}\ x_{jmet} \geq \sum_{r'\in R_j} k_{rr'}\ y_{jr'} \qquad \forall j \in J, r \in R_j \qquad (9)$$

Constraints (10) restrict forbidden task combinations to be assigned to the same caregiver on the same day.

$$\sum_{m\in M_j}\sum_{t\in S_{jmed}} x_{jmet} + \sum_{m\in M_{j'}}\sum_{t\in S_{j'med}} x_{j'met} \leq 1 \quad \forall e \in E, d \in D, (j, j') \in \tilde{J} \qquad (10)$$

Finally, Constraints (11)-(13) define the bounds on the decision variables.

$$x_{jmet} \in \{0, 1\} \qquad \forall j \in J, m \in M_j, e \in E, t \in S_{jme} \qquad (11)$$

$$y_{jr} \in \{0, 1\} \qquad \forall j \in J, r \in R_j \qquad (12)$$

$$z^-_{ed},\ z^+_{ed},\ idle_{ed} \geq 0 \qquad \forall j \in J, e \in E, d \in D \qquad (13)$$

4 Computational experiments

4.1 Data set and experimental setup

Given the lack of publicly-available benchmark instances for this problem, an instance generator was developed which allows to generate problem instances in a controlled manner. Several real-world problem characteristics which were observed in practice are included in the instance generator in order to obtain a realistic data set. Instances were generated with 5, 15 and 25 caregivers. All instances consider a scheduling period of one week in which daily caregiver availability was randomly generated to be between 9am and 5pm, with deviations of up to 30 minutes permitted. The skill level of caregivers was varied between 0.5 and 1.0, indicating the percentage of caregivers which are skilled for each task. The number of tasks in an instance is determined by two interacting parameters: task length and staffing ratio. Three task length categories were considered: long (2h30m to 5h), medium-length (1h15m to 2h30m) and short (30m to 1h) tasks. The staffing ratio is calculated as the fraction of total task demand over total caregiver availability. If this value is less than one there is overstaffing and it is possible to completely satisfy demand, otherwise there is understaffing which inevitably results in unassigned tasks. The data set consists of two instance classes which have staffing ratios of 1.1 (class A) and 1.6 (class B), thereby representing realistic scenarios. A higher staffing ratio is obtained by increasing the task frequency while keeping the task durations fixed. The weights in the objective function were set to $\omega_1 = 10000$, $\omega_2 = \omega_3 = \omega_4 = \omega_5 = 1$ to reflect priorities used in practice. All instances are publicly available at `https://people.cs.kuleuven.be/~pieter.smet/homecare.html` to encourage future research. Table 1 details an overview of the two instance classes.

All experiments were conducted on a Dell Poweredge T620, 2x Intel Xeon E5-2670 with 128GB RAM. Gurobi 7.0.2 was used as an integer programming solver, configured to use eight threads with a time limit of ten hours. The time granularity was set to $\phi = 15$ mins resulting in 672 time slots.

4.2 Computational results

Table 2 shows computational results for the instances in class A. Details concerning both the linear programming (LP) relaxation and integer linear programming (ILP) formulation are presented for each individual instance. If no feasible solution was obtained within the allowed time limit, a dash (-) is shown. Optimal solutions are highlighted in bold.

The results show that for instances with long tasks, optimal solutions are found within reasonable calculation time. Even for the more challenging realistically-sized instances with 25 caregivers, optimal solutions are found within 40 minutes. Note that for these instances, branching proved unnecessary and the optimal solutions were obtained by solving the root node. Instances with medium-length tasks were solved (close) to optimality, with only a single instance showing a significant gap of 3.2%. Finally, instances with short tasks presented the most challenging scenarios. With a limited number of caregivers (5-15), these instances could be solved to optimality or within a small gap of 0.6%. However, when considering 25 caregivers, feasible solutions were not obtained consistently. Overall, there is no clear trend regarding the influence of caregiver skill level on required calculation time.

Instance	No. of caregivers	No. of tasks	Skill level	Average frequency	Avg. no. of patterns	Avg. min. duration	Avg. pref. duration	Avg. no. of modes	Staffing ratio
A01	5	20	0.5	3.1	67.5	2h31m	4h59m	10.9	1.1
A02	5	20	0.7	3.1	67.5	2h31m	4h59m	10.9	1.1
A03	5	20	1.0	3.1	67.5	2h31m	4h59m	10.9	1.1
A04	5	30	0.5	4.0	94.3	1h15m	2h34m	6.3	1.1
A05	5	30	0.7	4.0	94.3	1h15m	2h34m	6.3	1.1
A06	5	30	1.0	4.0	94.3	1h15m	2h34m	6.3	1.1
A07	5	50	0.5	5.0	115.8	0h39m	1h13m	3.3	1.1
A08	5	50	0.7	5.0	115.8	0h39m	1h13m	3.3	1.1
A09	5	50	1.0	5.0	115.8	0h39m	1h13m	3.3	1.1
A10	15	60	0.5	3.1	66.9	2h28m	5h00m	11.1	1.1
A11	15	60	0.7	3.1	66.9	2h28m	5h00m	11.1	1.1
A12	15	60	1.0	3.1	66.9	2h28m	5h00m	11.1	1.1
A13	15	90	0.5	4.1	97.1	1h12m	2h30m	6.2	1.1
A14	15	90	0.7	4.1	97.1	1h12m	2h30m	6.2	1.1
A15	15	90	1.0	4.1	97.1	1h12m	2h30m	6.2	1.1
A16	15	150	0.5	5.3	119.2	0h37m	1h09m	3.2	1.1
A17	15	150	0.7	5.3	119.2	0h37m	1h09m	3.2	1.1
A18	15	150	1.0	5.3	119.2	0h37m	1h09m	3.2	1.1
A19	25	100	0.5	3.0	65.4	2h28m	5h03m	11.3	1.1
A20	25	100	0.7	3.0	65.4	2h28m	5h03m	11.3	1.1
A21	25	100	1.0	3.0	65.4	2h28m	5h03m	11.3	1.1
A22	25	150	0.5	4.2	98.7	1h16m	2h28m	5.8	1.1
A23	25	150	0.7	4.2	98.7	1h16m	2h28m	5.8	1.1
A24	25	150	1.0	4.2	98.7	1h16m	2h28m	5.8	1.1
A25	25	250	0.5	5.2	117.8	0h36m	1h10m	3.3	1.1
A26	25	250	0.7	5.2	117.8	0h36m	1h10m	3.3	1.1
A27	25	250	1.0	5.2	117.8	0h36m	1h10m	3.3	1.1
B01	5	20	0.5	4.0	99.0	2h30m	5h30m	13.0	1.6
B02	5	20	0.7	4.0	99.0	2h30m	5h30m	13.0	1.6
B03	5	20	1.0	4.0	99.0	2h30m	5h30m	13.0	1.6
B04	5	30	0.5	5.0	120.0	1h14m	2h59m	8.0	1.6
B05	5	30	0.7	5.0	120.0	1h14m	2h59m	8.0	1.6
B06	5	30	1.0	5.0	120.0	1h14m	2h59m	8.0	1.6
B07	5	50	0.5	6.0	127.0	0h37m	1h30m	4.5	1.6
B08	5	50	0.7	6.0	127.0	0h37m	1h30m	4.5	1.6
B09	5	50	1.0	6.0	127.0	0h37m	1h30m	4.5	1.6
B10	15	60	0.5	4.0	99.0	2h30m	5h30m	13.0	1.6
B11	15	60	0.7	4.0	99.0	2h30m	5h30m	13.0	1.6
B12	15	60	1.0	4.0	99.0	2h30m	5h30m	13.0	1.6
B13	15	90	0.5	5.0	120.0	1h14m	2h59m	8.0	1.6
B14	15	90	0.7	5.0	120.0	1h14m	2h59m	8.0	1.6
B15	15	90	1.0	5.0	120.0	1h14m	2h59m	8.0	1.6
B16	15	150	0.5	6.0	126.8	0h38m	1h29m	4.4	1.6
B17	15	150	0.7	6.0	126.8	0h38m	1h29m	4.4	1.6
B18	15	150	1.0	6.0	126.8	0h38m	1h29m	4.4	1.6
B19	25	100	0.5	4.0	99.0	2h28m	5h30m	13.1	1.6
B20	25	100	0.7	4.0	99.0	2h28m	5h30m	13.1	1.6
B21	25	100	1.0	4.0	99.0	2h28m	5h30m	13.1	1.6
B22	25	150	0.5	5.0	120.0	1h13m	2h59m	8.0	1.6
B23	25	150	0.7	5.0	120.0	1h13m	2h59m	8.0	1.6
B24	25	150	1.0	5.0	120.0	1h13m	2h59m	8.0	1.6
B25	25	250	0.5	6.0	126.9	0h37m	1h29m	4.5	1.6
B26	25	250	0.7	6.0	126.9	0h37m	1h29m	4.5	1.6
B27	25	250	1.0	6.0	126.9	0h37m	1h29m	4.5	1.6

Table 1: Data set characteristics

Table 3 presents computational results for the instances in class B with understaffing. The same computational details as before are presented.

The results for instances in class B follow the same general trend as those for class A: instances with long tasks are consistently solved to optimality, while this becomes increasingly difficult when short and medium-length tasks are present. The main difference, however, is that all instances in class B could be solved to optimality within the time limit. Branching was never required for obtaining an optimal solution, thereby confirming the strong LP relaxation of the time-indexed formulation. This is

Instance	LP relaxation		ILP formulation				
	Objective	Time (s)	LB	UB	Gap	Time (s)	Nodes
A01	3056.0	6	**3056.0**	**3056**	**0.0%**	**20**	**0**
A02	2835.0	7	**2836.0**	**2836**	**0.0%**	**27**	**0**
A03	2844.0	8	**2844.0**	**2844**	**0.0%**	**28**	**0**
A04	1876.9	12	**1877.8**	**1878**	**0.0%**	**2009**	**6315**
A05	1776.5	15	**1777.6**	**1778**	**0.0%**	**36000**	**233856**
A06	1766.1	18	**1766.4**	**1767**	**0.0%**	**36000**	**1256807**
A07	2349.2	14	**2349.2**	**2350**	**0.0%**	**36000**	**3015872**
A08	2009.2	20	**2009.2**	**2010**	**0.0%**	**36000**	**1117328**
A09	1969.2	27	**1969.2**	**1970**	**0.0%**	**36000**	**337705**
A10	8287.0	60	**8287.0**	**8287**	**0.0%**	**257**	**0**
A11	8286.0	121	**8286.0**	**8286**	**0.0%**	**461**	**0**
A12	8286.0	186	**8286.0**	**8286**	**0.0%**	**504**	**0**
A13	5147.6	274	5147.7	5183	0.7%	36000	1499
A14	5147.3	303	**5147.3**	**5148**	**0.0%**	**36000**	**4188**
A15	5147.0	339	**5147.1**	**5148**	**0.0%**	**36000**	**1083**
A16	5423.8	1784	5423.8	5425	0.0%	36000	62
A17	5423.8	2425	**5423.8**	**5424**	**0.0%**	**22483**	**0**
A18	5423.8	4290	**5423.8**	**5454**	**0.6%**	**36000**	**0**
A19	14366.0	226	**14366.0**	**14366**	**0.0%**	**752**	**0**
A20	14366.0	530	**14366.0**	**14366**	**0.0%**	**1572**	**0**
A21	14366.0	393	**14366.0**	**14366**	**0.0%**	**2484**	**0**
A22	8576.0	8750	8576.0	8863	3.2%	36000	0
A23	8576.0	14571	8576.0	8611	0.4%	36000	0
A24	8576.0	3450	**8576.0**	**8576**	**0.0%**	**15728**	**0**
A25	9073.8	14761	9073.8	9164	1.0%	36000	0
A26	9013.8	9411	9013.8	-	-	36000	0
A27	0.0	36000	0.0	-	-	36000	0

Table 2: Computational results for the instances in class A

also reflected in the required calculation time, which is, in general, significantly shorter compared to the time required for class A.

The influence of caregiver skill level on required calculation time is clearer for the instances in class B. As demonstrated in Figure 3, the general trend is that higher skill levels result in increased calculation time due to increased size of the ILP formulation. Furthermore, this comparison clearly shows the influence of task length on the solver's performance. For instances with long tasks, optimal solutions are found in less time than for instances with medium-length tasks, which in turn are solved faster than instances with short tasks.

5 Conclusions and future research

The present paper introduced an integer linear programming formulation for scheduling and assigning home care tasks to skilled caregivers in a multi-period setting. The proposed model includes controllable processing times and flexible task frequency, two problem properties which are common in practice but which have never before been considered by state of the art academic models. Incorporating these generalizations poses significant challenges in terms of both modeling and solving the problem.

A time-indexed formulation was proposed which incorporates these two novel features by modeling them as activity modes and task patterns. Activity modes are used

Instance	LP relaxation		ILP formulation				
	Objective	Time (s)	LB	UB	Gap	Time (s)	Nodes
B01	9855.0	13	**9855.0**	**9855**	**0.0%**	**35**	**0**
B02	9575.0	15	**9575.0**	**9575**	**0.0%**	**41**	**0**
B03	9575.0	25	**9575.0**	**9575**	**0.0%**	**57**	**0**
B04	10310.0	23	**10310.0**	**10310**	**0.0%**	**100**	**0**
B05	10210.0	42	**10210.0**	**10210**	**0.0%**	**127**	**0**
B06	10160.0	47	**10160.0**	**10160**	**0.0%**	**105**	**0**
B07	10975.0	31	**10975.0**	**10975**	**0.0%**	**90**	**0**
B08	10565.0	42	**10565.0**	**10565**	**0.0%**	**125**	**0**
B09	10505.0	54	**10505.0**	**10505**	**0.0%**	**151**	**0**
B10	28725.0	145	**28725.0**	**28725**	**0.0%**	**592**	**0**
B11	28725.0	264	**28725.0**	**28725**	**0.0%**	**1371**	**0**
B12	28725.0	185	**28725.0**	**28725**	**0.0%**	**1205**	**0**
B13	30450.0	631	**30450.0**	**30450**	**0.0%**	**2705**	**0**
B14	30450.0	742	**30450.0**	**30452**	**0.0%**	**7177**	**0**
B15	30450.0	3294	**30450.0**	**30450**	**0.0%**	**18772**	**0**
B16	30682.0	1954	**30682.0**	**30682**	**0.0%**	**2818**	**0**
B17	30682.0	1249	**30682.0**	**30682**	**0.0%**	**4363**	**0**
B18	30682.0	2278	**30682.0**	**30682**	**0.0%**	**8133**	**0**
B19	48125.0	4863	**48125.0**	**48125**	**0.0%**	**3488**	**0**
B20	48125.0	714	**48125.0**	**48125**	**0.0%**	**5400**	**0**
B21	48125.0	954	**48125.0**	**48125**	**0.0%**	**13939**	**0**
B22	50655.0	9981	**50655.0**	**50655**	**0.0%**	**32522**	**0**
B23	50655.0	4068	50655.0	-	-	36000	-
B24	50655.0	20812	50655.0	-	-	36000	-
B25	51289.0	36001	51349.0	51409	0.1%	36000	0
B26	51289.0	10553	51289.0	-	-	36000	-
B27	51289.0	21397	-	-	-	36000	-

Table 3: Computational results for the instances in class B

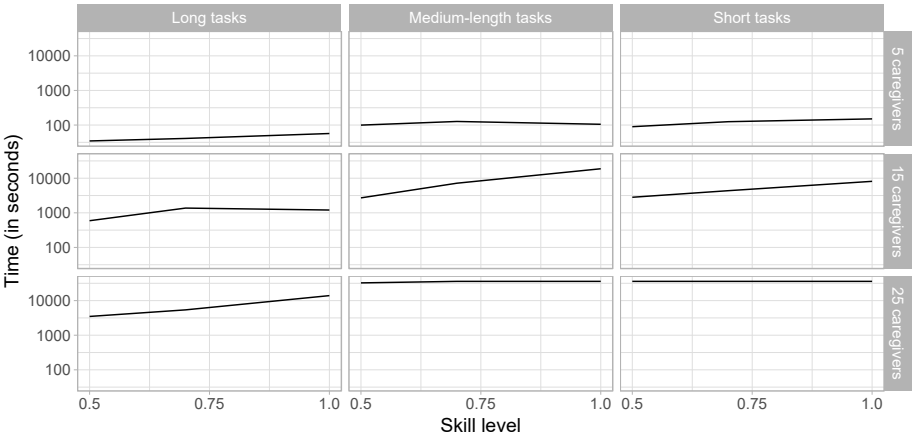


Fig. 3: Impact of skill level on calculation time for instances in class B

to model the different processing times for which a task may be scheduled. This enables the cost function to be of any form, however, typically, scheduling tasks with a shorter duration results in increased costs. Task patterns are the second modeling tool included in the proposed formulation. By selecting a pattern for a task, the days on which it is scheduled are determined, thereby enabling flexible modeling of task frequency and spreading throughout the scheduling period.

A series of computational experiments demonstrated that the proposed model could be solved to optimality for problem instances of realistic size with 25 caregivers and 100 tasks. Further analysis of the results revealed how the formulation's root node relaxation is very tight, often resulting in the optimal solution and thus avoiding any branching to obtain integer solutions. The benchmark data set used in this computational study has been made publicly available to stimulate further research on this challenging problem.

The present paper addressed the static, offline version of the problem. However, in practice, unforeseen events constantly affect the current planning. For example, caregivers become unavailable, task frequency increases or tasks are canceled. Future research should turn its attention towards addressing the re-scheduling problem in home care. This optimization problem imposes an additional restriction concerning available computation time given how new solutions are typically expected within a few minutes.

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Optimizing Production Schedule with Energy Consumption and Demand Charges in Parallel Machine Setting

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Abstract Environmental sustainability concerns, along with the growing need for electricity and associated costs, make energy-cost reduction an inevitable decision-making criterion in production scheduling. In this research, we study the problem of production scheduling on non-identical parallel machines with machine-dependent processing times and known job release dates to minimize total completion time and energy costs. The energy costs in this study include demand and consumption charges. We present a mixed-integer nonlinear model to formulate the problem. The model is then linearized and its performance is tested through numerical experiments.

1 Introduction

This paper proposes a new energy-aware parallel-machine production scheduling model in order to minimize total production completion time, energy consumption costs and peak power charges. The industrial sector uses 266 quadrillions BTU of energy, which accounts for 51% of total energy consumption in the world[1]. The breakdown of global energy consumption data reveals that 22% of the total amount of energy used in the industrial sector is electrical energy [1]. In the past 50 years, industrial electricity consumption has doubled [2]. In addition, the US Energy Information Administration (EIA) reports that the price of electricity is expected to

[1] https://www.eia.gov/tools/faqs/faq.cfm?id=447&t=1

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increase by 18% by 2040[2]. Currently, the cost of electricity for manufacturing in the United States exceeds 100 billion dollars [2], and this number will continue to increase the in future.

The surge in energy prices, along with the scarcity of natural sources, the growth of public awareness of environmental concerns, and the establishment of sustainability-based standards magnify the necessity of incorporating energy consumption and associated costs into planning and scheduling decisions at manufacturing facilities. The increase in energy demand causes difficulty for electrical energy providers, who must keep up with demand, which is particularly difficult during peak-demand periods. Time-of-Use (TOU) tariffs implemented by utility companies aim to shift demand from the expensive peak periods to less-expensive non-peak hours in order to flatten the load curve and decrease the deficit risks in supply.

In general, electricity charges can be categorized into two types: consumption charges and demand charges. Consumption charges are calculated according to the total amount of electrical energy consumed by a company during a given period, based on kilowatt-hour, and may vary throughout the day to motivate a shift of consumption away from peak hours. Demand charges try to address the overhead expenses that utility companies bear to provide the service. This charge is based on the highest level of power demanded over a given period of time during the billing period and is usually calculated as the highest "average fifteen-minute demand" for a month. Energy demand is measured in kilowatts (kW) and often represents a significant percentage of charges on utility bills for the industrial user.

Most of the existing research on energy-aware job scheduling does not differentiate between these two types of energy costs. In addition, in the majority of energy-aware job scheduling studies, the impact of various machine operating modes on decision making output is not considered. In a typical manufacturing system, the machines may be running idle for a significant amount of time waiting for the next job to arrive and be processed. One study showed that in a machining process, 85% of total energy consumption is used when the machine is idling, and only 15% is applied to the actual machining process [3]. The idle energy is used to run the auxiliary components. Therefore, it is critical to study the impact of various operating states on the production schedule, energy requirements, and cost planning. In the past few years, the number of studies investigating the energy-aware production scheduling has increased significantly. A literature survey of studies on energy efficiency in manufacturing companies is provided by [4]. This survey presents a breakdown of studies based on energy coverage (production system, internal and external conversion system), energy supply, energy demand (processing and non-processing energy demand), objective criteria (monetary, non-monetary), the system of objectives (multi/single objectives), the manufacturing model (single machine, parallel machines, flow shop, job shop/project scheduling, or hoist scheduling), the model type (linear, mixed integer linear, mixed integer quadratic constrained, mixed integer non-linear programing, queuing theory and simulation, and other analytical models such as Markov decision model), and solution approach (heuristic, exact, standard solver). To integrate energy concerns into classic scheduling problems, [5] investigated a single machine problem to minimize total energy consumption and maximum tardiness, with the possibility of machine shut-down between consecutive jobs following the break-even period. They considered only processing and idle energy consumption in their model. A bi-objective optimization problem to minimize weighted tardiness and non-process (idle and switch) energy consumption in a job-shop setting is proposed in [6]. Their model also allows for switching off a machine if the idle time is long enough, considering a breakeven time, and they solved the problem using Genetic Algorithm (GA). In another study, a job-shop scheduling problem with machine speed scaling to minimize makespan and energy consumption using GA was proposed by [7]. A job-shop problem with energy threshold and makespan minimization was investigated by [8] using a mixed integer linear model. They considered extra energy consumption at the beginning of the operation, and energy consumption was divided into "peak" and "processing" categories. An energy-aware scheduling model with tool selection and operation sequencing was introduced by [9]. Their bi-objective model minimized total energy consumption (idle, setup, and process

[2] http://www.eia.gov/forecasts/aeo/pdf/0383(2015).pdf

energy) and makespan in a flexible job-shop system. To incorporate TOU policy on energy aware scheduling, [10] minimized total electricity cost and number of machines based on TOU pricing in a uniform parallel-machine problem. In another study, [11] performed a job-machine assignment and scheduling in an unrelated parallel machine setting in order to minimize total energy costs according to TOU policy. In 2016, [12] minimized total energy consumption using TOU via job scheduling for a single machine problem.

In the existing research studies on energy-aware scheduling problem, the concurrent integration of operating mode-based energy consumption, TOU policies, and peak power demand is not well investigated in a parallel machine environment. The main contribution of this paper is to propose a new comprehensive framework to minimize total completion time, as well as time-dependent energy consumption and peak power charges simultaneously in a non-homogenous parallel-machine manufacturing system.

The remainder of this paper is organized as follows: Section 2 introduces the underlying assumptions of the model and presents the mathematical model. An illustration of the problem is presented via a case study in Section 3. Section 4 presents our numerical experiments as well as the results. Our conclusions are discussed in Section 5.

2 Problem Definition and Mathematical Modeling

This section describes the mathematical formulation proposed for a parallel machine scheduling problem where the total completion time of jobs, energy consumption, and power demand charges are minimized through determining the optimum sequence of jobs, job-machine assignment, and machine operating schedule. The proposed mixed-integer nonlinear programming (MINLP) model is built on the following underlying assumptions:

- Job processing times are known and the processing is non-preemptive.
- The machines are not identical, i.e., each machine has its own energy profile, and job processing times are machine-dependent. In other words, the processing time of a given jobs might vary on different machines.
- Machine energy consumption varies during different modes (states).
- Only one job can be processed on a given machine in each period.
- If there is no job to process on a machine in any given period, the machine will be idle and consuming idle energy. Idle mode is a very low-energy consuming mode.
- At the beginning of the scheduling horizon, the machines are off and might be turned on in an anticipation of an arriving job. The first job might arrive at the current period, or any other upcoming periods.
- The time to turn on the machines is assumed to be insignificant; therefore, it does not impact energy consumption significantly. However, the average power demand during the period at which the machine is turned on increases and is represented by *OP*. Note that *OP* is the average energy demand in the period at which the machine is turned on, accounting for power surge during the turn-on (start) process.
- When a machine switches to processing mode from idle, there will be a spike in power draw, called switch power (*SP*). The time to switch is assumed to be insignificant. As a result, when a switch to processing mode occurs in a given period, there will be an excess power demand during that period.
- The unit price of energy varies during peak/off-peak periods (TOU tariff). Demand charge is also a function of TOU and varies in different periods.
- The planning horizon is broken into *T* periods, such that the length of each period is the same as the interval used in energy demand charge calculations.

The parameters considered in the MINLP are as follows:

P_{jm} Processing time of job $j \in J$ on machine $m \in M$

IP_m^t Power consumption of machine $m \in M$ in idle mode during period $t \in T$

PP_m^t	Power consumption of machine $m \in M$ in processing mode during period $t \in T$
OP_m^t	Power consumption of machine $m \in M$ during turn-on process in period $t \in T$
SP_m^t	Power consumption of machine $m \in M$ during switch process from idle to processing mode in period $t \in T$
CP	Cost of maximum power demand
CE_t	Cost of energy consumption during period $t \in T$
L	Duration of each period
F_i	Objective function i, i = 1, 2, 3

The decision variables considered in the MINLP are as follows:

P_{max}	Maximum power demand
X_{jm}^t	1 if job $j \in J$ processing started on machine $m \in M$ at period $t \in T$; zero otherwise
W_{jm}^t	1 if job $j \in J$ is being processed on machine $m \in M$ at period $t \in T$; zero otherwise
Z_m^t	1 if machine $m \in M$ is turned on from off mode at period $t \in T$; zero otherwise
Y_m^t	1 if machine $m \in M$ is idle at period $t \in T$; zero otherwise
U_m^t	1 if machine $m \in M$ is switched from idle mode to processing mode at period $t \in T$; zero otherwise

The following is the proposed mixed-integer nonlinear programming model:

$$\text{Min} \quad \sum_{j \in J} \sum_{t \in T} \sum_{m \in M} \left(t + P_{jm} - 1\right) X_{jm}^t \tag{1}$$

$$\text{Min} \quad \sum_{t \in T} L \times CE_t \left(\sum_{m \in M} IP_m^t + \sum_{m \in M} PP_m^t \sum_{j \in J} W_{jm}^t \right) \tag{2}$$

$$\text{Min} \quad CP \times P_{\max} \tag{3}$$

Subject to

$$\sum_{m \in M} \sum_{t=1}^{T - P_{jm} + 1} X_{jm}^t = 1 \quad \forall j \in J \tag{4}$$

$$\sum_{j \in J} X_{jm}^t \le 1 \quad \forall m \in M, \forall t \in T \tag{5}$$

$$\sum_{j \in J} W_{jm}^t \le 1 \quad \forall m \in M, \forall t \in T \tag{6}$$

$$P_{jm} X_{jm}^t \le \sum_{\theta = t}^{t + P_{jm} - 1} W_{jm}^\theta \quad \forall j \in J, \forall m \in M, \forall t \in \left\{1, \ldots, T - P_{jm} + 1\right\} \tag{7}$$

$$W_{jm}^t = \sum_{\theta = 1}^{t} X_{jm}^\theta \quad \forall j \in J, \forall m \in M, \forall t \in \left\{1, \ldots, P_{jm} - 1\right\} \tag{8}$$

$$W_{jm}^t = \sum_{\theta = t - P_{jm} + 1}^{t} X_{jm}^\theta \quad \forall j \in J, \forall m \in M, \forall t \in \left\{P_{jm}, \ldots, T\right\} \tag{9}$$

$$\sum_{t \in T} Z_m^t \le 1 \quad \forall m \in M \tag{10}$$

$$\sum_{j \in J} X_{jm}^t - \sum_{\theta = 1}^{t} Z_m^\theta \le 0 \quad \forall m \in M, \forall t \in T \tag{11}$$

$$\sum_{\theta = 1}^{t} Z_m^\theta + \left(1 - \sum_{j \in J} W_{jm}^t\right) \le 1 + Y_m^t \quad \forall m \in M, \forall t \in T \tag{12}$$

$$1-\sum_{j\in J} W_{jm}^{t} \geq Y_{m}^{t} \quad \forall m \in M, \forall t \in T \tag{13}$$

$$\sum_{\theta=}^{t} Z_{m}^{\theta} \geq Y_{m}^{t} \quad \forall m \in M, \forall t \in T \tag{14}$$

$$Y_{m}^{t} - Y_{m}^{t+1} \leq U_{m}^{t+1} \quad \forall m \in M, \forall t \in \{1,\ldots,T-1\} \tag{15}$$

$$Z_{m}^{t} + U_{m}^{t} \leq 1 \quad \forall m \in M, \forall t \in T \tag{16}$$

$$\sum_{m\in M} IP_{m}^{t} Y_{m}^{t} + \sum_{m\in M} PP_{m}^{t} \sum_{j\in J} W_{jm}^{t} + \sum_{m\in M} \left(OP_{m}^{t} - PP_{m}^{t}\right) Z_{m}^{t} + \sum_{m\in M} \left(SP_{m}^{t} - PP_{m}^{t}\right) U_{m}^{t} + \sum_{m\in M} \left(PP_{m}^{t} - IP_{m}^{t}\right) Z_{m}^{t} \left(1-\sum_{j\in J} X_{jm}^{t}\right) \leq P_{\max} \quad \forall t \in T \tag{17}$$

In the proposed multi-objective model, the objective function (1) aims to minimize the total completion time. The second and third objective functions aim to minimize the cost of time-based energy consumption and maximum power demand, respectively.

Constraint set (4) – (9) are the job scheduling-based constraints: constraints (4) and (5) show that in a given period only one job can be "started" on each machine. Based on constraint (6), each machine can "process" at most one job in a given period. In other words, based on these constraints, there is a one-to-one assignment between job and machine. Note that a job can be processed after it is started, and based on constraint (7), the total number of processing periods for a job is determined by the job processing time. Constraints (8) and (9) show that the job processing is non-preemptive once started [13].

Constraint set (10) – (17) are machine-based constraints and address machine operation and energy planning: constraint (10) indicates that each machine is turned on (from the off mode) at most once during the planning horizon. Constraint (11) indicates that if a job processing is started on a machine in a given period, the machine might have been turned on either during that period or in any other prior periods. It is worth mentioning that for energy demand reduction purposes, a machine might be turned on in a period when there is no job to be processed. This strategy is helpful to flatten the overall peak power demand in parallel machine setting. Constraints (12) and (13) show that if a machine is on, with no job to process, it is in idle mode. According to constraint (14), a machine can be idle if it has been turned on in any of the previous periods. Constraint (15) explains the switch process from idle to processing mode between periods. Constraint (16) indicates that in a given period, either a switch or turn-on process occurs. Constraint (17) is the power demand capacity constraint and accounts for the power demand during processing and idle modes, and spikes during turn-on and switch process. There is an upper bound on total amount of power consumption to prevent supply shortage and over charging. The last term on the left hand side of constraint (17) is nonlinear, which leads to a nonlinear constraint. This equation can be linearized using the following set of constraints:

$$\sum_{m\in M} IP_{m}^{t} Y_{m}^{t} + \sum_{m\in M} PP_{m}^{t} \sum_{j\in J} W_{jm}^{t} + \sum_{m\in M} \left(OP_{m}^{t} - PP_{m}^{t}\right) Z_{m}^{t} + \sum_{m\in M} \left(SP_{m}^{t} - PP_{m}^{t}\right) U_{m}^{t} + \sum_{m\in M} \left(PP_{m}^{t} - IP_{m}^{t}\right) S_{m}^{t} \leq P_{\max} \quad \forall t \in T$$

such that

$$\begin{aligned} & S_{m}^{t} \leq Z_{m}^{t} \\ & S_{m}^{t} \leq 1-\sum_{j\in J} X_{jm}^{t} \\ & S_{m}^{t} \geq Z_{m}^{t} - \sum_{j\in J} X_{jm}^{t} \end{aligned} \tag{18}$$

3 Model Validation: Illustrative Case Study

This section presents an eight-job three-machine scheduling example with a planning horizon of 16 periods (Table 1) to illustrate the model performance. The unit price of energy ($/kWh) fluctuates in different periods and is given as follows {0.04, 0.04, 0.2, 0.04, 0.04, 0.2, 0.04, 0.2, 0.2, 0.2, 0.2, 0.04, 0.2, 0.04, 0.2, 0.2}. The duration of each period is assumed to be L = 0.5 hour. The machines are not identical, i.e., they have different power consumption amounts, and the job processing times vary on different machines. Since the machines have different capabilities, the job processing times can be different even though the processing power consumptions are the same. Table 1 shows the machines' power specifications and machine-based job durations. The *IP*, *PP*, *OP*, *SP* are power consumption in *kW*, and job processing times are given in periods. The model is solved using a weighted approach [15], as described in the next section, where, w_i represents the weight of each objective function.

Table 1 Illustrative case study data

	IP	***PP***	***OP***	***SP***	**J1**	**J2**	**J3**	**J4**	**J5**	**J6**	**J7**	**J8**
M1	0.8	4	8	4.8	3	1	3	4	2	5	2	2
M2	0.8	4	8	4.8	5	5	1	4	3	3	1	2
M3	1	5	15	6	5	3	2	4	3	4	5	2

*.

Figure 1 shows the solution output for the given example when the objectives are equally weighted. As shown, only *M1* and *M2* are selected, as they are the lowest-energy consuming machines. *M1* is turned on in the first period to process *J2* and then switches to an idle mode in period 2 at which *M2* is turned on. *M1* switches to an idle mode in period 2, considering the spike resulting from *M2* during the turn-on process, assisting in reducing peak power demand and the associated charges. The equally weighted multi-objective model tries to avoid concurrent turn-on processes, as it has a significant impact on peak demand.

The model yields P_{max}=8.8 *kW*, total completion time=41 periods (half-hour), and total energy consumption charges of $5.04/kWh. It should be noted that in industrial facilities, the unit price of power demand ($/*kW*) is significantly higher than unit energy consumption charges ($/*kWh*), and minimizing peak demand leads to considerable savings for companies. High power demand can also influence future contracts with utility providers, as sometimes they use the previous year peak-power demand data as a default for the power demand during the subsequent year. In this example, a weighted sum approach was used to solve the multiple-objective model. Without loss of generality, we assume that all three objectives are equally important, meaning that all have the same weight in a weighted-sum approach.

In order to illustrate the effect of energy-related objectives (i.e. objectives two and three), we analyzed the model considering only the first objective. The result is shown in Figure 2. In this case, all machines are turned on in the first period, making the completion time as small as its minimum value (=26 periods). The peak power is at its maximum, i.e., 31 *kW*, in the first period, which increases power demand charges significantly.

Periods

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
M1	J2		J5	J5	J1	J1	J1	J4	J4	J4	J4					
M2		J7	J3	J8	J8	J6	J6	J6								
M3																
Peak load	8	8.8	8.8	8	8	8	8	8	4.5	4.8	4.8	1.6	1.6	1.6	1.6	1.6

Turn-on Idle Switch to processing

Figure 1. Solution of illustrative case when all objectives are considered (w_1=w_2=w_3)

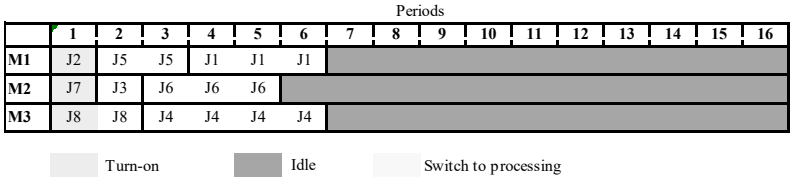

Figure 2. Solution of illustrative case when only completion time is minimized (w_1=1)

In the next scenario, we considered only the energy consumption charges objective function (second objective). The optimal value of the second objective is $3.52. In this case, the values of the other two objectives would be deteriorated. In this schedule, as shown in Figure 3, the total completion time is 52 periods (1 period = 30 minutes) and P_{max}=16 kW, which are higher in comparison with the equally weighted scenario. In this case, only two machines are utilized.

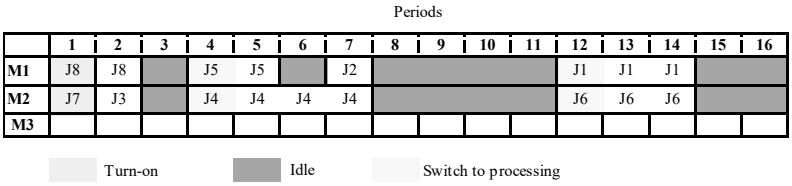

Figure 3. Solution of illustrative case when energy cost (objective 2) is minimized (w_2=1)

Finally, the model is studied considering only the third objective. In this case, the optimal value of the objective function is 88 (P_{max} = 8.8 kW), and the total completion time is 74 periods (Figure 4). Here only two machines are utilized, and the turn-on action and switches between modes occur at different periods in order to minimize power demand. It should be noted that in this schedule, *M2* is turned on in period 2 but it is kept idle until period 6.

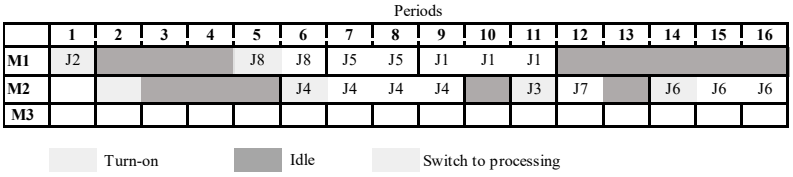

Figure 4. Solution of illustrative case when P_{max} is minimized (w_3=1)

4 Experimental Setup, Results, and Discussion

To show the effectiveness of the proposed mathematical model, we perform a numerical study in this section. For this purpose, instances were generated based on the parameters given in Table 2. To solve the generated instances, the mixed-integer linear program was implemented using C++, and the MILP solver of IBM ILOG CPLEX 12.5[3] was called to solve the instances on a desktop computer running Windows 64-bit operating system, an Intel i7-4790 CPU with eight 3.60 GHz cores, and 16 GB RAM.

[3] https://www.ibm.com/bs-en/marketplace/ibm-ilog-cplex

For the numerical study, five categories of instances were presented based on the number of machines (2 to 6 machines). Then, in each category, four random instances were generated based on the number of jobs, where the job durations and machines power consumption were generated using Table 2. The instances are solved in two ways. Firstly, each instance is solved with one objective at a time, and the optimal values of the objective functions along with the run times are reported in Table 3. The optimal values reported in Table 3 are used to find a compromise solution. From this numerical experiment, it can be seen that the run time is increasing from objective one to objective three, when the problem is solved with one objective at a time. This can be justified by the fact that the parallel machine with completion time can be solved to optimality in a polynomial time [14], while the P_{max} is a min-max objective function that increases the problem complexity.

Table 2. Parameters used to generate instances for the numerical study

Parameters	Possible Values
PP	{3, 4, 5, 6, 7, 8, 9}
IP	[0.2, 0.5]×*PP*
OP	[2, 3]×*PP*
SP	[1.2, 2] ×*PP*
CE	Pr (CE = 0.04) = Pr (CE = 0.2) = 0.5
L	0.5 hour
CP	10
P_{jm}	[1, 5] all integers
M	{2, 3, 4, 5, 6}
J	If *M*=2 or 3, then *M* + {1, 2, 3, 4} If *M*=4 or 5, then *M* + {7, 8, 9, 10} If *M*= 6, then *M* + {13, 14, 15,16}
T	16 = 8 hr

Table 3. Results for the first set of experiments

Instance			CPLEX Output					
#	*M*	*J*	Completion time	CPU time (sec)	2nd obj. (Energy cost)	CPU time (sec)	3rd Obj. P_{max}	CPU time (sec)
1	2	3	9	0	1.64	0	120	0
2		4	11	0	1.33	0	80	0
3		5	17	0	2.3	0	100	0
4		6	24	0	2.18	0	60	0
5	3	4	11	0	1.43	0	60	0
6		5	11	0	2.34	0	90	0
7		6	14	0	2.22	0	90	1
8		7	15	0	1.9	0	210	0
9	4	11	29	1	2.58	1	100	1
10		12	40	1	5.28	1	88	10
11		13	32	1	4.24	2	142	593
12		14	39	1	4.48	1	180	7
13	5	12	23	1	4.14	2	132	8
14		13	28	1	2.42	1	110	13
15		14	37	1	7.54	3	100	41
16		15	37	1	4.28	2	142	9,415
17	6	19	49	3	9.78	4	180	16
18		20	51	3	3.58	4	120	38
19		21	55	3	3.1	3	106	180
20		22	56	3	6.16	4	180	11
Average				1		1.5		516.7

In the second approach, the tri-objective model is solved, where the problem is converted to a single-objective using the compromised programming approach [15] to find the Pareto fronts. In this problem, the single objective is defined as follows:

$$F^{CP} = \sum_{i=1}^{3} \left(w_i \times \frac{F_i - F_i^*}{F_i^*} \right) \tag{19}$$

In Eq. (19), F^{cp} is the single objective, F^*_i, i =1, 2, 3 is the optimal value of objective i, and w_i, i = 1, 2, 3 is the weight of objective i, where $\sum_{i=1}^{3} w_i = 1$ and $0 \le w_i \le 1$. In this numerical study we set ($w_1 = w_2 = w_3$) and the results are given in Table 4.

Table 4. Results of the compromise approach ($w_1 = w_2 = w_3$)

Instance			CPLEX Output			
#	*M*	*J*	Obj 1	Obj 2	Obj 3	CPU time (sec)
1	2	3	19	3.17	120	0
2		4	14	2.23	98	0
3		5	30	3.2	100	0
4		6	41	2.88	60	0
5	3	4	21	2.2	100	1
6		5	28	3.07	90	0
7		6	24	3.06	96	1
8		7	26	4.62	250	0
9	4	11	56	4.29	110	2
10		12	64	6.8	118	37
11		13	58	5.56	190	63
12		14	84	8.76	240	104
13	5	12	54	9.05	202	120
14		13	85	6.54	110	200
15		14	82	9.58	130	160
16		15	84	6.46	190	162
17	6	19	92	12.01	270	150
18		20	117	6.62	170	41
19		21	118	4.81	180	116
20		22	119	10.92	240	212
Average						68.45

The comparison of results in Tables 3 and 4 reveals how the trade-offs among these three objectives can be made (Figure 5) and how the required time to achieve this can be affected. Moreover, by giving different weights to each objective by a decision maker, a set of solutions can be obtained. Then, the decision-maker decides which solution is more convenient depending on the circumstances and company policies. In addition, as shown in Tables 3 and 4, the solution time for the problems of this size, which are meaningful in practice, is negligible. This shows the effectiveness and applicability of the proposed model. However, as the dimension of the problem expands (larger number of machines, periods, and jobs), a more effective approach such as metaheuristics methods like NSGA-II is required to solve the problems in a more time-efficient manner.

A more detailed trade-off between objectives one and three is studied and depicted in Figure 5. In this set of experiment, instance 14 is considered as an example to be analyzed. Then, each objective one and three is given different combination of weights from a set of weights given by (w_1, w_3) = {(0.8, 0.1), (0.7, 0.2), (0.6, 0.3), (0.5, 0.4), (0.4, 0.5), (0.3, 0.6), (0.2, 0.7), (0.1, 0.8)} while w_2 is fixed at 0.1. The results in Figure 5 reveals the conflicts between these two objectives and shows how improving one will deteriorate the other.

5 Conclusion

In this paper, a mixed-integer nonlinear programming model is presented for a non-identical parallel machine scheduling problem with three objectives: total completion time, total energy cost, and maximum power demand charges to be minimized. This is the first study that

considers maximum power demand in each period as a decision variable where energy consumption is a function of operating modes, and energy costs are following TOU policy. Then, in order to find Pareto fronts, the compromise approach is used to help the decision-maker and production-scheduler to apply the best schedule. The proposed algorithm handles the practical size cases efficiently.

Different directions can be employed for future work. First, multi-objective techniques can be utilized to obtain a set of Pareto optimal solutions. Second, the model can be extended to other machine configurations. Third, the model can be modified to address some other scheduling objectives, such as makespan or tardiness minimization. Finally, a heuristic approach can be proposed to solve the large-scale problems in a more time-efficient manner.

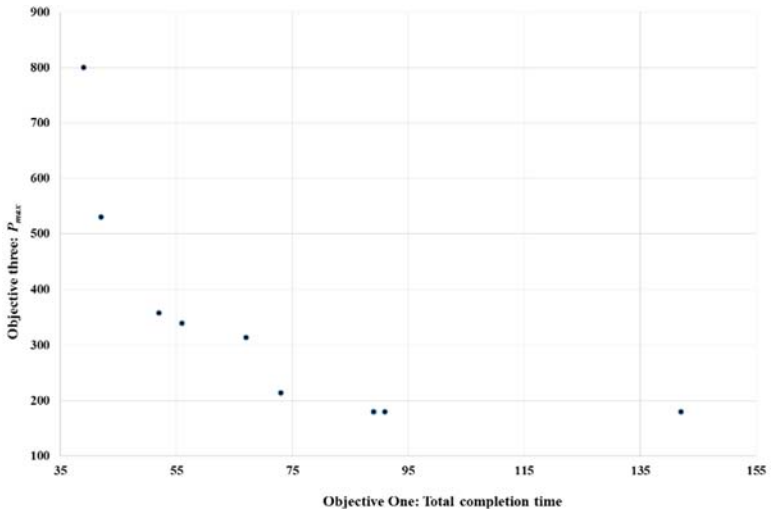

Figure 5. Values of objective one (total completion time) and objective three (P_{max}) of instance 14 where $w_2 = 0.1$, (w_1, w_3) = {(0.8, 0.1), (0.7, 0.2), (0.6, 0.3), (0.5, 0.4), (0.4, 0.5), (0.3, 0.6), (0.2, 0.7), (0.1, 0.8)}, and $w_1 + w_3 = 0.9$

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Evolutionary Bilevel Approach for Integrated Long-Term Staffing and Scheduling

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Abstract Determining size and structure of a company's workforce is one of the most challenging tasks in human resource planning, especially when considering a long-term planning horizon with varying demand. In this paper an approach for integrated staffing and scheduling in a strategic long-term context is presented by applying evolutionary bilevel optimization. For demonstration, the example of determining the number of employees in different categories over the period of one year in a midsized call center of a utility is used. In doing so, two contrary objectives were optimized simultaneously: reduce the overall workforce costs and retain a high scheduling quality. The results show that the proposed approach could be used to support corporate decision making related to strategic workforce planning, not only for call centers but for any other kind of workforce planning involving personnel scheduling.

1 Introduction and Related Work

Companies are challenged by the question of how to organize size and structure of their workforce in order to manage upcoming workload most cost-effectively. This is especially the case when entirely new business units are established, existing units are restructured or current and future demand strongly deviate from each other. Examples of a changing workload are found, among others, in the utility sector. Rising requirements for customer service combined with strong cost pressure require measures for utilities to create a cost-efficient workforce structure. Therefore, in this paper the problem of determining the ideal size and structure of a typical inbound call center of an utility is examined. However, it may be noted that the methodology applied in this paper is not limited to the considered call center, but rather can be applied to problems of other companies and business units.

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When considering size and structure (e.g. skill-mix and contract types) of a company's workforce, the purpose of staffing is to determine the adequate future number of employees needed in different categories. As this is already not an easy task with only one type of employee, it gets even more challenging when looking at a heterogeneous workforce due to different types of skills or contracts [7]. Scheduling, as another crucial task in workforce planning, is concerned with getting the right people to the right place at the right time. It is easily recognizable that staffing decisions have direct impact on scheduling quality. Hence, it is reasonable not to look at staffing and scheduling decisions as two consecutive tasks but to employ an integrated planning approach.

In the literature, the number of contributions addressing integrated staffing and scheduling is rather limited, especially compared to literature concentrated on staffing or scheduling problems [7, 16, 25]. There are, however, several approaches differing in methodology and considered planning horizon. Avramidis et al. [2] for example provide a simulation-based algorithm that simultaneously optimizes the staffing and scheduling over one day in a multi-skill call center. They focus on how many agents of each type are needed based on the arrival rates and type of calls at a given day. In the model presented by Brunner and Edenharter [8] a column generation based heuristic is applied to identify the weekly demand of physicians with different experience levels. Even though the authors are targeting a long-term planning horizon of one year, they solve each week independently. Beliën and Demeulemeester [3] propose a branch-and-price approach considering a planning horizon of four weeks with the aim of reducing staffing costs by integrating both processes, operation room scheduling, which determines the required nurse staffing level, and nurse scheduling. A branch-and-price methodology with a planning horizon of four weeks was also used in the integrated model developed by Maenhout and Vanhoucke [16], with the purpose of identifying optimal staffing and scheduling policies in a hospital. In a more recent contribution published by Beliën et al. [4], an enumerative MILP algorithm is proposed for optimizing the team sizes of an aircraft maintenance company in order to minimize the overall labor costs for a period of six weeks.

The proposed methods already deliver detailed insight into the needed workforce at a given day, week or month and therefore a necessary basis for further workforce planning. However, the planning horizons considered are short when a strategic perspective is taken. Thus, the approaches so far cannot provide information about the required overall workforce, especially when considering a long-term period, e.g. one year, with varying demand as well as factors like overtime/flextime and holidays of employees. To fill this gap, in this paper an approach for integrated staffing and scheduling in a strategic long-term context using evolutionary bilevel optimization is presented.

Bilevel optimization can be seen as a form of hierarchical optimization problem. More specifically, an upper-level optimization problem has another optimization problem within its constraints and therefore is dependent on the results of the lower-level problem. This hierarchical relationship is closely related to the problem of Stackelberg [22], where a follower (lower-level problem) optimizes his objective based on the given parameters determined by the leader (upper-level problem). The leader, on the other hand, optimizes his own objective under consideration of the follower's possible reactions [10]. In the case of integrated staffing and scheduling, staffing will be treated as upper-level problem with the objective to minimize the overall labor costs, i.e. adjusting number and qualification of employees, but at the same time maximizing the quality of personnel schedules. Scheduling, as the lower-level optimization problem, has the objective to maximize scheduling quality based on the staffing decisions made at the upper-level. The quality in this case is assessed by a fitness function that considers the match of staffing demand and allocation of employees with certain skills at given time intervals as well as employee overtime.

Evolutionary bilevel optimization was successfully applied in various practical applications in fields such as economics, transportation, engineering and management, but, to the best of our knowledge, not yet in workforce planning and scheduling problems (see [20] for a comprehensive review). Evolutionary Algorithms (EA) are a common metaheuristic

approach to compute good solutions in an acceptable amount of time, especially when working with real world problems that otherwise cannot be solved to optimality within reasonable computation time [17]. This also applies to workforce planning and scheduling problems, with Genetic Algorithms (GA) as the most often used class of metaheuristics in this domain [7, 25]. Due to its widespread usage and successful application to similar problems, GA were chosen in our case, both to solve the upper-level staffing and the lower-level scheduling problem.

The remainder of the paper is structured as follows: In Section 2 the problem of integrated staffing and scheduling is presented. Section 3 describes the applied evolutionary bilevel approach. In Section 4 the computational results will be discussed. Finally, the conclusions and suggestions for further research are presented in Section 5.

2 Problem Description

In this section, the problem of integrated staffing and scheduling in the environment of a German utility is presented. We consider a strategic context in which the company has to make its overall workforce planning one year in advance to assure that all required employees with the right qualification are available. Due to internal restrictions of the utility, the presented problem is derived and abstracted from a real world problem commonly found in strategic workforce planning.

2.1 Bilevel Optimization

Bilevel optimization problems are proven to be strongly NP-hard [14] and can generally be formulated as follows [10, 20, 24]:

$$\begin{aligned} &\min_{x \in X} F(x, y) \\ &\text{subject to} \quad G(x, y) \leq 0 \\ &\qquad\qquad \min_{y \in Y} f(x, y) \\ &\qquad\qquad \text{subject to} \quad g(x, y) \leq 0 \end{aligned} \tag{1}$$

where x is the vector of decision variables determined by the upper-level problem and y is the vector determined by the lower-level problem. Besides, $F(x, y)$ and $f(x, y)$ are the objective functions and $G(x, y)$ and $g(x, y)$ the constraints of the upper- and lower-level problem. For each vector x, y will be the optimization result of the lower-level problem $\min f(x, y)$. Therefore, $\min f(x, y)$ could also be denoted as $y(x)$ [27]. Thus, the result of the upper-level problem is dependent on the result of the lower-level problem, which in turn is dependent on the vector x given by the upper-level problem.

For the considered problem of integrated staffing and scheduling, x will be the staffing decision made at the upper-level determining the number of employees of each type (combination of skill set and contract type). Based on the given workforce structure, the personnel scheduling will be conducted yielding schedules for each day of the planning horizon. Hence, the objective function at the upper-level $\min F(x, y)$ depends on the costs due to staffing decisions as well as the quality $y(x)$ of the created schedules at the lower level.

2.2 Staffing Problem

The here considered call center has a need of three different skill types $s \in S$ with $S = \{\text{agent}, \text{support}, \text{supervisor}\}$. The skills are considered to be categorical, which means they determine the tasks that can be performed by each employee. However, it is possible to cross-train employees so they can perform more than one type of task [7]. The qualification of an employee can therefore be seen as set of different skill combinations $q \subseteq S$. The contract type $t \in T$ of an employee determines his average weekly working time.

Within its staffing decision, the company has to predefine feasible employee types $\bar{E}$ (see Table 1). Each employee type $\bar{e}_{qt} \in \bar{E}$ is defined by its qualification q and contract type t. Furthermore, each employee type $\bar{e}_{qt}$ is linked to costs $c_{\bar{e}qt}$ that arise for employing one employee of this type over the considered planning horizon. Here, the costs of each employee type are represented by a relative factor summing up annual wages, payroll taxes, overhead and training costs. The number of employees of each type is represented by the decision variable $x_{\bar{e}qt}$. The setting of the staffing problem is shown in Table 1.

The objective here is, as part of the upper-level problem, to minimize the overall staffing costs (2a) subject to the output of the lower-level problem (2b).

Table 1 Setting of the staffing problem

Contract type	Qualification	Costs
40 h	agent	1
20 h	agent	0.6
40 h	support	1.1
20 h	support	0.65
40 h	agent - support	1.3
20 h	agent - support	0.75
40 h	supervisor	1.4

Parameters (staffing)

S	set of skills (index s)
q	qualification of an employee ($q \subseteq S$)
T	set of contract types (index t)
$\bar{E}$	set of employee types (index $\bar{e}_{qt}$)
$c_{\bar{e}qt}$	costs for an employee of type $\bar{e}_{qt}$

Decision variable (staffing)

$x_{\bar{e}qt}$	number of employees of type $\bar{e}_{qt}$

Staffing problem (upper level)

$$\min_{x} F\left(\sum_{\bar{e}_{qt} \in \bar{E}} x_{\bar{e}qt}\, c_{\bar{e}qt}\,, y(x)\right) \tag{2a}$$

with

$x_{\bar{e}qt} \geq 0 \; and \; integer \quad \forall\, \bar{e}_{qt} \in \bar{E}$

$q \subseteq S, t \in T$

subject to

(3a) – (3i) (2b)

2.3 Scheduling Problem

The scheduling problem presented in this paper considers the daily staff scheduling of a call center over a planning horizon of one year. Each week of the planning horizon $w \in W$ is partitioned into periods $p \in P$, representing the operating days of the call center. Moreover, each operating day again is segmented into time intervals $i \in I$. In this practical case, a planning horizon $W = \{1, \ldots, 52\}$ with operating days $P = \{1, \ldots, 5\}$ and, due to the strategic context, hourly planning intervals $I = \{8, \ldots, 17\}$ were chosen, representing the operating times 8 a.m. to 6 p.m.

The set of employees E is determined by the staffing decision at the upper-level with a concrete employee for each $x_{\bar{e}qt}$. It is assumed that the company has a predefined set of possible shift patterns M (see Table 2) with b_{mi} determining whether a shift pattern is covering a specific time interval. In addition, variable n_{es} determines if an employee's qualification contains skill s. It is assumed that each employee has six weeks of holidays each year. Therefore it is possible for employees not to be available at certain periods which is determined by variable a^e_{pw}.

Table 2 Possible shift patterns

Shift start (a.m.)	8	8	8	10	10	10	12
Shift duration (h)	4	8	10	4	6	8	4

The assignment of an employee $e \in E$ to a shift m on day p in week w with skill s is controlled by using the binary decision variable y^{es}_{mpw}. An employee can only be assigned if he is available and has the required skill (3b) - (3c). Furthermore, one employee can only be assigned to one shift each day (3d).

For each time interval i on day p in week w and each skill s a certain staffing level d^s_{ipw} has to be satisfied. The number of planned employees of each skill at time interval i is determined by variable e^s_{ipw} (3e). If a deviation $|e^s_{ipw} - d^s_{ipw}|$ arises from the staffing target, penalty points are generated by the function P_d (3f). An additional penalty is added if no employees are planned but required or vice versa.

To compensate overtime and minus hours, each employee has a flextime account u_{ew}, which is updated on a weekly basis. Therefore, the deviation of the employee's actual working time l_{ew} (3g) and the average weekly working time h_e is added to his flextime account (3h). However, to provide an equal workload distribution and to ensure that employees are staffed according to their contract types, the penalty function P_u generates penalty points based on how far employees exceeded or fall below their average weekly working time u_{ew}/h_e (3i). The weekly penalty is calculated by multiplying the absolute flextime value times the percentage of deviation.

It has to be noted that both penalty functions have to be carefully balanced, as otherwise employees might fall far beyond their contractual working hours (if over-/understaffing is too expensive) or, on the other side, flextime will not be used at all.

The objective here is to minimize the overall penalty points over the considered planning horizon (3a) subject to given constraints (3b) – (3i) described above.

Parameters (scheduling)

W	set of weeks in planning horizon (index w)
P	set of periods in planning week w (index p)
I	set of time intervals in planning period p (index i)

E	set of employees (index e) determined by the upper-level decision variable $x_{\bar{e}qt}$
S	set of skills (index s)
M	set of shift patterns (index m)
b_{mi}	1 if shift m covering time interval i, 0 otherwise
n_{es}	1 if employee's qualification contains skill s, 0 otherwise
a^e_{pw}	1 if employee e is available on day p in week w, 0 otherwise
d^s_{ipw}	demand of skill s at time interval i on day p in week w
e^s_{ipw}	number of planned employees with skill s at time interval i on day p in week w
P_d	demand penalty function
u_{ew}	flextime account of employee e in week w
l_{ew}	actual working time employee e in week w
h_e	average weekly working time of employee e
P_u	working time penalty function

Binary decision variable (scheduling)

y^{es}_{mpw}	1 if employee e is assigned to a shift m on day p in week w with skill s, 0 otherwise

Scheduling problem (lower level)

$$\min_y P_d + P_u \tag{3a}$$

with

$$n_{es}, b_{mi}, a^e_{pw}, y^{es}_{mpw} \in \{0,1\}$$
$$\forall\, e \in E, s \in S, m \in M, p \in P, w \in W$$

subject to

$$y^{es}_{mpw} \le a^e_{pw} \quad \forall\, e \in E, s \in S, m \in M, p \in P, w \in W \tag{3b}$$

$$y^{es}_{mpw} \le n_{es} \quad \forall\, e \in E, s \in S, m \in M, p \in P, w \in W \tag{3c}$$

$$\sum_{s \in S} \sum_{m \in M} y^{es}_{mpw} \le 1 \quad \forall\, e \in E, p \in P, w \in W \tag{3d}$$

$$e^s_{ipw} = \sum_{e \in E} y^{es}_{mpw}\, b_{mi} \qquad \forall\, s \in S, m \in M, i \in I, p \in P, w \in W \tag{3e}$$

$$P_d = \sum_{s \in S} \sum_{e \in E} \sum_{i \in I} \sum_{p \in P} \sum_{w \in W} \left| d^s_{ipw} - e^s_{ipw} \right| * \gamma_d\,, \qquad with$$
$$\gamma_d = \begin{cases} 500, & e^s_{ipw} > 0 \text{ and } d^s_{ipw} = 0 \\ 500, & d^s_{ipw} > 0 \text{ and } e^s_{ipw} = 0 \\ 1, & otherwise \end{cases} \tag{3f}$$

$$l_{ew} = \sum_{i \in I} \sum_{p \in P} y_{mpw}^{es} \, b_{mi} \qquad \forall \, e \in E, m \in M, w \in W \tag{3g}$$

$$u_{ew} = u_{e(w-1)} + (l_{ew} - h_{ew}), \qquad \text{with } u_{e0} = 0 \quad \forall \, s \in S, m \in M, i \in I, p \in P, w \in W \tag{3h}$$

$$P_u = \sum_{e \in E} \sum_{w \in W} \frac{{u_{ew}}^2}{h_e} \tag{3i}$$

3 Evolutionary Bilevel Approach

3.1 Genetic Algorithms

GA are population-based metaheuristics and rely on three basic principles. First, there is a set of solutions (population). Each solution (individual) is evaluated based on its quality (fitness) by applying an objective function (fitness function). Second, variation operators are applied in the process of creating new solutions (reproduction). This can be done by crossover (recombining two or more individuals) and/or mutation (random variation of an individual). Both variation operators are probabilistically applied and exist in many different variants. Finally, individuals with high fitness values are more likely to be selected for reproduction by a selection procedure (see [18, 19, 23] for more detailed information on metaheuristic optimization in general and GA in particular). The GA applied in this paper are based on the basic version shown in Algorithm 1.

The individuals of the here applied GA are represented by matrices, with each row corresponding to an abstract employee type (upper-level algorithm) respectively a concrete employee (lower-level algorithm). The rows at the upper-level are encoded as 4-bit Gray strings, allowing a number between 0 and 15 employees for each type. At the lower-level, 3-bit Gray encoding is used to determine one of seven possible shift patterns for an employee's working day or absence of the employee.

For reproduction, one-point, uniform and two types of n-point crossover are used, each with a probability p=0.25. The first type of n-point crossover randomly selects half of the rows of each matrix and interchanges the entire rows between the two individuals. The second type interchanges one n-bit block of random size per row between the individuals. Moreover, bit flip mutation is used with each bit flipping with the probability of the given mutation rate (see Section 4.1).

3.2 Multi-Objective Optimization

Within the here discussed problem of integrated staffing and scheduling, two objectives have to be optimized. However, both objectives are in conflict with each other, as for example hiring multi-skilled, flexible part-time employees will yield high quality schedules but also increase the staffing costs and on the other hand, reducing the number of employees will reduce labor costs but also the scheduling quality. The resulting multi-objective problem can be solved by using the concept of Pareto efficiency, which will yield a set of Pareto optimal solutions (Pareto front). The final solution to be selected will therefore be a trade-off among the two considered objectives staffing costs and scheduling quality (see [9, 6, 19] for more detailed information on multi-objective optimization).

Algorithm 1 Overview of GA in pseudocode

```
1: popsize ← desired population size
2: generations ← number of generations to be evaluated
3: P ← build initial population of random individuals with size popsize
4: Best ← select best individual according to fitness of initial population
5: for generations times do
6:     for each individual P_i ∈ P do
7:         if Fitness(P_i) > Fitness(Best)
8:             Best ← P_i
9:         end if
10:    end for
11:    P' ← { }
12:    for popsize times do
13:        Parent P_a ← SelectIndividual(P)
14:        Parent P_b ← SelectIndividual(P)
15:        Child C ← Crossover(P_a, P_b)
16:        P' ← P' ∪ {Mutate(C)}
17:    end for
18:    P := P'
19: end for
20: return Best
```

3.3 Nested Bilevel Genetic Algorithm

Within the context of integrated (long-term) staffing and scheduling problems, Maenhout and Vanhoucke [16] point out that most researchers (e.g. [1, 15, 11, 26, 3]), including themselves, iteratively alternate between the staffing and the scheduling problem as they are creating and evaluating personnel schedules based on certain staffing decisions. This was also noted by more recent research [12]. This basic procedure also applies for the evolutionary bilevel approach.

Following the taxonomy given by Talbi [24], the here presented procedure can be defined as a nested constructing approach with metaheuristics on both levels. In this type of bilevel model, an upper-level metaheuristic calls a lower-level metaheuristic during its fitness assessment. In doing so, the upper-level heuristic determines the decision variable x (here the number of employees for each type) as input of the lower-level algorithm, which in turn determines the decision variable y. Both variables are subsequently used to solve the bilevel problem at the upper-level. By the existence of a multi-objective optimization problem, non-dominated sorting is used to evaluate the fitness of each individual at the upper-level GA [21]. As a result, a Pareto front will be built of all non-dominated solutions evaluated at the upper-level (see Algorithm 2, line 10 and 11). An overview of the nested bilevel GA applied in this paper is shown in Algorithm 2.

Algorithm 2 Overview of nested bilevel GA in pseudocode

```
1: initialization (see Algorithm 1, lines 1-3)
2: Best ← { }
3: for generations times do
4:     for each individual P_i ∈ P do
5:         call lower-level GA with P_i as input (see Algorithm 1)
6:     end for
7:     Fitness(P)
8:     A ← ParetoFront(P) ∪ Best
9:     Best ← ParetoFront(A)
10:    reproduction (see Algorithm 1, lines 11-18)
11: end for
12: return Best
```

One major issue when applying bilevel optimization are the long computation times. Preliminary experiments showed that after the evaluation of the first four weeks (of 52 in total) at the lower-level GA, it was already roughly possible to determine the quality of the staffing decision. Once the fitness of two individuals showed a deviation of at least 100% in the fourth week, the fitness development of both individuals did not tend to change. This behavior was used to implement termination criteria, which were applied at different points during the fitness assessment of each individual at the lower-level problem. Once one criterion applies, the evaluation of the individual at the lower-level is terminated.

In general, at each termination checkpoint the assessed individual is compared to all individuals included in the current $Best$ Pareto front (upper-level problem). The first two checkpoints are set after the fourth and eighth week. Here, the assessment is terminated if the overall penalty of the assessed individual is higher by a factor of d=2 compared to any of the solutions of the $Best$ Pareto front at the given weeks. However, as there is no point in keeping solutions with lower fitness and higher costs but, on the other hand, solutions with lower fitness and lower costs could be interesting, the termination only applies if the costs of the assessed individual are higher or equal to the compared individual. The last termination checkpoint is set after week twelve. Here, the assessment is terminated if the overall penalty deviates by a factor of d=3 regardless of the solution's costs. To avoid termination due to outliers, two consecutive weeks are checked within the termination checkpoints.

By applying these criteria it is possible to early identify irrelevant staffing decisions (e.g. too many employees or only one employee type) and to concentrate on more promising solutions. The experiments showed that by implementing these three termination checkpoints, the performance already increased significantly. However, to identify subsequent deviations there could also be implemented more checkpoints during the entire fitness assessment.

4 Computational Results and Discussion

4.1 Experimental Setup

The parameters of both GA were set based upon preliminary studies. For the upper-level GA, a population size of 20, a generation number of 40 and n=10 restarts were chosen, with each restart having a random initial population. The lower-level GA was configured with a population size of 50 and a generation number of 80. On both levels the mutation rate was set to $1/v$, with v being the number of bits of the encoded individual. The fitness at the upper-level was evaluated by Eq. (2a), for the fitness evaluation at the lower-level Eq. (3a) was used.

The optimization software was written in Julia [5] and all experiments were executed on Windows 10 machines with Intel Core i5-2400K processors (4 cores, maximum clock rate 3.1 GHz) and 4 GB RAM. By applying the termination criteria the computation time could be reduced by 50%, however, despite parallel computation within the fitness assessment at the upper-level, each restart of the upper-level algorithm took about twelve hours.

For this experimental study, the following scenario is assumed. The call center has an initial staffing level based on the estimated demand for the year 2017 (see Table 3, solution 1). For the coming year, the company expects a 20% increase in demand. The estimated demand for both years (aggregated agent hours) is shown in Fig. 1. Furthermore, Fig. 2 shows the demand fluctuations in hourly resolution for an exemplary day. The data used for the experimental study was created by using a demand generator relying on slightly modified real world data (see [13] for more details on the used demand generator). The demand for the other two skills is calculated based on staffing ratios. In this concrete case, one support for each four and one supervisor for each eight agents is required.

Moreover, it is assumed that each employee has six weeks of holidays. As the generation of employees is done automatically during the optimization procedure, the holidays of each employee are assigned randomly on a weekly basis. However, while doing so it is assured that at maximum 30% of each skill type can be on holidays at the same time, the holidays are

equally distributed over the whole year and there are two consecutive weeks of holidays during the summer period (June until September).

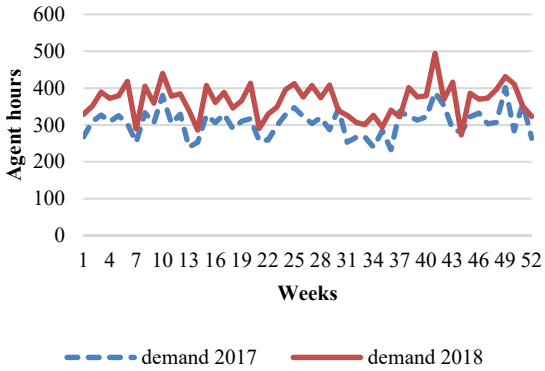

Fig. 1 Weekly demand of agent hours over the planning horizon

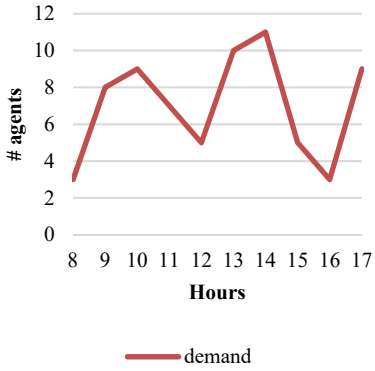

Fig. 2 Exemplary demand of a single day

4.2 Results and Discussion

For the purpose of demonstration and because the Pareto fronts tend to show similar behavior over multiple runs, the upper-level optimization algorithm was executed n=10 times, with each restart yielding a set of Pareto optimal solutions. The combined solutions of all Pareto fronts are shown in Fig. 3. Without the consideration of outliers and solutions that did not pass the termination criteria, a total of 134 possible staffing decisions with different combinations of costs and scheduling penalties were found (based on the demand for 2018). For further discussion, 15 solutions along the new Pareto front were selected. A detailed description of the selected staffing decisions can be found in Table 3.

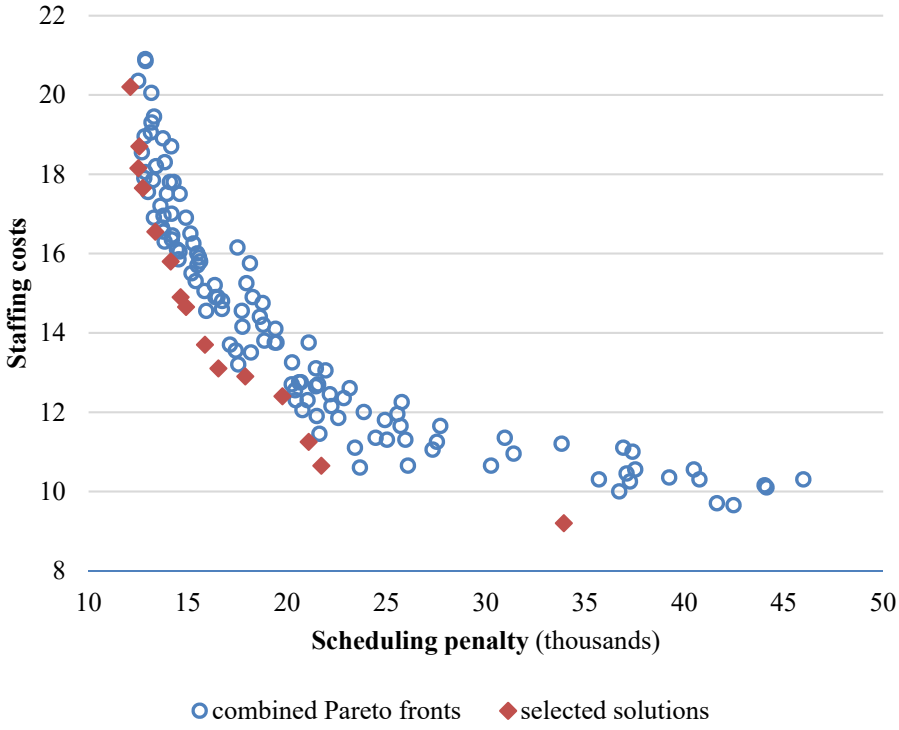

Fig. 3 Possible staffing decisions (n=10 optimization runs)

Table 3 Staffing decisions and corresponding scheduling quality

Solution	Agent		Support		Agent - Support		Supervisor	Penalty			Staffing costs
	40 h	20 h	40 h	20 h	40 h	20 h	40h	Overstaffing	Understaffing	Flextime	
1 (init. 2017)	3	4	1	1	1	1	3	3,500	7,451	4,080	13.4
2 (init. 2018)	3	4	1	1	1	1	3	2,529	10,734	4,942	13.4
3	1	5	0	1	7	3	3	4,991	4,942	2,174	20.2
4	3	4	1	2	4	2	3	4,504	5,387	2,660	18.7
5	2	2	2	0	6	1	3	4,259	6,036	2,212	18.15
6	3	1	0	0	7	1	3	3,903	6,570	2,260	17.65
7	8	0	1	3	1	0	3	4,408	6,231	2,724	16.55
8	6	1	1	0	3	0	3	3,735	7,344	3,056	15.8
9	3	2	1	0	3	2	3	2,900	9,251	2,482	14.9
10	5	2	2	0	1	1	3	3,168	8,647	3,092	14.65
11	3	0	1	0	3	2	3	2,610	10,811	2,440	13.7
12	3	4	2	0	1	0	3	2,580	10,827	3,130	13.1
13	2	5	1	0	2	0	3	2,393	11,505	3,996	12.9
14	2	2	2	0	1	2	3	2,433	12,484	4,842	12.4
15	3	2	2	1	0	0	3	2,542	13,463	5,076	11.25
16	2	2	0	1	2	0	3	2,143	14,918	4,658	10.65
17	2	1	1	2	0	0	3	2,284	16,839	14,804	9.2

The first two solutions shown in Table 3 represent the initial staffing level, optimized with the demand for 2017 and 2018. Solutions 3 to 17 represent the optimized staffing levels based on the demand for 2018. When looking at the results of solution 2, due to the rising demand and therefore an insufficient number of employees, the penalty for understaffing and flextime increased and the overstaffing penalty decreased (compared to solution 1). The resulting scheduling quality of solution 2 now is comparable to the quality of solutions 11 to 13, which furthermore show approximately the same cost level. In case the company wants to retain its service level of 2017 (considering the understaffing penalty of solution 1), the current workforce should be developed towards the staffing level of solution 8. Taking a more general look on the results, it can be noted that no solution below the cost level of 9.2 passed the termination criteria within the fitness assessment. The low costs can simply be explained by the insufficient staffing level, resulting in high understaffing and flextime penalties. Looking at the staffing decisions in Table 3 from bottom to top, it can be seen that the scheduling quality grows with an increasing number of employees and increasing contractual working hours, and, hence, rising staffing costs. Furthermore, when comparing solutions 8 and 7 with 6 to 3, an increased employment of cross-trained and part-time workers can be noticed yielding the highest scheduling quality. However, no solution was found above the cost level of 20.85. This may indicate that there is a point at which scheduling quality cannot be increased under the given staffing and scheduling policies. Thus, measures to further improve the scheduling quality could be, for example, the introduction of new contract types (e.g. contracts with a working time of 30 or 12 hours per week) or more flexible shift patterns to better compensate varying demand over the day (e.g. short 2h shifts).

5 Conclusion and Future Research

In this paper, a model for strategic long-term staffing was presented considering varying demand, different types of employees regarding skills and contractual working times as well as compensation of overtime due to flextime policies. For this purpose, an evolutionary bilevel algorithm with GA on both levels was applied, optimizing the staffing decision at the upper-level and simultaneously evaluating the resulting workforce structure by the creation of personnel schedules over a planning horizon of 52 weeks. This integrated staffing and scheduling approach was demonstrated by the example of the yearly workforce planning of a midsized call center. The computational results indicate that the proposed procedure could be used to support corporate decision making related to strategic workforce planning. Due to the nested structure and independent formulation of the staffing and the scheduling problem, both problems could generically be replaced. Therefore, the model is not limited to the considered call center problem but could be used for any other kind of (strategic) workforce planning involving personnel scheduling.

However, an important limitation arises from the fact that the optimization problem at the lower-level only was executed once, which leads to noisy results. Thus, there is a risk of discarding a possibly good solution due to one “unlucky” optimization run at the lower level. Moreover, it is hard to compare solutions being close to each other (e.g. solutions 4 to 6). These issues could be solved by restarting the lower-level algorithm multiple times, which in turn will lead to a massive increase of computation time. As this is a general challenge when applying bilevel optimization, further research should be aimed at executing the bilevel algorithm more efficiently, e.g. by applying more precise termination criteria, using distributed computation or approximation of the lower-level model. Other opportunities for further research are seen in comparing the here proposed approach to the methods presented in Section 1 (e.g. regarding speed and quality) as well as the consideration of uncertainty and unplanned events during the optimization procedure, such as illness, fluctuation or infra-annual hiring of employees.

As stated in Section 4.2, there may be the need not only to optimize staffing decisions, but also staffing and scheduling policies. The study presented in this paper was limited to optimize the staffing decision as input of the scheduling problem. However, further research

should be conducted addressing the possibility to optimize all framework conditions related to personnel scheduling, such as shift types, overtime and break regulations or any other type of adjustable constraints. This could potentially increase the solution quality attainable, but would in turn raise the complexity of the optimization problem considerably.

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A constraint programming approach for the energy-efficient job shop scheduling problem

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Abstract Optimising the energy consumption is one of the most important issues in scheduling nowadays. In this work we consider a multi-objective optimisation for the well-known job-shop scheduling problem. In particular, we minimise the makespan and the energy consumption at the same time. We consider a realistic energy model where each machine can be in off, stand-by, idle or working state. We design a constraint-programming approach that also uses a piecewise linear programming step to further optimise the energy consumption of the solutions. Experimental results illustrate the potential of the proposed method, outperforming the results of the current state of the art in this problem.

1 Introduction

The job shop is a scheduling problem widely studied in the literature due to the fact that it is a model which is close to many real production environments. It is proven that the job shop is NP-hard, and so its resolution is very complex. In the literature we can find many different solving approaches for the job shop, from exact methods to all kinds of meta-heuristic algorithms.

Although the makespan is the most studied objective function (see for example [1], [4] or [18]), energy considerations are increasingly important nowadays, mainly for economical and environmental reasons. In fact, we can find many recent approaches that tackle different scheduling problems with energy considerations. For example, in [5] the authors solve a flexible flow shop scheduling problem with energy costs by using a genetic-simulated annealing method. In [17] a single machine problem is studied,

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where the machine can be switched on and off. Other papers, as for example [10], even consider shifting energy costs.

There are also some papers addressing the energy-efficient job shop. For example, in [20] the authors try to minimise both the weighted tardiness and the energy consumption in a job shop where the processing mode of operations can be modified. Another approach is that of [14], where the authors consider a simple energy model where the machines can only be in *Working* or in *Idle* state. In [9] the authors improve the results reported in [14] by using a hybrid evolutionary meta-heuristic and also a constraint-programming approach. One problem with the last two papers is that the considered energy model is not too realistic. The model proposed in [15] is much more interesting, as the machines can be either in the *Idle*, *Working*, *Off*, or switched to a *Stand-by* state.

In this paper we consider this last energy model and try to minimize at the same time the makespan and the energy consumption in a job shop. Although some multi-objective works consider weighted or lexicographical approaches, probably the most interesting approaches are those based on the Pareto Front.

In particular, we have designed a set of constraint-based procedures to minimise both the makespan and the energy consumption, within a well-studied multi-objective optimisation method to generate the whole Pareto (i.e., the ϵ-constraint method [16]). The contribution of the paper is twofold: first, we design a constraint-based model where we add as decision variables the states of the machines during the no-working periods (i.e., *Idle*, *Off*, or *Stand-by* states); second, in order to take into account the non-regularity of the energy objective function, we design a piecewise-linear programming approach to *post-process* a full input solution and minimise the energy consumption within the same makespan.

This paper is organised as follows: Section 2 formulates the problem at hand and Section 3 describes the solving methods. Then, in Section 4 we analyse our proposals and we compare them with the state-of-the-art algorithms [15], and finally in Section 5 we report the conclusions of our work and remark some ideas for future work.

2 Problem formulation

The job shop scheduling problem (JSP) consists on scheduling a set of N jobs, $J = \{J_1, \ldots, J_N\}$ in a set of M machines or resources, $R = \{R_1, \ldots, R_M\}$. Each of the jobs J_i consists of n_i tasks $(\theta_{i1}, \ldots, \theta_{in_i})$ that must be scheduled exactly in that particular order. Each task requires a given resource during all its processing time. Additionally, no preemption is allowed, so when a resource starts processing a task, it cannot be interrupted until it ends. Moreover, resources can at most process one task at a time. The objective of the problem is to minimise some objective functions subject to the described precedence and capacity constraints. Although we have denoted the tasks as θ_{ij} in this problem definition, in the following we will denote them by a single letter, if possible, in order to simplify the expressions. We denote by Ω the set of tasks, by p_u the processing time of task u, by r_u the resource required by task u, and by s_u the starting time of task u (which needs to be determined).

As we have seen, the JSP has precedence constraints, defined by the routing of the tasks within the jobs, that translate into linear inequalities: $s_u + p_u \leq s_v$, where v is the next task to u in the job sequence. The problem has also capacity constraints, as the resources can only process one task at a time, and they translate into disjunctive

constraints: $(s_u + p_u \leq s_v) \vee (s_v + p_v \leq s_u)$, where u and v are tasks requiring the same resource. The objective is to build a feasible schedule, i.e. determine a starting time for each task such that all constraints are fulfilled. In the following, given a feasible schedule, we will denote with PJ_v and SJ_v the predecessor and successor of v, respectively, in the job sequence, and with PM_v and SM_v the predecessor and successor of v, respectively, in its resource sequence. In addition, we will denote with α_k and ω_k the first and last operations respectively on machine R_k in the considered schedule.

The goal of the present analysis is the minimisation of both the energy consumption and the overall completion time, or *makespan*. In general, for a minimization problem with two objective functions f_i $(i = 1, 2)$, a solution S is said to be *dominated* by another solution S', denoted $S' \prec S$, if and only if for each objective function f_i, $f_i(S') \leq f_i(S)$ and there exists at least one i such that $f_i(S') < f_i(S)$. However, the possibly conflicting nature of these two objectives may prevent the existence of a unique solution S^* that is optimal w.r.t. both the objectives. Therefore, in this work we are interested in the set of all optimal "tradeoffs", which are known as the *Pareto optimal* solutions. A Pareto optimal solution is a solution such that the improvement of one objective necessarily implies the worsening of the other objective. The Pareto front PS^* is the set of solutions S, such that for each $S \in PS^*$ there is no solution S' which dominates S $(S' \prec S)$.

The makespan is the first objective function and corresponds to the maximum completion time of the schedule, that is

$$\max_{u \in \Omega} \{s_u + p_u\} \tag{1}$$

About the second objective the energy model is taken from [15], where it is supposed that a resource can be in five different states: *Off*, *Stand-by*, *Idle*, *Setup* or *Working*. However, May et al. in their experiments from [15] consider together the times and energy consumption of the *Working* and *Setup* states; as a consequence, we can consider a total of four possible states (see Figure 1). The power consumption in each state for a given resource R_k is denoted by P_k^{idle}, $P_k^{stand\text{-}by}$ and $P_k^{working}$, whereas if the machine is *Off* it consumes no power. Additionally, we assume that the machine can instantly switch from *Idle* to *Stand-by*, *Off* or *Working*, consuming no power. On the other hand, switching from *Off* to *Idle* requires an amount of $T_k^{ramp\text{-}up\text{-}off}$ time units, whereas switching from *Stand-by* to *Idle* requires $T_k^{ramp\text{-}up\text{-}stand\text{-}by}$ time units. In both cases, the power consumed when ramping up is denoted by $P_k^{ramp\text{-}up}$. In Figure 1 we show the considered state diagram, which is the same for each machine. Also, we assume that all machines do not consume any energy before the processing of its first task assigned. It is easy to see that in the job shop scheduling problem, each resource must always process the same set of tasks, and so the working energy consumption is the same in every possible schedule. Therefore, following [15], in order to reduce the energy consumption we consider the WEC (Worthless Energy Consumption) measure as the second objective function to minimize, which is defined as follows:

$$\begin{aligned} WEC = & \sum_{k=1,\ldots,M} [P_k^{idle}\, t_k^{idle} + P_k^{stand\text{-}by}\, t_k^{stand\text{-}by}] + \\ & \sum_{k=1,\ldots,M} P_k^{ramp\text{-}up} \left(n_k^{ramp\text{-}up\text{-}standby}\, T_k^{ramp\text{-}up\text{-}standby} + n_k^{ramp\text{-}up\text{-}off}\, T_k^{ramp\text{-}up\text{-}off}\right) \end{aligned} \tag{2}$$

where t_k^{idle} is the total amount of time spent by R_k in *Idle* state, $t_k^{stand\text{-}by}$ is the total amount of time spent by R_k in *Stand-by* state, $n_k^{ramp\text{-}up\text{-}standby}$ is the number of times

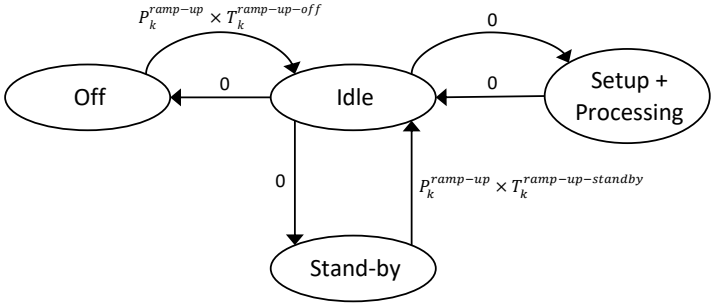


Fig. 1 State diagram for a machine, indicating the energy consumed in each transition

that resource R_k transitions from *Stand-by* to *Idle* state, and finally $n_k^{ramp\text{-}up\text{-}off}$ is the number of transitions from *Off* to *Idle*.

To the aim of assessing how the power consumption of the machines may vary depending on the different states to which they are allowed to transition, we follow the analysis performed in [15], taking into account two different machine behavior *policies*, which we will respectively call $P3$ and $P4$ as described in the following. The $P3$ policy is implemented by switching the machines on at their first operation and switching them off at their last, with the possibility to switch them on and off from the *Idle* state, between any pair of consecutive tasks belonging to the production batch (see Figure 1). The $P4$ policy is similar to the previous one, with the addition of the *Stand-by* state. According to the $P4$ policy, each machine can transition from the *Idle* state to the *Stand-by* state during the production batch, whenever such transition is energetically convenient over both switching the machine on and off again, and leaving it in the *Idle* state. In [15] two more policies called $P1$ and $P2$ are investigated, but such policies are not taken into account in this work because they are very simple and hence not of great interest for our purposes.

According to [3], the makespan is a regular performance measure, which means that it can be increased only by increasing at least one of the completion times in a given schedule. To optimize regular measures it is enough to consider "left-shift schedules", i.e. schedules that are built from a partial ordering of the tasks, in such a way that each operation starts in the earliest possible time after all the preceding tasks in the partial ordering. As opposed to the makespan, the WEC is a non-regular measure, and it can sometimes be decreased by increasing the completion time of some tasks while leaving the other tasks unmodified.

2.1 Solution example

To better illustrate the problem, in this section we present a small toy example. Consider an instance with 3 jobs (with 3 tasks for each job) and 3 resources. The processing times are the following: $p_{\theta_{11}} = 4$, $p_{\theta_{12}} = 5$, $p_{\theta_{13}} = 2$, $p_{\theta_{21}} = 2$, $p_{\theta_{22}} = 5$, $p_{\theta_{23}} = 3$, $p_{\theta_{31}} = 4$, $p_{\theta_{32}} = 7$, $p_{\theta_{33}} = 3$. The required resources are the following: $r_{\theta_{11}} = R_1$, $r_{\theta_{12}} = R_2$, $r_{\theta_{13}} = R_3$, $r_{\theta_{21}} = R_1$, $r_{\theta_{22}} = R_3$, $r_{\theta_{23}} = R_2$, $r_{\theta_{31}} = R_2$, $r_{\theta_{32}} = R_1$, $r_{\theta_{33}} = R_3$. Also, consider the following values for every machine $k \in$

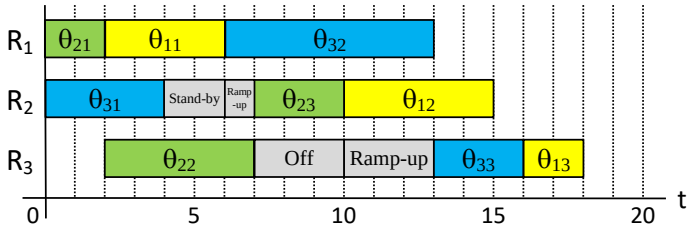


Fig. 2 Feasible solution for an example instance using a "left-shift schedule".

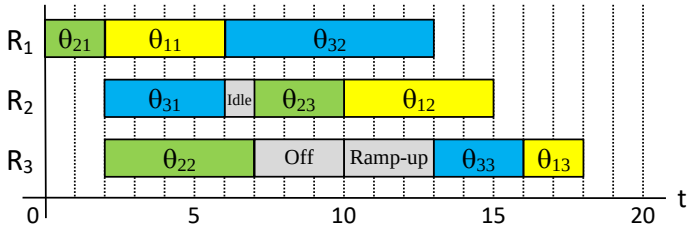


Fig. 3 Improving the solution of figure 2 by delaying one task.

$\{1,2,3\}$: $P_k^{working} = 10kW$, $P_k^{idle} = 6kW$, $P_k^{stand\text{-}by} = 4kW$, $P_k^{ramp\text{-}up} = 8kW$, $T_k^{ramp\text{-}up\text{-}off} = 3$ and $T_k^{ramp\text{-}up\text{-}stand\text{-}by} = 1$.

Figure 2 shows a feasible solution for this instance. In fact it is a "left-shift schedule", i.e. every task starts as soon as possible in the considered partial ordering. This schedule has a makespan of 18 and a WEC of 40 (16 from R_2 plus 24 from R_3). In resource R_2 we have decided to switch the machine to *Stand-by* state between the end of θ_{31} and the beginning of θ_{23}, because in this case it adds 16 units to the WEC, whereas if switched *Off* it would add 24 units and if it remained *Idle* it would add 18 units. Using the same reasoning we decided to switch R_3 off between the end of θ_{22} and the beginning of θ_{33}.

It is easy to see that these "left-shift schedules" can be easily improved by delaying some tasks. Figure 3 shows the same solution after delaying task θ_{31}. Now there is only one time unit between the end of θ_{31} and the beginning of θ_{23}, and so the best option is to leave the machine in *Idle* state. The makespan is still 18 but the WEC is reduced from 40 to 30.

3 The proposed solving method

As we have seen in the previous section, the WEC is a non-regular performance measure. Moreover, the work [15] only considers "left-shift schedules", while we have seen that they can be improved by delaying some tasks, in order to reduce the total energy consumption. In the next section we describe a two-step procedure that takes into account the non-regularity of the WEC objective such that an approximation of the Pareto front is generated by a Constraint Programming (CP) procedure (first

step), which is further improved by a Piecewise Linear Programming post-processing optimization (PO) procedure (second step). It is worth noting that the proposed CP approach is in principle able to find an optimal WEC value if given sufficient computational time (we do not provide any formal proof about this property). Nonetheless, in our *heuristic* approach we use the post-processing step to improve the WEC (non-regular) measure because it allows to reach better solutions in a shorter time, provided the solution it inherits from CP is good enough. As the experiments confirm, the PO procedure is able to give better performance over the single-step CP approach.

3.1 Energy optimisation procedure: a Constraint Programming approach

Constraint Programming (CP) is a declarative programming paradigm [2] suitable for solving constraint satisfaction and optimisation problems. A constraint program is defined as a set of *decision variables*, each ranging on a discrete domain of values, and a set of *constraints* that limit the possible combination of variable-value assignments. After a *model* of the problem is created, the solver interleaves two main steps: *constraint propagation*, where inconsistent values are removed from variables domains, and *search*.

Constraint Programming is particularly suited for solving scheduling problems where the decision variables are associated to the problem operations. In particular, each operation variable a is characterised at least by two features: s_a representing its start time, and p_a representing its duration. For scheduling problems, a number of different *global constraints* have been developed, the most important being the `unary-resource` constraint [19] for modelling simple machines, the `cumulative` resource constraint [13] for modelling cumulative resources (e.g., a pool of workers), and the `reservoir` [11] for modelling consumable resources (e.g., a fuel tank). In particular, given `unary-resource(`A`)`, the constraint holds if and only if all the operations in the set A never overlap at any time point. A number of propagation algorithms are embedded in the `unary-resource` constraint for removing probably inconsistent assignments of operation start-time variables.

We describe a Constraint Programming (CP) model based on the problem defined in Section 2, where the main *decision variables* are the start times s_a of the operations $a \in \Omega$ characterized by a processing time p_a. Each start time s_a ranges in the interval $[0, H-p_a]$, where H is the problem's horizon. The decision variables set is then extended with the start times s_{OnOff_k} of the $OnOff_k$ intervals, where each $OnOff_k$ interval is defined as spanning over all the operations executed on machine k. Hence, the s_{OnOff_k} variable represents the first instant when machine k is turned on. The model, whose utilisation will be described in the experimental section (Section 4), is built on top of the IBM-ILOG CPLEX Optimization Studio CP Optimizer and its details are presented below.

Let O_k be the set of problem operations assigned to machine $k = 1, \ldots, M$ and U_k a set of auxiliary *unit-duration operations*, assigned to a dummy unary machine mirroring k (it is worth noting that the two sets O_k and U_k represent separate processing orders of activities). The introduction of the auxiliary set of operations U_k [1] is necessary to represent the position of each activity $a \in O_k$ in the processing orders imposed among

[1] We were inspired to adopt this solution by a post on a discussion board on the website `www.or-exchange.com` about the explicit representation of an interval position in a OPL `sequence`. At the date of the writing of this note, this discussion board does not seem available anymore.

the operations assigned to each machine $k \in R$. More concretely, the auxiliary unit-duration operations indirectly implement the definition of a *successor function* SM_a (returning the successor of each operation a for each total order imposed on the set of operations O_k assigned to a machine k). To the best of our knowledge, this workaround is necessary because we want to use the native OPL construct to implement the global constraints `unary-constraint(`O_k`)` for efficiency reasons, and the successor function is not natively present in the OPL language (see IBM ILOG CPLEX Optimization Studio OPL Language Reference Manual, Version 12 Release 7.1).

Operationally, the set of *unit-duration* operations $u \in U_k$ can be assigned to the dummy machine k (in the same fashion of the operations a) so that, for each processing order imposed on a machine k, $a_0 \prec a_1, \ldots, \prec a_i, \ldots \prec a_M$, an identical order is imposed on the unit-duration operations $u_0 \prec u_1, \ldots, \prec u_i, \ldots \prec u_M$. In this manner, the position i of the operation a_i coincides with the start-time value of the unit-duration operation u_i. For the reasons above, the starting times s_u of the operations $u \in U_k$ must be added to the model as additional set of *decision variables*.

$$SM_p = \begin{cases} q & \exists v^{(q)} \in U_k : s_{v^{(q)}} = s_{u^{(p)}} + 1 \\ nil & \text{otherwise} \end{cases} \tag{3a}$$

$$\begin{aligned} E^k_{pq} = min\{ & P^{idle}_k \, d_{pq}, \\ & P^{stand\text{-}by}_k \, (d_{pq} - T^{ramp\text{-}up\text{-}standby}_k) + P^{ramp\text{-}up}_k \, T^{ramp\text{-}up\text{-}standby}_k, \\ & P^{ramp\text{-}up}_k \, T^{ramp\text{-}up\text{-}off}_k \} \end{aligned} \tag{3b}$$

$$WEC = \sum_{k=1,\ldots,M} \sum_{\substack{p \in O_k, \\ q = SM_p, \, q \neq nil}} E^k_{pq} \tag{3c}$$

$$C_{max} = \max_{a \in \Omega} \{ s_a + p_a \} \tag{3d}$$

The definition (3a) represents the successor function SM_p such that the position of the operation $p \in O_k$ coincides with the start-time value $s_{u^{(p)}}$ of its corresponding unit-duration operation $u^{(p)} \in U_k$, and the successor q (if exists) corresponds to the unary activity $v^{(q)} \in U_k$, such that $s_{v^{(q)}} = s_{u^{(p)}} + 1$. Whereas, according to Section 2, the energy objective *WEC* (3c) is the sum of the unload energy consumption E^k_{pq} (i.e., when a machine is *Idle*, switched *Off*, or switched to a *Stand-by* state) of each pair of contiguous operations (p, q) assigned on the same machine k (3b), where $d_{pq} = s_q - e_p$ is the difference between q's start time and p's end time. The makespan objective C_{max} is described at line (3d), and follows the classical definition. Once all the necessary definitions have been provided and all the variables have been introduced, we present

the CP model (optimisation criteria and constraints).

$$\text{lex min}\,(WEC, C_{max}) \tag{4a}$$

s.t. :

$$C_{max} \leq C_\epsilon \tag{4b}$$

$$s_v + p_v \leq s_{SJ_v} \quad v \in \Omega \setminus \{\theta_{1n_1}, .., \theta_{Nn_N}\} \tag{4c}$$

$$\texttt{span}(OnOff_k,\, O_k) \quad k = 1, 2, .., M \tag{4d}$$

$$ed_p \in \{0, 1, 2\}\ p \in \Omega \tag{4e}$$

$$SM_p = q \wedge ed_p = 0 \Rightarrow s_q - e_p \leq T_k^{idle\text{-}standby} \tag{4f}$$

$$SM_p = q \wedge ed_p = 1 \Rightarrow s_q - e_p > T_k^{idle\text{-}standby} \wedge s_q - e_p \leq T_k^{standby\text{-}off} \tag{4g}$$

$$SM_p = q \wedge ed_p = 2 \Rightarrow s_q - e_p > T_k^{standby\text{-}off} \tag{4h}$$

$$\texttt{same-sequence}(O_k, U_k)\ k = 1, \ldots, M \tag{4i}$$

$$s_u \leq |O_k| - 1\ \forall k \tag{4j}$$

$$\texttt{unary-constraint}(O_k) k = 1, \ldots, M \tag{4k}$$

$$\texttt{unary-constraint}(U_k) k = 1, \ldots, M \tag{4l}$$

Line (4a) represents the lexicographic minimisation of the objective pair (WEC, C_{max}) with the energy WEC as primary objective. According to the implemented ϵ-constraint method [16] for calculating the Pareto set we optimise the energy WEC, while we impose an upper bound to the other objective C_{max} in the form $C_{max} \leq C_\epsilon$ (see (4b)). The constraints in (4c) represent the linear orderings imposed on the set of operations Ω by the set of jobs J. Constraints (4d) impose to the set O_k of operations requiring machine k to be contained in the *spanning* operations $OnOff_k$, $k = 1, \ldots, M$. More specifically, for each operation $v \in O_k$, the following constraints $s_{OnOff_k} \leq s_v$ and $s_v + p_v \leq s_{OnOff_k} + p_{OnOff_k}$ hold, such that operation $OnOff_k$ starts together with the *first* present operation in O_k according to the order imposed on the k-th machine, and ends together with the *last* present operation according to the same order.

Constraints (4f), (4g), (4h), impose respectively, for each pair of contiguous activities (p, q) on a resource k, the temporal constraint $s_q - e_p \leq T_k^{idle\text{-}standby}$, $s_q - e_p > T_k^{idle\text{-}standby} \wedge s_v - e_u \leq T_k^{standby\text{-}off}$, or $s_q - e_p > T_k^{standby\text{-}off}$. In turn, such constraints guarantee that the minimal unload energy state is respectively *Idle*, *Standby* or *Off* between (p, q). To this purpose, we introduce a set of decision variables $ed_p \in \{0, 1, 2\}$, $p \in \Omega$ representing the unload state (i.e., 0 when machine is *Idle*, 1 when is switched to a *Stand-by* state, and 2 when switched *Off*) imposed on every pair of contiguous activities (p, q) on the same machine. We note that, under the hypothesis $P_k^{stand\text{-}by} \leq P_k^{idle} \leq P_k^{ramp\text{-}up}$ and $T_k^{ramp\text{-}up\text{-}standby} \leq T_k^{ramp\text{-}up\text{-}off}$, two cut-off values, $T_k^{idle\text{-}standby}$ and $T_k^{standby\text{-}off}$ can be calculated such that the minimal energy state will be *Idle* if $s_v - e_u \in [0, T_k^{idle\text{-}standby}]$, *Stand-by* when $s_v - e_u \in (T_k^{idle\text{-}standby}, T_k^{standby\text{-}off}]$, and *Off* otherwise; see Figure 4 for a graphical representation of the three energy intervals determined by the two cut-off values. The constraints in (4i) impose the same order between the activities in the two sets O_k and U_k by means of the global constraints $\texttt{same-sequence}(O_k, U_k)$. The constraints in (4j) bound the start-time value of each unit-duration operation u to $|O_k| - 1$ operations assigned to the machine k. Finally, (4k) and (4l) represents the non-overlapping constraints imposed by the machines M to

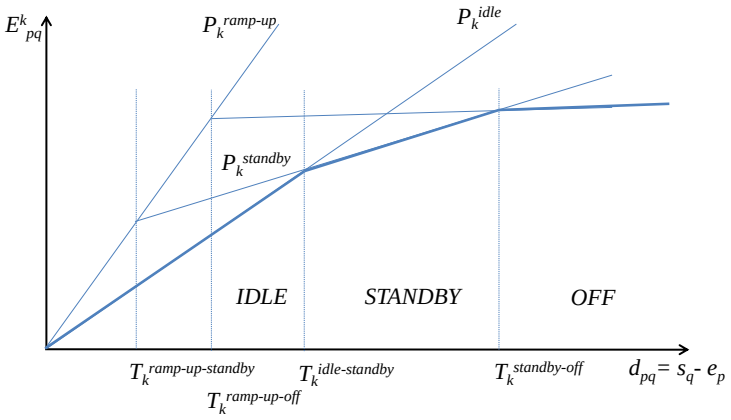


Fig. 4 Minimal energy consumption between two consecutive operations (p, q) on the same machine.

Algorithm 1 Bi-criterion ϵ-constraint method

Require: The objective $\mathbf{f}$, the bounds $f^{(2)}_{min}$ and $f^{(2)}_{max}$, and the decrement value δ
  $P \leftarrow \emptyset$;
  $\epsilon \leftarrow f^{(2)}_{max}$;
  while $\epsilon \geq f^{(2)}_{min}$ **do**
    $S \leftarrow \mathrm{CP}(\mathbf{f}, \epsilon)$;
    if $(S \neq nil) \wedge (\nexists S' \in P : S' \prec S)$ **then**
      $P \leftarrow (P \cup \{S\}) \setminus \{S' \in P : S \prec S'\}$;
    end if
    $\epsilon \leftarrow \epsilon - \delta$;
  end while
  return P

the operations in O_k and U_k, through the global constraints `unary-constraint(`O_k`)` and `unary-constraint(`U_r`)`, respectively.

A well-known multi-objective optimization method to generate the Pareto front is the ϵ-constraint method [16]. It works by choosing one objective function as the only objective and properly constraining the remaining objective functions during the optimisation process. Through a systematic variation of the constraint bounds, different elements of the Pareto front can be obtained.

Algorithm 1 presents the ϵ-constraint method for the case of a bi-criterion objective function $\mathbf{f} = (f^{(1)}, f^{(2)})$. The algorithm is used in the experimental section of the work and takes the following inputs: (i) the objective $\mathbf{f}$, (ii) the bounds $f^{(2)}_{min}$ and

$f^{(2)}_{max}$ on the second component of the objective, and (iii) the decrement value δ. As previously mentioned, the method iteratively leverages a procedure provided in input to solve constrained single-objective problems (the CP() procedure corresponding to the constraint programming model previously described). The algorithm proceeds as follows: after initializing the constraint bound ϵ to the $f^{(2)}_{max}$ value, a new solution S is computed by calling CP() at each step of the *while* solving cycle. If S is not dominated by any of the existing solutions in the current Pareto front P, then S is inserted in P, and all the solutions possibly dominated by S are removed from P. The rationale behind this method is to iteratively tighten the constraint bound by a pre-defined constant δ at each step of the solving cycle. A similar procedure is the *adaptive* ϵ-constraint method described in [12]; the main difference with respect to our algorithm is that, as soon as a new solution S is found at each iteration, the constraint bound is tightened by the value $f^{(2)}(S)$. We will test this procedure in a future and extended version of this work.

3.2 Post-optimization: a piecewise linear programming approach

The solution returned by the energy optimisation procedure described in the previous Section 3.1, in case the WEC value in not optimal, can be further improved if we keep the processing ordering of the operations on all the machines and find a different assignment of the starting times of the operations in order to possibly obtain a reduction in the *WEC* energy consumption. As checking for all these possibilities is computationally expensive, we choose to apply this idea only to the solutions in the approximate Pareto front returned by the proposed CP approach. To this end, given the problem definition of Section 2 and an input solution S, we consider the following piecewise linear programming problem [8].

$$\min WEC = \sum_{k=1,\dots,M} \sum_{\substack{p \in O_k, \\ q = SM_p,\, q \neq nil}} E^k_{pq}$$

$$s.t.: \quad s_v + p_v \leq s_{SJ_v} \quad v \in \Omega \setminus \{\theta_{1n_1}, .., \theta_{Nn_N}\} \tag{5a}$$

$$s_v + p_v \leq s_{SM_v} \quad v \in M_k \setminus \{\omega_k\},\ k = 1, .., M \tag{5b}$$

$$0 \leq s_{\theta_{i1}} \quad i = 1, .., N \tag{5c}$$

$$s_{\theta_{in_i}} + p_{\theta_{in_i}} \leq C_{max} \quad i = 1, .., N \tag{5d}$$

Decision variables are the starting times of the operations s_v with $v \in \Omega$. Constraints (5a) represent the linear orderings imposed on the set of operations Ω by the jobs J, note that they hold for each operation $v \in \Omega$ except when v is the last operation of a job J_i. The processing orderings on the machines in S are represented by constraints (5b), note that, for each machine k, we do not consider the last activity ω_k of the imposed total ordering. Constraints (5c) impose to the first operation θ_{i1} of each job J_i to start after the reference value 0, whereas Constraints (5d) impose to the last operations θ_{in_i} of each job J_i to end before the makespan value C_{max} of the input solution S. It is worth noting that under some restrictive hypotheses, the previous optimisation problem can be handled through a pure linear programming (LP) approach (e.g., concave piecewise linear functions in maximization problems) [7]. However, these hypothesis are not met in our case, and the above optimisation problem must be transformed into an integer-linear program (ILP), e.g., see Chapter 17 in [8]. This is the approach used

in the following experimental section to implement the aforementioned piecewise linear program, using the IBM-ILOG CPLEX Optimization Studio. We propose to apply the given piecewise linear programming approach to all solutions of the Pareto front obtained with the proposed CP approach as a post-optimization (PO) step, in order to further improve its final results.

4 Experimental results

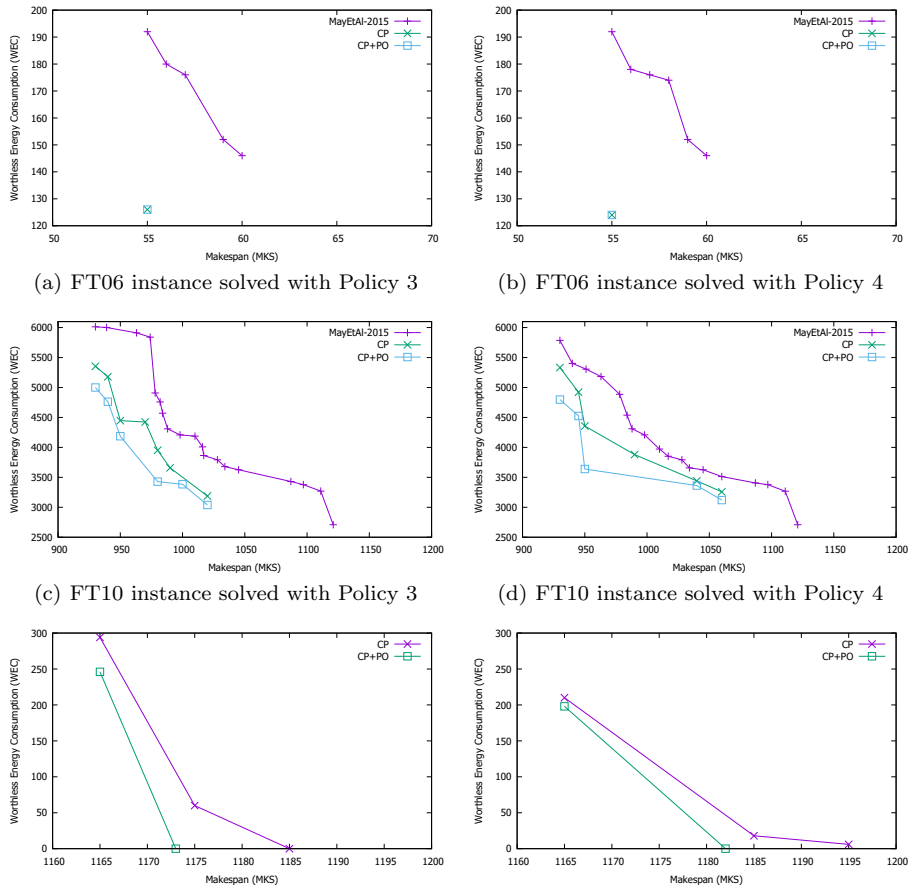
In this section we will analyze the results we have obtained with our CP procedure, and compare such results with the state of the art in [15]. In our work, we test our model against three well-known JSP instances called, respectively, FT06, FT10 and FT20 (as considered in [15]). These instances were introduced by Fisher and Thompson [6], and are characterized by different dimensions both for the number of jobs and for the number of machines. In particular, the FT06 instance has 6 jobs and 6 machines, the FT10 instance has 10 jobs and 10 machines, and the FT20 instance has 20 jobs and 5 machines. In the literature we can find the optimal makespan of these instances, which is 55, 930 and 1165, respectively.

In our tests, we have mainly compared our results with those present in [15] and related to the machine behavior policies $P3$ and $P4$ introduced in Section 2, as these are the most interesting from the energy minimization standpoint. From the analysis performed in Section 2, it is expected that the solutions obtained with the $P4$ policy will exhibit lower energy consumptions that those obtained with the $P3$ policy.

Figure 5 graphically presents a comparison of the obtained results. The figure shows 6 plots organized in 3 rows (one row for each problem instance) and 2 columns (the first column depicts the P3 policy results, while the second column depicts the P4 policy results). In particular, the plots labelled "*MayEtAl-2015*" describe the Pareto front reported in [15], while the plots labelled "*CP+PO*" and "*CP*" describe the results obtained with our CP model, with and without the Post-optimization (PO) procedure described in Section 3.2, respectively. Relatively to the last row (FT20), we limited ourselves to comparing our results with and without post-optimization, as the authors of [15] did not extend their analysis to the FT20 instance case.

In these tests, for the CP phase we allowed for a maximum 5 minute for each $FT06$ solution and a maximum 15 minute for each $FT10$ and $FT20$ solution, while for the PO phase we allowed for a maximum 2 minute for every solution (though the optimum was reached within an average of 10 seconds for almost all instances). Both the CP and the PO models have been implemented on the IBM-ILOG CPLEX Optimization Studio V. 12.7.1.0, a state-of-the-art commercial CP development toolkit, and executed on a Core(TM)2 Duo CPU, 3.33 Ghz under Windows 10 Operating System.

As the Figure 5 shows, our CP model demonstrates a significant improvement over the existing results, for both the FT06 and the FT10 instances, and for both the P3 and the P4 policy. In particular, the advantage of employing the post-optimization procedure is clearly visible, especially for the FT10 and FT20 instances, where the complexity of the solutions and the difficulty of the problem instances leave more room for further optimization and readjustment of the activity start times associated to the machine sequences. Relatively to the FT06 instance, the single solutions obtained with the CP procedure in both policies clearly "kill" the Pareto obtained in [15], and no further optimization margin is left for the post-optimization procedure (the single



(a) FT06 instance solved with Policy 3

(b) FT06 instance solved with Policy 4

(c) FT10 instance solved with Policy 3

(d) FT10 instance solved with Policy 4

(e) FT20 instance solved with Policy 3

(f) FT20 instance solved with Policy 4

Fig. 5 The obtained results organized in 6 different plots, by problem instance (rows) and considered policy (columns)

solutions obtained are exactly the same). This is probably due to the fact that, given its small size, the FT06 instance can be easily solved to optimality.

To conclude the section, the exact numerical figures related to the Pareto fronts shown in Figure 5 are reported in Tables 1 and 2, respectively for the P3 and P4 policies. Overall, if we compare policies 3 and 4, we can observe that the latter usually obtains solutions with lower energy consumption. This means that, as expected, the additional possibility of switching the machine to stand-by state is indeed beneficial. For example, in the FT06 instance we were able to reduce the WEC from 126 to 124, while maintaining the optimal makespan of 55. Another example is instance FT10, where the solution with the optimal makespan (930) presents a WEC of 5000 using policy 3 and 4798 when using policy 4. Also, in the solution with the optimal makespan (1165) of instance FT20, the WEC is reduced from 246 to 198.

Table 1 Pareto sets data relative to Figure 5 (Policy P3)

Problem	Pareto Set - set of pairs (MKS, WEC)
FT06	MayEtAl-2015: { (60, 146), (59, 152), (57, 176), (56, 180), (55, 192) }
	CP: { (55, 126) }
	CP+PO: { (55, 126) }
FT10	MayEtAl-2015: { (1121, 2708), (1111, 3270), (1097, 3378), (1087, 3430), (1045, 3626), (1034, 3678), (1028, 3792), (1017, 3864), (1016, 4008), (1010, 4188), (998, 4208), (988, 4310), (984, 4570), (982, 4758), (978, 4908), (974, 5840), (963, 5912), (939, 6001), (930, 6013) }
	CP: { (1020, 3188), (990, 3658), (980, 3950), (970, 4424), (950, 4446), (940, 5178), (930, 5354) }
	CP+PO: { (1020 3038), (1000 3384), (980 3428), (950 4186), (940 4762), (930 5000) }
FT20	CP: { (1185, 0), (1175, 60), (1165, 294) }
	CP+PO: { (1173, 0), (1165, 246) }

Table 2 Pareto sets data relative to Figure 5 (Policy P4)

Problem	Pareto Set - set of pairs (MKS, WEC)
FT06	MayEtAl-2015: { (60, 146), (59, 152), (58, 174), (57, 176), (56, 178), (55, 192) }
	CP: { (55, 124) }
	CP+PO: { (55, 124) }
FT10	MayEtAl-2015: { (1121, 2708), (1111, 3268), (1097, 3378), (1087, 3406), (1060, 3512), (1045, 3626), (1034, 3658), (1028, 3792), (1017, 3852), (1010, 3972), (998, 4208), (988, 4310), (984, 4538), (978, 4886), (963, 5182), (951, 5307), (940, 5402), (930, 5786) }
	CP: { (1060, 3258), (1040, 3440), (990, 3880), (950, 4356), (945, 4922), (930, 5332) }
	CP+PO: { (1060 3120), (1040 3362), (950 3638), (945 4524), (930 4798) }
FT20	CP: { (1195, 6), (1185, 18), (1165, 210) }
	CP+PO: { (1182, 0), (1165, 198) }

5 Conclusions

In this paper we have considered a bi-objective optimization in the job shop scheduling problem. We minimise at the same time the makespan and the energy consumption. To this end, we consider an energy model in which each machine can be off, stand-by, idle or working. To solve this complex, although interesting and realistic problem, we designed a constraint-programming approach that also uses a piecewise linear programming approach as post-optimization procedure. Our proposal is analyzed and compared against the current state-of-the-art algorithm, obtaining better results.

For future work we plan to consider even more realistic energy models. For example if we do not consider the setup and working states together we can have a more realistic model, specially if the setup times are sequence-dependent. Additionally, considering a flexible environment, i.e. a task can be performed by several machines, each one with different energy consumptions and/or processing times, would lead to a more realistic model. It is also possible to consider shifting energy costs, as in [10].

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Towards improving tenders for Higher Education timetabling software

"Uncovering the selection criteria of HEIs when choosing timetabling applications, using ERP as a reference"

Rudy Oude Vrielink • Erik Jansen • Ewout Gort • Jos van Hillegersberg • Erwin Hans

Abstract Higher Education Institutions (HEIs) are under constant competitive pressure, resulting in the increased importance of achieving both efficiency and effectiveness in such organizations. This intensifies the importance of selecting a suitable timetabling software application, which can be considered to be at the heart of the organization, as it supports organizing the primary process. The timetable software regulates the scheduling of teachers, students and staff, and thus significantly impacts on their effectiveness and efficiency. The selection of such an important application is an essential first step for managing and controlling the schedules.

The contribution of this paper is threefold. First, we decide on a method for comparing the criteria found in tenders to software selection theory. We select and analyze an existing model for selecting ERP software from the literature. Second, we evaluate public tenders submitted in several northwest European countries from 2003 to 2016 and demonstrate that HEIs use a varied and incomplete set of tender criteria. Third, we apply the ERP software selection model to the selection criteria of timetabling applications in HEIs. We present and discuss the model as the approach for HEIs to select timetabling applications in a more structured and consistent way, intended to lead ultimately to use resources more effectively and efficiently.

Keywords Higher education, timetabling, public tenders, selection criteria, ERP

1 Introduction

Timetabling applications are essential for HEIs to control the effective and efficient deployment of teachers, staff and other resources (SURF, 2014), as HEIs are in a constant race to lower costs while attracting more students at both the national and international levels (Jacob, 2015). This results in an increasing demand for flexibility, meaning more student- or individual-centered timetabling practices (Cook-Sather, Bovill, & Felten, 2014) (Oude Vrielink, Schepers, & Jansen, 2016). Selecting a suitable timetabling application that maximizes the efficiency and effectiveness of a HEI is therefore critical.

Timetabling applications are concerned with "the allocation of resources to specific objects being placed in space and time, in such a way as to satisfy as nearly as possible a set of desirable objectives, subjected to constraints" (Wren, 1996). A timetable is not acceptable when any hard constraint is violated, whereas it is considered feasible when no hard constraint, but only some of the soft constraints are violated, which is usually the case. Timetabling tries to approximate optimal solutions as it is an NP hard problem (Moura & Scaraficci, 2010) (Bettenelli, Cacchiani, Roberti, & et al., 2015), meaning that for large instances only feasible rather than optimal solutions can be found in limited time. Three categories of university timetabling can be distinguished: Examination Timetabling, Post-Enrolment-Based Course Timetabling and Curriculum-Based Course Timetabling (Second International Timetabling Competition, 2007) (Di Gaspero, McCollum, & Schaerf, 2007). We consider software applications dealing with one or more of these categories to be timetabling software applications.

HEIs regularly re-evaluate their current timetabling software and make a decision on whether they should keep it as-is, modify it or replace it. A public tender must be issued when the decision is made to acquire a new application and the value of the contract exceeds the threshold that is laid down in EU regulations. This threshold was €209,000 in 2016 for the total costs of purchase, implementation, maintenance and other additional costs, combined over a time period of 5 years (Europa.eu, 2016). This means that HEIs will generally have to issue a tender when acquiring new timetabling software. At the end of such a tender process, the contract is awarded to the vendor who best meets the requirements set out in the tender.

This paper first surveys the literature on selecting timetabling software applications for HEIs. Second, a suitable model is proposed incorporating criteria for selecting timetabling software applications. For this purpose, we analysed ERP theory, as timetabling is considered to be a form of an ERP process (Rabaa'i, Bandara, & Gable, 2009), and, where in contrast with timetabling theory and practices, there is an extensive knowledge base. Third, this paper examines the currently used selection criteria by HEIs in selecting timetabling software applications and compares these criteria to the theoretical model. Finally, we demonstrate that the model is applicable when combining the findings from literature and looking at practical relevance. We propose that applying this specific ERP model in the selection process of timetabling applications is a good step towards improving timetabling application selection and consequently towards improving the competitiveness of the HEI as a whole. This results in the following research question:

What can be learnt about current practices in timetabling application selection issued by HEIs, when comparing tenders to ERP system selection theory?

To find answers to this question, we search for answers to the following sub-questions:

- What model for tendering ERP software can also apply to tendering timetabling software?
- To what extent are selection criteria used by HEIs in tenders similar, when selecting a timetabling application?
- To what extent does ERP theory capture timetabling tender criteria in HEIs?

Figure 1 shows the roadmap of the research process to help improve the tenders for timetabling software.

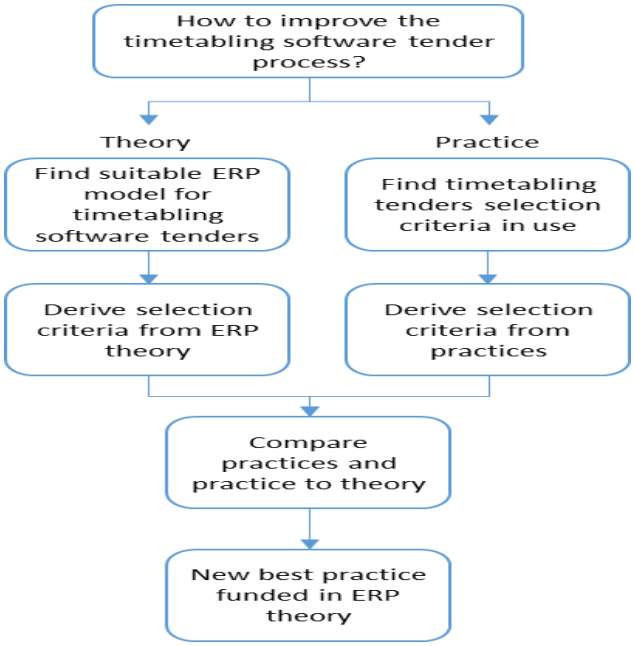

Figure 1 Roadmap of research process to improve tenders for timetabling software in HEIs

Section 2 discusses the literature about timetabling in higher education. Section 3 addresses the selection of a suitable ERP model for evaluating public tenders. Section 4 gives an outline of all the selection criteria used by HEIs in their tendering processes when selecting a timetabling application and compares them to each other. Section 5 analyzes these selection criteria by comparing them to ERP theory. All selection criteria set out in the tenders are compared to determine to what extent the ERP theory matches these criteria. In the final section, we present our conclusions and recommend further research.

2 Literature

Searching in Google Scholar, Web of Science and in Scopus reveals that little research has yet been performed on the subject of the selection process for timetabling software applications for HEIs. We used the following search query to search for papers and other theory on software selection in higher education in the areas of computer science, business, decision support or economics:

TITLE-ABS-KEY("Software Selection" AND "Higher Education") AND (LIMIT-TO (SUBJAREA,"COMP ") OR LIMIT-TO (SUBJAREA,"BUSI ") OR LIMIT-TO (SUBJAREA,"DECI ") OR LIMIT-TO (SUBJAREA,"ECON "))

Figure 2 shows that there are only a very small, but increasing, number of papers concerning the selection process of timetabling software in HEIs. We also found that these papers are only occasionally cited and, perhaps for that reason, they are limited in their relevance. For instance, one paper is concerned with the comparison of two implementations of SAP R/3 scheduling modules, which does not provide any scientific grounding for improving the tendering process. Furthermore, we searched for articles that cover the selection of software within the higher education sector, which limits the number of results but increases the relevance of the search results.

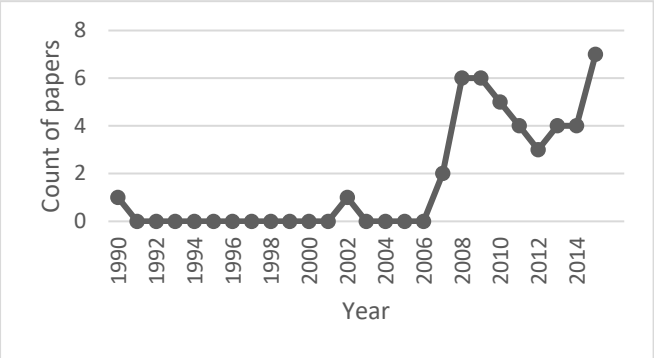

Figure 2 Count of papers found on the selection process of timetabling software in HE

Enterprise resources planning (ERP) has its roots in the field of manufacturing resources planning, but has come to have a more central position and influence on the enterprise-wide operations of an organization (Chen, 2001). Timetabling is considered to be a subset of ERP (Rabaa'i, Bandara, & Gable, 2009). Even though timetabling is only a part of the much wider field of ERP, both ERP and timetabling systems are central systems that greatly influence almost all aspects of an organization. However, the field of ERP research is a much more mature field of research than the field of timetabling in HEIs, and can therefore be used as a reference to evaluate the tenders collected from HEIs. Better insight into the selection criteria used by HEIs in selecting timetabling applications and evaluating the process, can be achieved in a systematic way by comparing ERP system selection with the actual tenders for selecting timetabling software applications.

3 Selection of an ERP model for timetabling software tenders

We used the literature from the field of ERP to establish the model as there are, to the best of our knowledge, no theoretical models of this kind in the specific field of selecting timetabling software. However, in the ERP literature there is a high number of models for selecting software systems, based on scientific research. We compared ERP software selection models with each other in order to find an established model consisting of a set of useful selection criteria and also influencing system selection when considering the weight of the key criteria. Figure 3 shows the outline of the search for potentially relevant ERP theories. The search with only the first three criteria led to 40 relevant papers, and we found that most models are based on earlier models. Of these, one paper is by far the most cited, is considered well-established and uses weighted selection criteria for system selection applicable in higher education.

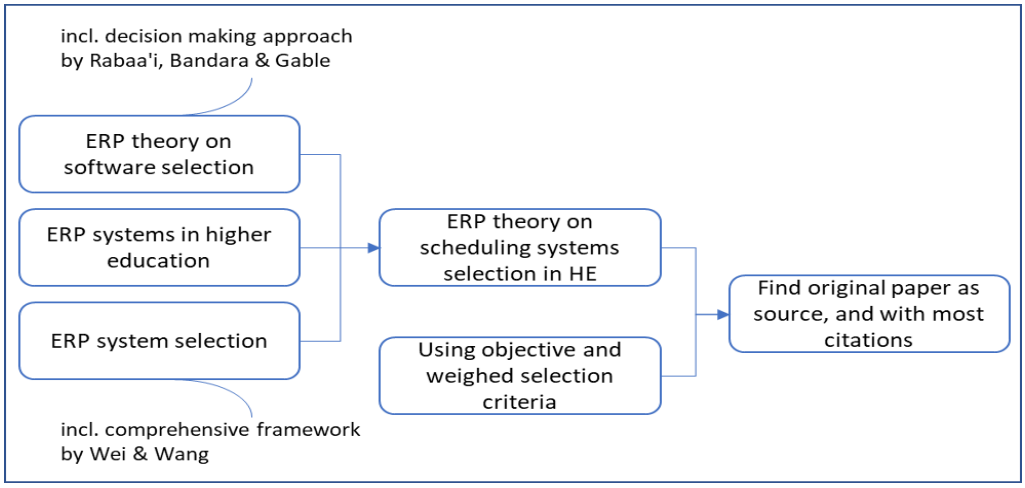

Figure 3 Search process for papers on the selection process of ERP software

This most-relevant model we found is the ERP system selection model proposed by Wei, Chien and Wang (2005). It covers the selection criteria for ERP system selection used in other literature such as Van Everdingen, Van Hillegersberg, & Waarts (2000), Verville and Halingten (2002), Kumar et al. (2003) and Hecht (1997). In the academic search engines Google Scholar, Scopus and Web of Science, this article is frequently cited on this subject. It is referenced as a key source for many other papers on ERP and therefore is a useful source from which to further explore and derive theory for timetabling system selection in HEIs. The paper may seem outdated because it was published many years ago, but that only strengthens the idea that it is the original paper with the original theory on ERP system selection, and the basis of many other papers on this subject.

When selecting software, one could use multi-criteria decision making or multi-criteria heuristics. Korkonen et al. (1992) wrote an, in our opinion, excellent article that provides a review of multi-criteria decision support. Wallenius et al. (2008) wrote a follow-up to this article. We have personally been involved in several tenders and, to the best of our knowledge, HEIs do not use multi-criteria decision support systems when selecting timetabling software. This provides further support to our proposal to use the model by Wei et al. (2005) as the best suited model to evaluate the selection process.

That model consists of two main parts, in which both the system itself and the supplier are assessed. The first part includes categories of selection criteria for selecting the most suitable ERP software system for a particular organization, while the second part encompasses categories of selection criteria for selecting the most suitable ERP vendor for a particular organization. The model has nine categories of selection criteria, which are taken into account when selecting an appropriate ERP system, divided into the two main parts system and vendor. The categories of selection criteria according to Wei et al. (2005) are set out in the table below.

A.	***Selecting the most appropriate system***	***B.***	***Selecting the best vendor***
1.	Minimising Total Costs	7.	Having Good Reputation
2.	Minimising Implementation Time	8.	Providing Good Technical Capability
3.	Having Complete Functionality	9.	Supplying Ongoing Service
4.	Having User-Friendly Interface and Operations		
5.	Having Excellent System Flexibility		
6.	Having High System Reliability		

These categories have each been given a specific weighting, indicating their relative importance.

3.1 Most suitable system selection criteria

We first discuss the six categories for selecting the most suitable system.

3.1.1. Minimizing Total Costs

This category of selection criteria encompasses factors contributing to the costs of the system. The model makes a distinction between (1) price, (2) maintenance costs, (3) infrastructure costs, and (4) consulting expenses. We consider price to be the direct cost of gaining the right to use the software. Maintenance costs are considered the costs brought about by repairs and fixes to keep the system performing as expected, not to be confused with infrastructure costs which are the costs of the support systems that enable the software to run. Consulting expenses are the costs of consultancy, mainly during the implementation phase.

3.1.2. Minimizing Implementation Time

This category includes selection criteria concerned with the implementation time of the system. It contains criteria related to the (1) planning and (2) implementation timeframe for the system in the organization.

3.1.3. Having Complete Functionality

This category contains criteria contributing to ensuring complete functionality of the system. The model makes a distinction between (1) module completeness, (2) function fitness, and (3) security. Module completeness criteria ensure that the system contains all the modules the HEI expects it to have. Function fitness ensures that the implementation fits within the current timetabling process. For instance, the criterion that a system can import student data is therefore a module completeness criterion, while the criterion that the system should be able to handle at least 40,000 students is considered to be part of function fitness. The final subcategory of these selection criteria, namely security, contains criteria which ensure the security of the data held and produced in the system in terms of both unlawful external access and unlawful internal access.

3.1.4. Having a User-Friendly Interface and Operations

This category of selection criteria encompasses factors contributing to the user-friendliness of both the interface and the operations of the system. The model makes a distinction between (1) ease of operation, and (2) ease of learning. Ease of operation means that operations within the system can be done in a sufficiently easy and quick manner. Ease of learning means the effort that users of the system -especially new users- have to put in to learn to use the system.

3.1.5. Having Excellent System Flexibility

This category encompasses all the selection criteria contributing to the flexibility of the system. The model makes a distinction between (1) ease of integration, (2) upgrade ability and (3) ease of in-house development. Ease of Integration concerns the connectivity of the system to other systems already in place. Upgrade Ability deals with the ease of upgrading, such as the ability to develop and implement upgrades. Ease of In-House development concerns the extent to which the system can also be upgraded and adapted by the HEI itself.

3.1.6. Having High System Reliability

This category encompasses all the factors contributing to the reliability of the system. The model makes a distinction between (1) stability, and (2) recovery ability. Stability concerns the selection criteria ensuring that the system will not stop functioning when faced with unexpected internal and external influences. This in contrast to recovery ability which concerns the criteria that will ensure the system is able to recover back to a functioning state after it stopped functioning.

3.2 Best vendor selection criteria

Next, we discuss the three categories of selection criteria that involve choosing the most suitable vendor.

3.2.1. Having Good Reputation

This category of selection criteria encompasses all the factors contributing to the reputation of the vendor. The model makes a distinction between (1) the scale of the vendor, (2) financial condition, and (3) market share. Scale of vendor criteria are concerned with the size of the vendor. Financial condition criteria are concerned with the financial standing of the vendor. Market share criteria concern the number of other organizations using the system.

3.2.2. Technical Capability

This category encompasses all the factors contributing to the perceived technical capabilities of the vendor. The model makes a distinction between (1) research and development ability, (2) technical support capability, and (3) implementation ability. The ability of the vendor to research and develop new technologies is classified as a Research and Development criterion. Technical support capability criteria are concerned with the ability of the vendor to deal with technical difficulties while implementation ability criteria are concerned with the ability of the vendor to implement agreed and specified functionality.

3.2.3. Service

This category encompasses all the factors contributing to the vendor providing ongoing services. The model makes a distinction between (1) warranties, (2) consultancy services, (3) training services and (4) service speed. Warranty criteria are concerned with the warranties the vendor provides in case the system or the implementation process do not meet the promised levels. Consultancy services cover the criteria ensuring the number of consultants and experience of the consultants working at the vendor. Criteria concerned with the amount of training time and the quality of the trainings are bundled into training service criteria. Service speed is concerned with the required response time of the various services.

4 Selection of tenders used in higher education

This section is concerned with the evaluation of the tenders and grouping the information in these tenders to find out the selection criteria used in tenders for timetabling software in HE. First, we searched for suitable tenders. Then, we listed all demands and requirements in these tenders and grouped them. We searched for similarities and differences used in these actual tenders from HEIs and compared them to the ERP model to find out to what extent tenders can be improved by learning from each other and from theory. We considered tenders that comply with the following three rules:

1. The tender is for a timetabling application (may also be termed a timetabling system)
2. To achieve comparability, the tender is issued by an HEI located in the North-West region of Europe (i.e. Benelux, Scandinavia, Germany, UK and Ireland)
3. Not only the RFP, but also more explanation in accompanying tender documents are available.

The tenders were gathered by searching the “online version of the 'Supplement to the Official Journal' of the EU, dedicated to European public procurement” (TED, n.d.). The selection criteria were then extracted from the tenders in order to be able to compare them to each other and analyze them using the ERP selection model.

4.1 Analysis of the categorized selection criteria

The categorized selection criteria were analyzed to determine to what extent the requirements of the actual tenders can be labelled employing the criteria of the ERP software selection model, and, vice versa, to what extent the categories of the ERP software selection model can be found in the tenders. To achieve this in both directions, means that we maximized the opportunities to learn from both theory and practice. Possible categories presented in the tenders that are not present in the ERP software selection model, were found by evaluating the newly created category 'Miscellaneous', which contained selection criteria from tenders that could not immediately be allocated to an existing category. In addition, the weight of the categories relative to each other was analyzed and compared to those used in our ERP evaluation model.

5 Analysis of timetabling tenders

Eighteen tenders were collected for this research, from The Netherlands, Belgium, the United Kingdom, Ireland and Norway, and which were published between 2003 and 2016. The process of extracting the selection criteria from the tenders was difficult because of the very different formats and structures of the tenders. It became apparent that they lack consistency and do not use any kind of general framework.

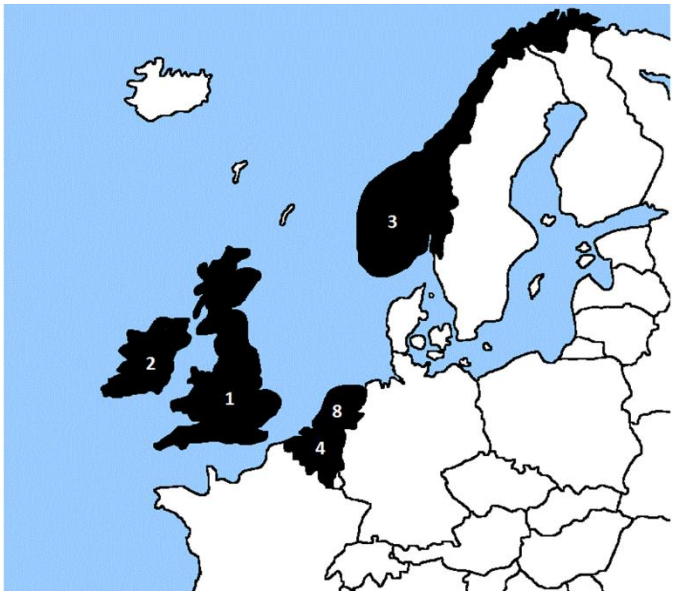

Figure 4 Map of the origin and the number of tenders found on timetabling in HEIs

5.1 Cleaning the data

A first indicator for the quality of a tender is the number of categories of selection criteria it addresses. Figure 5 shows how many tenders addressed how many categories. Of the tenders evaluated, one outlier only addressed two categories. The remaining 17 tenders addressed on average 7.4 of the 9 categories from the model, ranging from 5 up to the full 9 out of 9 categories. The outlier is therefore eliminated from the dataset as we suspect there is incomplete documentation. The remaining dataset consisting of 2,190 selection criteria was divided into the 9 different selection categories from the ERP model, and 46 selection criteria did not fit in any of these categories and were thus classified as 'Miscellaneous'. This accounts for an average of 132 selection criteria per tender for a total of 17 usable tenders.

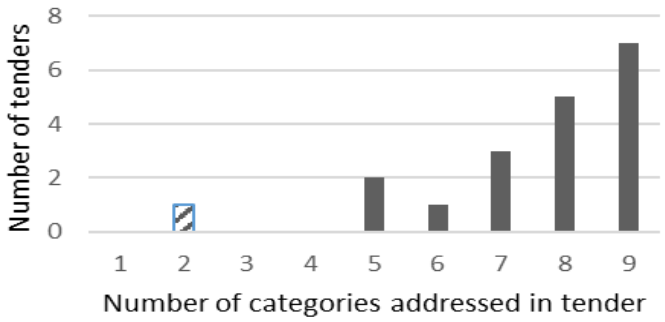

Figure 5 Count of tenders sorted by how many categories they address

After the criteria from the tenders were classified, a closer look was taken at the criteria that were left in the category 'Miscellaneous'. Although roughly 76% of the tenders have one or more selection criteria that were labelled Miscellaneous, this category only contains 46 of the 2,190 selection criteria in total, which makes this category marginal (2.1%). As almost all criteria could be related to existing tender categories in the model, there is no need for new categories of system selection. This suggests that current practices of tendering for timetabling software in HEIs do not use other selection criteria or categories not yet known to the ERP system selection theory proposed by Wei et al. (2005).

5.2 Count of categories addressed per tender

Figure 6 shows the tender and the ERP categories. All tenders have selection criteria in the Flexibility, Functionality and User-Friendliness categories. The Reliability category is similar to these categories with 94% of the tenders having selection criteria in this category. After these four categories, a large drop is seen in the number of categories that the tenders include. These first four categories combined are therefore considered to be a consistent part of timetabling software selection tenders in practice.

Criteria in the Service and Reputation categories are mentioned in 82% of the tenders, and criteria in the Technical Capability and Costs categories are mentioned in 76% of the tenders. Most tenders thus have selection criteria in these categories, although a notable number of tenders do not. This suggests that these tenders could have been improved by adding selection criteria in these not-yet-covered categories. It is remarkable to find 24% of the tenders not addressing total costs.

The least number of tenders, at 59%, contained selection criteria from the Implementation Time category. Thus, of all the 9 categories, adding criteria concerning the implementation time seems to offer the most potential for the improvement of future tenders.

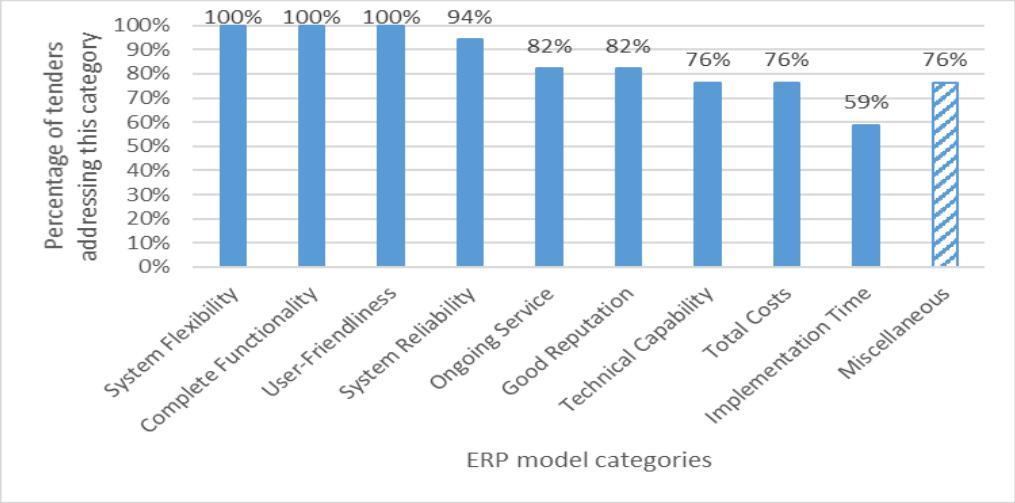

Figure 6 Percentage of tenders from HEIs addressing ERP software selection criteria

5.3 Determination of the relative importance of the selection criteria

Weights to determine the relative importance of criteria were not defined in the tenders we found. This, to us, is a mayor improvement point for tenders. For the analysis, we had to find a way to differentiate in terms of importance between the various selection criteria in tenders from HEIs. We assume that the difference in the number of selection criteria between categories can be seen as an indicator for their relative weight. This assumption is based on the idea that elements with a higher importance are mentioned more often, either because the same criteria are mentioned several times in different parts of the tender, or because the criteria in a category are of a higher detail resulting in more criteria in the same category. Either way, more importance for a criterion leads to it being mentioned more often in the tender, meaning more weight is given to it. With this in mind, a comparison can be made between the weight of categories given in public tenders and the weight of categories as given in the ERP model. This results in Figures 7 and 8, where Figure 7 shows the comparison between the weight of the system selection criteria between the ERP model and the tenders found, and Figure 8 shows the same for the vendor selection criteria.

As most notable differences, we can identify those where the relative count of criteria is less than half or more than double the weight given by the ERP model. For the system selection, these are System Flexibility (5 vs 19%), Implementation Time (15 vs 2%), System Reliability (24 vs 7%) and User-Friendly Interface and Operations (called: User Friendliness) (4 vs 8%). System Flexibility and User-Friendliness are considered to be less important by the ERP model than they are valued in tenders for timetabling software issued by HEIs. On the other hand, Implementation Time and System Reliability are valued as more important by the ERP model than by tenders for timetabling software issued by HEIs.

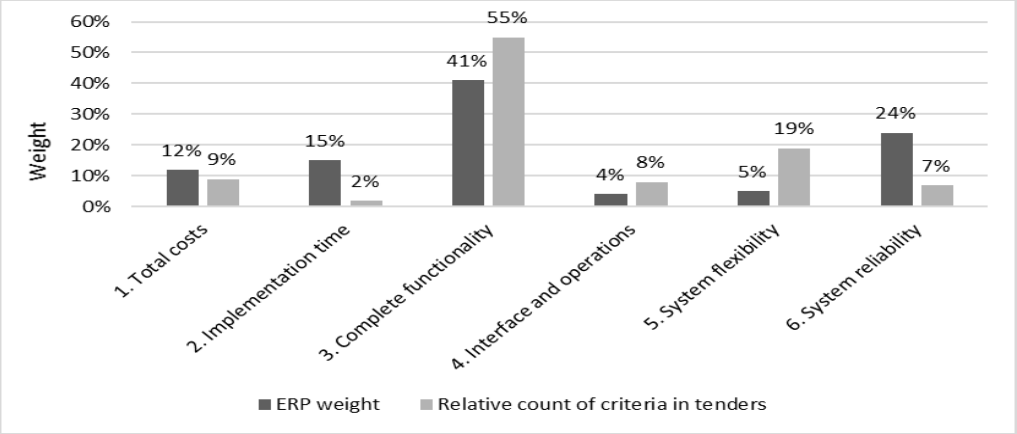

Figure 7 The weights of system selection criteria categories compared between model and practice

This means that, according to the model proposed by Wei et al., the practice of these tenders could be improved by giving more attention to implementation time and system reliability, and perhaps less to system flexibility and a user-friendly interface and operations. Figure 8 shows the comparison between the weight of the vendor selection criteria between the ERP model and the selected tenders.

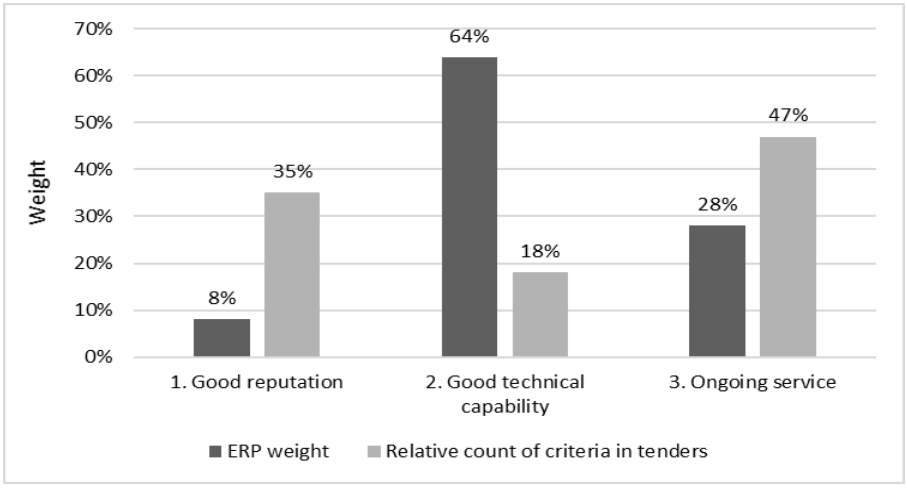

Figure 8 The weights of vendor selection criteria categories compared between model and practice

Good Reputation and Good Technical Capability are the most notable criteria categories in the vendor selection factors. Good Reputation is valued as less important by the ERP model than it is valued by tenders for timetabling software issued by HEIs. However, Good Technical Capability is valued as more important by the ERP model than by tenders for timetabling software issued by HEIs. This means that in tenders, HEIs focus more on reputation than on the technical capability of the supplier, as compared to the ERP theory.

5.4 Weight of the subcategories

Figure 9 shows the sub-criteria for all the nine categories. The category Costs consists of the sub-criteria Consultancy, Infrastructure, Maintenance, Price, and Miscellaneous. Infrastructure criteria are a negligible part of costs criteria in tenders. Price criteria are most frequently mentioned and make 36% of the Total Costs criteria. The Consultancy, Maintenance and

Miscellaneous subcategories each form about 20%. The fact that the sub-criterion 'Miscellaneous' is still as large as it is, suggests that there are missing subcategories within the Costs category. The Implementation Time category was excluded from further analysis, as the ERP model does not provide sub-categories for it.

The System Flexibility category consists of Ease of In-house Development, Ease of Integration, Upgrade ability and Miscellaneous. Sub-criterion Ease of integration is by far the largest subcategory, accounting for 76% of the Flexibility criteria. This indicates that this subcategory might be usefully split up, giving room for more detail in this category when HEIs are tendering for timetabling software.

The remainder of the System flexibility sub-criteria are equally spread accounting for about 8% each. This is a low number which could be the result of the Integration subcategory being too large, but could also indicate that these sub-criteria should be widened and be made more general.

The System Reliability category consists of the Recovery Ability, Stability, and Miscellaneous labels. These subcategories are all fairly evenly distributed. The Miscellaneous subcategory could indicate a need for new subcategories that are currently missing.

The Reputation category consists of the Financial condition, Market share, Scale of vendor and Miscellaneous labels. Financial condition and Market share make up 23% and 22% respectively of the criteria in the Reputation category. Scale of vendor is a smaller subcategory accounting for 14%. However, the Miscellaneous category contributes 41% of the Reputation criteria, indicating that there may be sub-categories missing.

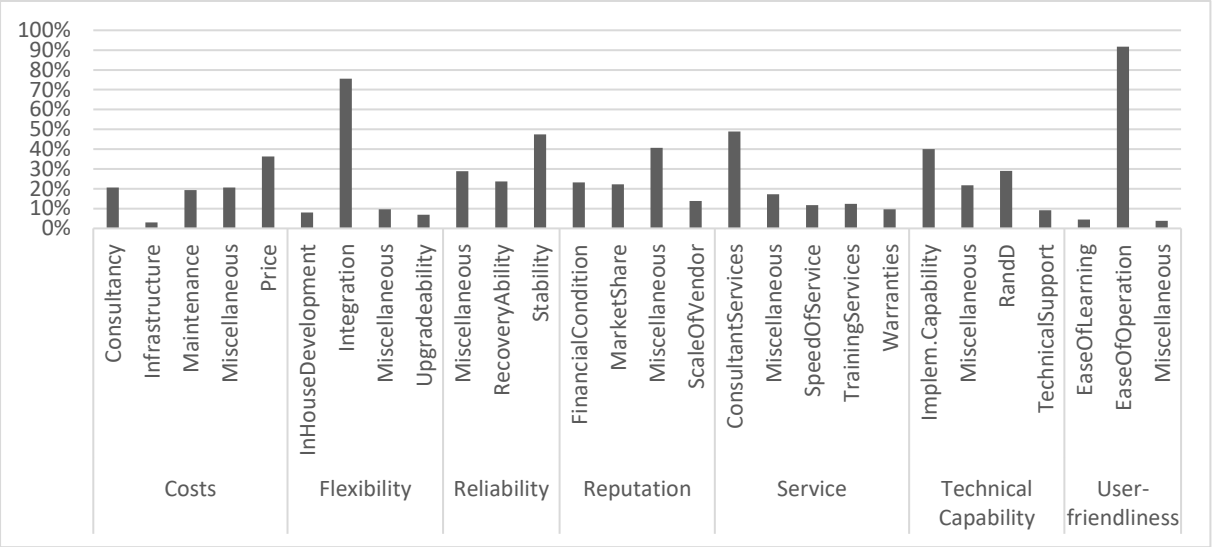

Figure 9 Relative weight of sub-category criteria per category

The Ongoing Service category consists of the Consultant Services, Training Service, Warranties and Miscellaneous labels. By far the biggest subcategory in the Service category is Consultant Services at 49%. This is a good indication that this subcategory can be split up. The Speed of Service, Training Services and Warranties subcategories each account for about 11%. The Miscellaneous category accounts for 17% of the Service category. This also indicates possible missing subcategories.

The Good Technical Capability vendor selection category consists of the Implementation Capability, Research and Development, Technical Support and Miscellaneous labels. Implementation Capability and Research and Development account for 40% and 29% respectively of the selection criteria. Technical Support accounts for only 9%, making it the

smallest subcategory. The Miscellaneous category accounts for 22% of the criteria, indicating a possible missing subcategory.

The User Friendliness, or "Having User Friendly Interface and Operations" category, consists of the Ease of Learning, Ease of Operation and Miscellaneous labels. Ease of operation accounts for 92%, by far the biggest subcategory of User Friendliness, which indicates that this subcategory can be more nuanced by splitting it up into more detailed subcategories concerning ease of operations. Ease of Learning and Miscellaneous both account for 4% of the User Friendliness criteria.

6 Conclusions and discussions

Analyzing the data produced several findings that can be summarized in the following five points:

1. The ERP system selection model proposed by Wei et al. (2005) provides a suitable reference for current tenders for timetabling software in higher education, as no new categories of selection criteria were needed to label the selection criteria found in the tenders evaluated. Tenders evaluated address on average 7.4 of the 9 categories provided by the ERP software selection model.
2. The Flexibility, Functionality, User Friendliness and Reliability selection criteria can be found in all selected tenders, while Reputation, Service, Technical Capability and Costs selection criteria are found in considerably fewer tenders, at about 80%. The Implementation Time selection criteria are found in the least number of tenders, at about 60%.
3. The tenders put more weight on the Flexibility and User Friendliness system categories, and on the Reputation vendor category than Wei's model for ERP system selection does.
4. The tenders put less weight on the Implementation Time and Reliability system categories and on the Technical Capability vendor category than Wei's model does.
5. Wei's ERP model provides a set of subcategories for each category to be used in evaluating systems. Several of these subcategories are probably too general, namely: Flexibility-integration, Service-consultant service and User Friendliness-Ease of Operation. Some subcategories were found to be too narrow in definition, namely: Costs-Infrastructure and User Friendliness-Ease of Learning. Also, indications were found for several categories where subcategories are missing, namely: Costs, Flexibility, Reliability, Reputation, Service and Technical Capability.

Overall, the tenders seemed to be of a reasonable level of completeness, with several categories of selection criteria identifiable in all the analyzed tenders. However, there are several categories of selection criteria which are not yet optimally integrated in current timetabling application tenders. The Implementation Time category provides the biggest opportunity for improvement. The system categories were identified in descending order of the number of tenders in which they have appeared: Flexibility and Functionality and User Friendliness, Reliability, Costs and Implementation Time. Subsequently, the vendor categories were identified in descending order of the number of tenders in which they appeared: Service and Reputation and Technical Capability.

Timetabling application tenders issued by HEIs seem to have a higher interest in the flexibility of the system and reputation of the vendor than would be expected from the ERP literature. The high need for flexibility can possibly be explained by the fact that the timetabling application often is one of the core systems in an HEI and that the system is often linked to multiple databases and portals. However, reputation is a category for which its higher weight is more difficult to explain. Timetabling tenders by HEIs seem to have less interest in the implementation time, and the reliability of the system and the technical capability of the vendor than would be expected from the ERP literature. The low importance placed on implementation time could be caused by the nature of the category, as often only a few criteria are needed to cover its domain. However, this raises the question of whether implementation time deserves to

be a category on its own. The apparent low interest in reliability and technical capability of the vendor is surprising, and cannot easily be explained. This probably indicates an opportunity for the improvement of HEI timetabling application tenders.

This paper aimed to provide insight in the selection criteria used by HEIs to select timetabling applications. This can be the first step in improving this selection process, leading to a better understanding of ways to control effectiveness and efficiency in education. The paper identified the various categories of selection criteria appearing in public tenders for timetabling applications in HEIs located in North-Western Europe. This was done by comparing these tenders with a well-established and well-regarded ERP software selection model. This paper also provided some further insight into the relative weights given to the categories. Finally, a first critical view of possible subcategories was made.

7 Further research

Possible additional subcategories were discussed in the conclusion and discussion. Further labelling of the dataset could provide a more thorough insight in the subcategories specific to tenders for timetabling application by HEIs. When comparing the tenders that were researched, we found large differences between them. A generally accepted framework seems to be missing, which would provide a great opportunity to increase the efficiency and effectiveness of the timetabling application tender processes within HEIs. A framework encompassing all the various aspects of the tendering processes, including the selection criteria, should be established. The findings of this paper can be a good starting point for such a framework.

Further steps for future research would be first to collect more tenders from other countries to facilitate a more accurate analysis. Second, tenders from other parts of the world, such as North-America, Australia and New Zealand, would have to be looked at to see differences in tendering between countries in different parts of the world. Third, a close look at which supplier actually won which tender would be helpful for further analysis. Conclusions can be drawn from the outcomes of the tendering process, and the contents and quality of winning tenders.

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Scheduling Models for Multi-Agent Path Finding

Roman Barták · Jiří Švancara · Marek Vlk

Abstract Multi-agent path finding (MAPF) deals with the problem of finding a collision free path for a set of agents. The agents are located at nodes of a directed graph, they can move over the arcs, and each agent has its own destination node. It is not possible for two agents to be at the same node at the same time. This paper suggests to model the MAPF problem using scheduling techniques, namely, nodes are seen as unary resources. We present three models of the problem. One model is motivated by network flows, another model uses classical unary resource constraints together with path constraints, and the last model works with optional activities. We compare the efficiency of models experimentally.

1 Introduction

There exist numerous practical situations, where a set of agents is moving in a shared environment, each agent having its own destination. For example, traffic junctions and large warehouses are typical examples of congested environments, where agents are moving between locations while sharing paths. In the era of autonomous systems, it is important to have efficient solutions for coordinating such agents.

The above problem is known as multi-agent path finding (MAPF) or cooperative path finding (CPF) [8]. The problem can be formalized as a (directed) graph, where agents are initially distributed at some nodes, each agent having a destination node to reach, and the task is to find a plan of movements for each agent to reach the destination node while not being at the same node as another agent at the same time. A frequent abstraction assumes that agents are moving synchronously and distances between the nodes are identical. Then, at each time step, each agent either moves to a neighboring node or stays in the current node. Grid worlds (such as the famous Lloyd 15-puzzle) are satisfying this assumption. This model makes it natural to use solving techniques based on Boolean satisfiability or state-space search, which are currently two leading approaches to solve MAPF. On the other hand, such an abstraction might

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be too restrictive as distances between the nodes might be important (and different) in some practical applications.

In this paper, we suggest models of MAPF that borrow ideas from scheduling and routing problems. We see the nodes (and possibly also the arcs) as resources with limited capacity, which is one in this particular setting but could be larger in future applications. We model the movements of agents using various techniques, namely as network flows, as paths, and as optional activities. The motivation is supporting richer (in comparison to traditional MAPF) temporal and capacity constraints, which makes the models closer to reality. On the other side, there is one extra restriction of our current models with respect to traditional MAPF formulation - the models are designed such that no agent visits the same node more than once.

2 Background on Multi-Agent Path Finding

The MAPF problem is formulated by a graph and a set of packages (agents) sitting at certain nodes. The task is to transport packages to their destination nodes – each package moves itself – while satisfying some capacity constraints, namely no two packages meet at the same node at the same time. The difference from usual MAPF definition is that in the rest of the paper, we will also assume that no package enters any node more than once.

Let $G = (V, E, w)$ be a directed arc-weighted graph and P be a set of packages. The weight $w(a)$ indicates the duration of moving a package over the arc a. In many MAPF formulations, this duration is expected to be one. For each package p we denote $orig(p) \in V$ the original location (node) of the package and $dest(p) \in V$ its destination node. Let $InArcs(x)$ be the set of incoming arcs to x and $OutArcs(x)$ be the set of outgoing arcs from x. Formally,

$$InArcs(x) = \{(y, x) \mid (y, x) \in E\},$$
$$OutArcs(x) = \{(x, y) \mid (x, y) \in E\}$$

The solution for MAPF problem as described above is a sequence of positions in time for each package that satisfies the condition that no two packages are at the same node at the same time. In this paper, we will focus on solutions that are makespan optimal – the total time until the last package reaches its destination is minimized.

The classical MAPF is usually solved by algorithms that can be divided into two categories:

1. **Reduction based solvers.** Many solvers reduce MAPF to another known problem such as SAT [10], inductive logic programming [12] and answer set programming [3]. These approaches are based on fast solvers that work very well with unit cost parameters.
2. **Search-based solvers.** On the other hand, many recent solvers are search-based. Some are variants of A* over a global search space – all possibilities how to place agents into the nodes of the graph [9]. Other make use of novel search trees [7, 2, 6].

3 Flow Model

The *Flow model* is motivated by the model for the closely related problem of multiple-origin multiple-destination problem [1]. The model consists of two parts, a logical one and a numerical one. The logical part describes a valid path for each package using the idea of network flows. The numerical part describes temporal and resource constraints, namely that paths for different packages do not overlap in time and space.

3.1 The Logical Part (Modeling Paths)

For each package $p \in P$ and for each arc $a \in E$ we introduce a Boolean decision variable $Used[a, p]$ that indicates whether or not arc a is used to transport package p. For each package $p \in P$ and for each vertex $x \in V$ a Boolean variable $Flow[x, p]$ indicates whether or not the transport of package p goes through the vertex x.

To model a transport path for a package we specify the flow preservation constraints. These constraints describe that each package must leave its origin and must arrive at its destination, and if the package goes through some vertex then it must enter the vertex and leave it (both exactly once). In the case of origin, the package only leaves it and, similarly, in the case of destination, the package only enters it. Formally, for each package $p \in P$ we introduce the following flow preservation constraints (recall that domains of all the variables are Boolean, that is, $\{0, 1\}$):

$$\forall a \in InArcs(orig(p)) : Used[a, p] = 0 \tag{1}$$

$$\forall a \in OutArcs(dest(p)) : Used[a, p] = 0 \tag{2}$$

$$Flow[orig(p), p] = 1 \tag{3}$$

$$Flow[dest(p), p] = 1 \tag{4}$$

$$\forall x \in V \setminus \{orig(p)\} : \sum_{a \in InArcs(x)} Used[a, p] = Flow[x, p] \tag{5}$$

$$\forall x \in V \setminus \{dest(p)\} : \sum_{a \in OutArcs(x)} Used[a, p] = Flow[x, p] \tag{6}$$

3.2 The Numerical Part (Modeling Nodes as Resources)

The numerical part specifies non-overlapping constraints, namely two packages do not meet at the same node at the same time, and travel time between the nodes that is expressed by weights of arcs. To model the time interval when a package $p \in P$ stays in a node $x \in V$, we introduce two numerical variables $InT[x, p]$ and $OutT[x, p]$ modeling the time when the package enters the node and when it leaves the node respectively. We can describe the travel time of package p between the nodes x and y through the arc a as follows:

$$Used[a, p] \Rightarrow OutT[x, p] + w(a) = InT[y, p]. \tag{7}$$

If the package p is going through the node x then the package cannot enter the node before it leaves it:

$$InT[x, p] \leq OutT[x, p]. \tag{8}$$

Let $sp(x, y)$ be the length of the shortest path from node x to node y. Then we can calculate bounds of the time variables as follows:

$$\forall x \in V \setminus \{orig(p)\} : Flow[x, p] \Rightarrow OutT[orig(p), p] + sp(orig(p), x) \leq InT[x, p] \quad (9)$$

$$\forall x \in V \setminus \{dest(p)\} : Flow[x, p] \Rightarrow OutT[x, p] + sp(x, dest(p)) \leq InT[dest(p), p] \quad (10)$$

Let $MKSP$ be the time when each package must be in its destination - it corresponds to makespan of the schedule. Then we set the times in package's origin and destination as follows:

$$InT[orig(p), p] = 0 \quad (11)$$

$$OutT[dest(p), p] = MKSP \quad (12)$$

Finally, to model that two packages p_1 and p_2 do not meet at the same node x, we need to specify that their times of visit do not overlap:

$$(Flow[x, p_1] \wedge Flow[x, p_2]) \Rightarrow (OutT[x, p_1] < InT[x, p_2] \vee OutT[x, p_2] < InT[x, p_1]) \quad (13)$$

3.3 Model Soundness

It is easy to prove that the Flow model is sound, that is, every consistent instantiation of variables defines a solution to the MAPF problem. The constraints (1)-(6) define a single path from origin to destination for each package, i.e., the variables *Flow* and *Used* are equal to one for nodes and arcs used on the path and equal to zero for all other nodes and arcs. The origin and destination must be on the path due to constraints (3) and (4). The path must continue from origin due to (6) and must reach the destination due to (5). The path cannot start and cannot finish in any other node due to constraints (5) and (6). The flow constraints allow a loop to be formed in the graph, but such loops are forbidden by temporal constraints (7) and (8). Each package starts its tour at time zero (11) and finishes at time $MKSP$ (12) and two packages cannot meet at the same node at the same time due to constraint (13). Hence each solution to the above constraint satisfaction problem defines conflict free paths for all packages.

4 Path Model

The disjunctive non-overlap constraint (13) from the *Flow model* is a classical expression of a unary (disjunctive) resource. In constraint programming, these disjunctive constraints are known to propagate badly and special global constraints modeling resources have been proposed [11]. Hence it seems natural to exploit such constraints in a model, where the presence of a package at a node is modeled as an activity. These activities must be connected via temporal constraint to define a path from origin to destination.

Formally, for each package $p \in P$ and each node $x \in V$, we introduce an activity $N[x, p]$ describing time that the package p spends in the node x. We denote $StartOf(N[x, p])$ the start time of the activity - it corresponds to $InT[x, p]$ in the Flow model - and similarly $EndOf(N[x, p])$ denotes the end time of activity corresponding to $OutT[x, p]$ in the Flow model. The start time of activity corresponding to

the origin of the package is set to zero, while the end time of activity corresponding to the destination of the package is set to $MKSP$, which is the makespan of the schedule:

$$StartOf(N[orig(p), p]) = 0 \tag{14}$$
$$EndOf(N[dest(p), p]) = MKSP. \tag{15}$$

4.1 The Path Part

To model the path from origin to destination, we will use a double-link model describing predecessors and successors of activities. The real path will be completed to form a loop by assuming that the origin directly follows the destination. The activities (nodes) that are not used in the path will form self-loops (the node will be its own predecessor and successor).

Formally, for each package $p \in P$ and for each node $x \in V$ we will use two variables $Prev[x, p]$ and $Next[x, p]$ describing the predecessor and successor of node x on the path of package p. The domain of the variable $Prev[x, p]$ consists of all nodes y such that $(y, x) \in E$ plus the node x. Similarly, the domain of variable $Next[x, p]$ consists of nodes z such that $(x, z) \in E$ plus the node x. To ensure that the variables are instantiated consistently, we introduce the constraint:

$$Prev[x, p] = y \Leftrightarrow Next[y, p] = x. \tag{16}$$

To close the loop, we will use the following constraints:

$$Prev[orig(p), p] = dest(p) \tag{17}$$
$$Next[dest(p), p] = orig(p). \tag{18}$$

It remains to connect information about the path with the activities over the path, namely to properly connect times of the activities so they are ordered correctly in time. This will be realized by the constraint:

$$EndOf(N[x, p]) + w(x, Next[x, p]) = StartOf(N[Next[x, p], p]), \tag{19}$$

where $w(x, y)$ is the length of arc from x to y. We set

$$w(x, x) = -1 \tag{20}$$
$$w(dest(p), orig(p)) = -MKSP. \tag{21}$$

In order to prune the search space, we add for all $x \in V \setminus \{orig(p)\}$ the following constraints:

$$Next[x, p] \neq x \Rightarrow EndOf(N[orig(p), p]) + sp(orig(p), x) \leq StartOf(N[x, p]), \tag{22}$$

and for all $x \in V \setminus \{dest(p)\}$, we add:

$$Next[x, p] \neq x \Rightarrow EndOf(N[x, p]) + sp(x, dest(p)) \leq StartOf(N[dest(p), p]). \tag{23}$$

4.2 The Resource Part

For each node $x \in V$, we add the following constraint encoding that the visits of the node x are not overlapping:

$$NoOverlap(\bigcup_{p \in P} N[x,p]). \tag{24}$$

4.3 Model Soundness

Any solution to the Path constraint model defines a solution of the MAPF problem and vice versa. For each package, each node (activity) has some predecessor and successor and they are defined consistently thanks to constraint (16), i.e., if x is a predecessor of y then y is the successor of x. It means that all nodes of the graph are covered by loops. Moreover, the origin and destination nodes are part of the same loop due to constraints (17) and (18). All other loops must be of length one due to constraints (19) and (20). Note that durations of activities are only restricted to be positive numbers and as regular arcs also have positive lengths, the only way to satisfy the constraints (19) over the loop is to include an arc with a negative length. Only the arcs (x,x) and $(dest(p), orig(p))$ have negative lengths as specified in constraints (20) and (21). Finally, each path starts at time zero (14) and finishes at time *MKSP* (15) and no two paths overlap at any node at any time due to constraint (24). Note that activities that are not used at any path (they are part of loops of length one) are still allocated to unary resource modeling the node. The duration of such activities is one due to constraints (19) and (20). However, as their start and end times are not restricted by bounds 0 and *MKSP*, such activities can be shifted to future (after *MKSP*).

5 Opt Model

The *Path model* uses classical activities. Some of them are used on the packages' paths from origins to destinations, while others are not necessary (those that are part of loops of length one). These are dummy activities that are part of the model as we do not know in advance which activities will be necessary (which nodes will be visited). In scheduling there exists a concept of *optional activities* that is used to model exactly the same problem. We will exploit optional activities in the *Opt model*. Now, unlike in the Path model, we do not use variables $Next$ and $Prev$ in order to find the path, but the succeeding and preceding nodes will be entailed by whether or not an activity corresponding to the arc and the package is present. All the activities in this model are optional.

Formally, for each package $p \in P$ and each node $x \in V$, we introduce three optional activities $N[x,p]$, $N^{out}[x,p]$, and $N^{in}[x,p]$. As in the Path model, the activity $N[x,p]$ corresponds to the time of a package p spent at node x. The activities $N^{in}[x,p]$ and $N^{out}[x,p]$ describe the time spent in the incoming and outgoing arcs. Next, for each package $p \in P$ and each arc $(x,y) \in E$, we introduce an optional activity $A[x,y,p]$. Again, we use an integer variable *MKSP* to denote the end of schedule (makespan).

5.1 The Path Part

The idea is that the path of a package corresponds to the activities that are present in the solution and that in turn correspond to the nodes and arcs in the path. In the terminology of hierarchical scheduling, it can be conceived such that each activity $N^{out}[x,p]$ has the activities $A[x,y,p]$ corresponding to the arcs outgoing from the node x as its children, and symmetrically, $N^{in}[x,p]$ has the activities $A[y,x,p]$ corresponding to the arcs incoming to the node x as its children. Hence each activity $A[x,y,p]$ has two parents: $N^{out}[x,p]$ and $N^{in}[y,p]$ as the arc (x,y) is an outgoing arc for node x and an incoming arc for node y.

Formally, for each package $p \in P$, the following logical constraints are introduced:

$$PresenceOf(N[orig(p),p]) = 1 \tag{25}$$

$$PresenceOf(N[dest(p),p]) = 1 \tag{26}$$

$$PresenceOf(N^{in}[orig(p),p]) = 0 \tag{27}$$

$$PresenceOf(N^{out}[dest(p),p]) = 0 \tag{28}$$

$$\forall x \in V \setminus \{orig(p)\} : PresenceOf(N[x,p]) \Leftrightarrow PresenceOf(N^{in}[x,p]) \tag{29}$$

$$\forall x \in V \setminus \{dest(p)\} : PresenceOf(N[x,p]) \Leftrightarrow PresenceOf(N^{out}[x,p]) \tag{30}$$

$$\forall x \in V \setminus \{orig(p)\} : Alternative(N^{in}[x,p], \bigcup_{(y,x)\in E} A[y,x,p]) \tag{31}$$

$$\forall x \in V \setminus \{dest(p)\} : Alternative(N^{out}[x,p], \bigcup_{(x,y)\in E} A[x,y,p]) \tag{32}$$

The constraint *Alternative* enforces that if the activity given as the first argument is present, then exactly one activity from the set of activities given as the second argument is present. In addition, it ensures that the start and end times of the present activities are equivalent. Since this implication goes only in one direction, we have to impose the following constraints in order to find the path:

$$\forall (x,y) \in E : PresenceOf(A[x,y,p]) \Rightarrow PresenceOf(N^{in}[y,p]) \tag{33}$$

$$\forall (x,y) \in E : PresenceOf(A[x,y,p]) \Rightarrow PresenceOf(N^{out}[x,p]) \tag{34}$$

The processing times of activities $A[x,y,p]$ are fixed to the weights of the arcs $w(x,y)$, whereas the processing times of activities N, N^{out}, and N^{in} are to be found. Thanks to the *Alternative* constraints, the processing times of activities N^{out} and N^{in} will span over the child activity A that will be present, and for the rest, the following constraints need to be added:

$$\forall x \in V \setminus \{orig(p)\} : StartOf(N[x,p]) = EndOf(N^{in}[x,p]) \tag{35}$$

$$\forall x \in V \setminus \{dest(p)\} : EndOf(N[x,p]) = StartOf(N^{out}[x,p]) \tag{36}$$

$$StartOf(N[orig(p),p]) = 0 \tag{37}$$

$$EndOf(N[dest(p),p]) = MKSP \tag{38}$$

Again, in order to prune the search space, we add the following constraints:

$$\forall x \in V \setminus \{orig(p)\} : EndOf(N[orig(p),p]) + sp(orig(p),x) \leq StartOf(N[x,p]) \tag{39}$$

$$\forall x \in V \setminus \{dest(p)\} : EndOf(N[x,p]) + sp(x,dest(p)) \leq StartOf(N[dest(p),p]) \tag{40}$$

5.2 The Resource Part

Exactly as in the Path model, we need to introduce the constraint precluding the packages from occurring at the same node at the same time, that is, for each node $x \in V$, we add:

$$NoOverlap(\bigcup_{p \in P} N[x, p]) \tag{41}$$

5.3 Model Soundness

The solution of the Opt constraint model consists of selection of activities and their time allocation. The activities corresponding to origins and destinations of packages must be selected due to constraints (25) and (26). The constraints (29)-(34) ensure that if a node is used on some path then there must be exactly one incoming and one outgoing arc selected (except for the origin, where no incoming arc is used due to (27), and for the destination where no outgoing arc is selected due to (28)). No activity outside the path is selected as such activities would have to form a loop due to constraints (29)-(34), but that would violate the temporal constraints (35) and (36). Finally, each path starts at time zero (37) and finishes at time *MKSP* (38) and activities in nodes are not overlapping (41).

6 Experimental Results

We implemented the models in the IBM CP Optimizer version 12.7.1 [5]. The only parameters that we adjusted are `DefaultInferenceLevel`, which was set to `Extended`, and `Workers`, which we set to 1. The experiments were run on a Dell PC with an Intel® Core™ i7-4610M processor running at 3.00 GHz with 16 GB of RAM. We use a cutoff time of 100 seconds per problem instance.

6.1 Implementation Details

For all three models, we compute the all-pairs-shortest-path matrix *sp* using the *Floyd-Warshall* algorithm [4] as the preprocessing phase. We set the lower bound on makespan to be the longest path of the packages' shortest paths from their origins to their destinations, and the upper bound on makespan *UB* is set to be the sum of the shortest paths from the origins to the destinations of all the packages. Further, if for a package $p \in P$ and a node $x \in V$, $sp(orig(p), x) > UB$, it means that the node x cannot be passed through by the package p, and thus we omit creating variables associated with the node x and the package p.

To represent the activities in the Path model and the Opt model, we use the *Interval Variables* of the CP Optimizer, which are tailored for the scheduling problems and support specialized constraints such as Alternative and NoOverlap. The only issue is that the NoOverlap constraint works with non-strict inequalities, whereas if a package leaves a node at time t, another package is allowed to enter the same node no sooner than at time $t + 1$. In fact, the times spent by packages at nodes are mostly zero-length. Hence, the NoOverlap constraint is given a so-called *Transition Distance* matrix

containing all ones, which ensures that the time distances between two consecutive visits of a node are at least one. Consequently, instead of constraint (20) in the Path model, we set $w(x, x) = 0$, and as we omit the constraints (19) between the nodes $dest(p)$ and $orig(p)$, the constraints (21) can be also omitted.

The bounds of the intervals and other time variables are limited using the sp matrix. Note that in the Path model, all the intervals must be scheduled and non-overlapping even when the package is not passing through the associated node, so that we use the time upper bound $UB + |P|$.

As to the implementation of the constraints (19) in the Path model, one option is to use the specialized *Element* constraint. Another option is to use constraints in the form of implications for each possible value of $Next[x, p]$:

$$Next[x, p] = y \Rightarrow EndOf(N[x, p]) + w(x, y) = StartOf(N[y, p])$$

The implications turned out to be much more efficient than using the *Element* constraint so that the implications are used in the experiments.

We also tested the models without the constraints for pruning the search space (9)-(10), (22)-(23), and (39)-(40), which led to increase in average runtime for the Flow, Path, and Opt model roughly by 26 %, 37 %, and 20 %, respectively. For the Path model, we also tried adding the constraints $Next[x, p] = x \Leftrightarrow StartOf(N[x, p]) \geq UB$, which turned out to be counterproductive.

6.2 Problem Instances

The problem instances are simple four connected grid maps with unit-length edges. To ensure interaction between agents, impassable walls are introduced in the grid graph. These walls create two types of graphs - a grid that has an obstacle in the middle that the agents have to go around, and a grid that has a bottleneck that the agents have to squeeze through. To create different complexity of the instances we incrementally increase the grid size from 5 by 5 to 9 by 9 as well as we vary the number of agents from 2 to 9 for each size of the graph.

Different origin and destination positions are also included in the experiments. Both can be either randomly scattered across the whole graph or grouped in one place. This yields four different combinations of origin and destination positions. Each of the instances described above was generated five times. Hence, we generated 1600 instances in total.

6.3 The Results

The Figure 1 shows the overall comparison in the form of a cactus graph. It shows the number of problems solved (x-axis) within a given time (y-axis). For simple instances, the Flow model is the best one. Then the middle complexity instances are solved best by the Path model, but the overall winner is the Opt model that can solve the largest number of instances. This is an interesting behavior, in particular, that the Flow model is better than the Path model for simpler and for more complex instances, but not for the middle-complexity instances.

We compared the models also based on parameters of the instances. Recall, that two types of worlds (maps) were generated - one with an obstacle to go around it

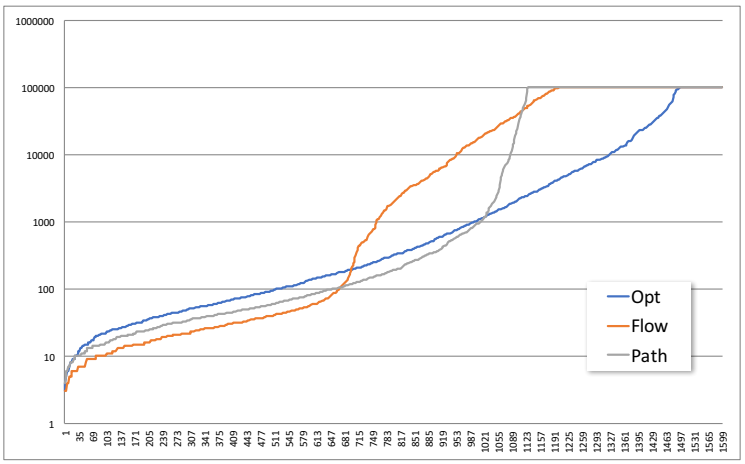

Fig. 1 Dependence of the number of problems solved on time (logarithmic scale; time measured in milliseconds).

and one with a bottleneck that the agents have to squeeze through. Figure 2 shows the comparison in the form a cactus graph. The Opt model is overall the best model independently of the map. The bottleneck maps seem to favor the Flow model over the Path though the trend for the obstacle maps seems similar and maybe if a larger cutoff time is used, the behavior of models will be similar.

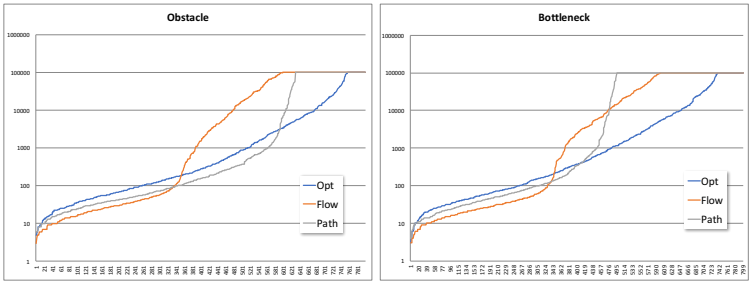

Fig. 2 Dependence of the number of problems solved on time for two types of maps (logarithmic scale; time measured in milliseconds).

We also studied the behavior of models based on the size of instances. The size can be measured by the size of the map or by the number of agents. Figure 3 shows the comparison of models for different sizes of maps. It is clear that for small maps, the Flow model works very well but as the size increases the Path model works better. Again, the Opt model demonstrates the most stable behavior. Regarding the number of agents, it seems that the behavior of models corresponds to the overall behavior and the number of agents does not favor any of the models. Figure 4 shows the comparison for selected numbers of agents.

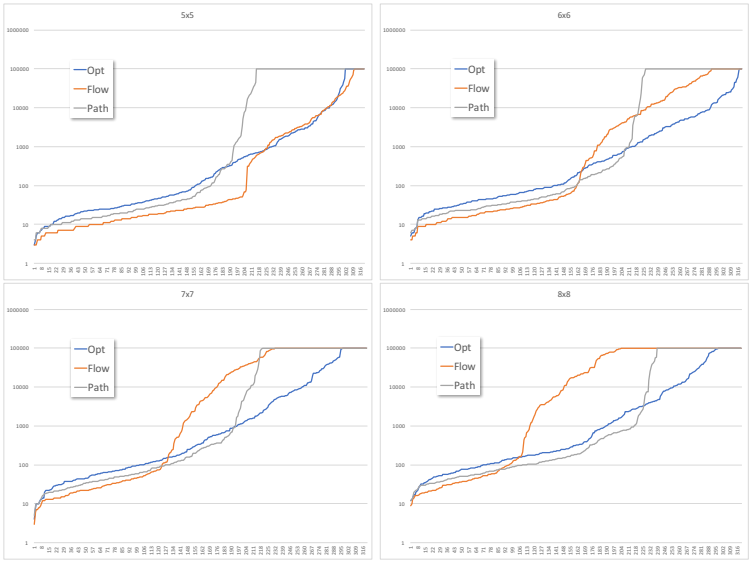

Fig. 3 Dependence of the number of problems solved on time for different sizes of maps (logarithmic scale; time measured in milliseconds).

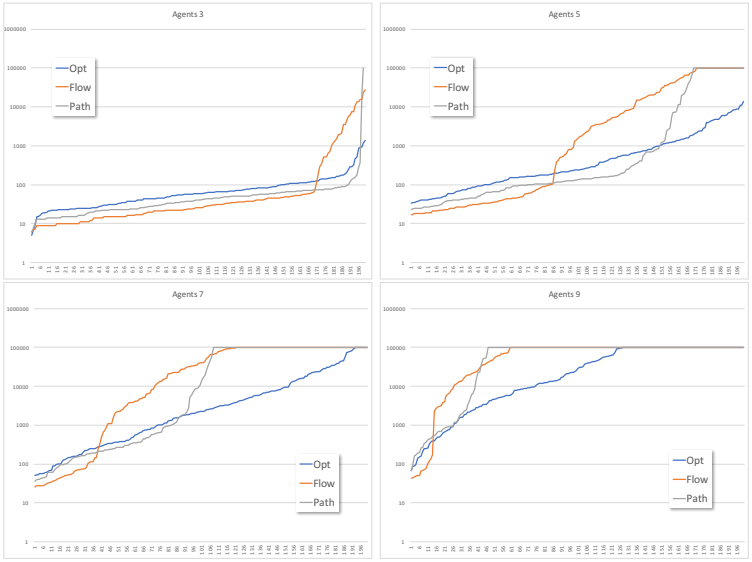

Fig. 4 Dependence of the number of problems solved on time for different numbers of agents (logarithmic scale; time measured in milliseconds).

7 Conclusions

In this paper, we proposed three scheduling models for multi-agent path finding problems. The major motivation was to exploit techniques developed for scheduling problems in a new area, where they have not been used so far. This should allow easier

solving of more realistic problems with various resource and temporal constraints such as non-uniform distances between the nodes and various capacities of nodes (and arcs). The model with optional activities seems the most stable, in particular when the problems are becoming larger. There is an interesting behavior of the Flow model, which is the best for small instances, then it is the worst model for middle-size instances, but the runtime increase seems smaller for larger instances in comparison to other models. This model is more influenced by the size of the graph than the other two models.

There is one significant restriction of the presented models - no agent (package) can return to any node. A future research can study how to extend the models to allow re-visits of the nodes, which is supported by existing solving approaches to MAPF.

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Mathematical Formulation for Minimizing Total Tardiness in a Scheduling Problem with Parallel Machines

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Abstract This paper addresses the NP-hard parallel machine scheduling problem with sequence and machine-dependent setup times for minimizing total tardiness. Mathematical models for this problem often use a constant known as big-M on account of the disjunctive constraints. This yields very weak lower bounds that make it difficult to obtain the optimal solution, even for small-size instances. To address this problem, we propose a mathematical formulation that does not use the big-M constant. To this end, we present an approach that uses dummy jobs instead of the big-M constant. Additionally, an optimality condition method that reduces the solution space of the problem is proposed. Experiments conducted on two instance types produced computational proof of the superiority of the proposed model compared to models based on Wagner's (1959) and Manne's (1960) formulations. The proposed model produced 153 optimal solutions compared to 81 and 42 of Wagner's and Manne's models, respectively, and it was up to three orders of magnitude faster in the 180 instances that were tested.

1 Introduction

In today's competitive business environment of manufacturing and services, efficient scheduling is one of the most critical issues [1]. The parallel machine scheduling problem (PMSP) is broadly applied in many manufacturing and service systems. Therefore, it has been a subject of continuing interest for researchers and practitioners [2]. Many types of PMSPs have been proposed in the literature. They can be classified into identical, uniform, and unrelated parallel machine categories [3]. Of these types, the PMSP which includes the machine and sequence-dependent setup times and total tardiness as criterion has received less attention than other PMSPs [4]. However, with the adoption by companies of the just-in-time philosophy, an increasing amount of research in the past two decades has involved tardiness [5]. Nevertheless, tardiness is a difficult criterion with which to work, even in the single-machine environment [6]. Applications of all PMSP types are common in many industries,

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including painting, plastic, textile, glass, semiconductor, chemical, and paper manufacturing [7].

Exact mathematical programming approaches for scheduling problems use two distinct types of formulations [8]: (1) formulations whereby the job sequence is represented by binary variables and completion times are denoted by continuous variables; and (2) time-indexed formulations, whereby the completion time of each job is represented by binary variables indexed over a discrete time horizon [9]. The formulations of type (2) are known to yield tight linear relaxations; however, they cannot be directly applied to many instances on account of their pseudo-polynomially large number of variables. The formulations of type (1) are compact in that they involve a polynomial number of variables and constraints. On the other hand, they yield poor linear relaxations. This is notoriously due to the big-M constant used to linearize the disjunctive constraints [10]. The formulations of type (2), on the other hand, do not use this constant.

Avalos-Rosales et al. [11] proposed several mixed integer formulations of type (1) for a PMSP to minimize the makespan. These formulations outperform the previously published formulations in terms of the instance size and computational time for reaching optimal solutions. Using these models, it is possible to solve instances up to 60 jobs and five machines that are six times larger than was previously solved. In addition, they enable attainment of optimal solutions for instances of the same size up to four orders of magnitude faster. This is only possible because those authors proposed an additional constraint to calculate the makespan that does not use the big-M constant. We emphasize that these formulations still use this constant in the disjunctive constraints. Unfortunately, these formulations thus cannot be used when the criterion is the minimization of total tardiness once the new linearization applies only to computing the makespan. To the best of our knowledge, it does not exist a formulation of type (1) for PMSPs with tardiness as a criterion that does not use the big-M.

Inspired by the performance achieved by the formulation of Avalos-Rosales et al. [11], we propose a mathematical formulation for the problem under study that does not use the big-M constant. To this end, we employ dummy jobs instead of the big-M constant to linearize the computation of the total tardiness of the jobs. We additionally propose an optimality condition that reduces the solution space of the problem. Computational results showed that the proposed model obtained tight linear relaxations, more optimal solutions, and smaller runtimes when compared to models from the literature. These are the main contributions of this paper.

The remainder of this paper is organized as follows. Section 2 reviews the solution approaches for PMSPs. Section 3 presents two mathematical models from the literature and a new mathematical formulation is proposed. Section 4 describes the computational experiments comparing the mathematical formulations from the literature, and the proposed formulation, and the results are reported. In Section 5, the conclusions are presented.

2 Literature review

This section reviews the previous studies on applying PMSPs. Our review is restricted to PMSPs with the due date and setup times because these features are considered in this paper. For more details, on parallel machine scheduling problems considering due date as a criterion, and setup time, see [12], [13], and [14].

Most previous studies have been conducted on identical or uniform PMSPs only with sequence-dependent setup times. Lee and Pinedo [15] suggest a three-phase heuristic using the apparent tardiness cost with setups (ATCS) rule, a dispatching rule, and a simulated annealing algorithm for minimizing the sum of the weighted tardiness. For minimizing the total tardiness, Park et al. [16] improve the dispatching rule using look-ahead parameters calculated by a neural network. Bilge et al. [17] propose a tabu search approach, whereby the candidate list strategies, tabu classifications, tabu tenures, and intensification/diversification strategies are investigated. Anghinolfi and Paolucci [18] propose a hybrid metaheuristic that incorporates the

core features of simulated annealing, tabu search, and variable neighborhood search. Armentano and de França Filho [19] propose GRASP (*Greedy* Randomized Adaptive Search Procedure)-based search heuristics that incorporate adaptive memory principles.

For the unrelated PMSP with only sequence-dependent setup times, Kim et al. [20] suggest a simulated annealing algorithm with various interchange and insertion methods for minimizing the total tardiness. For minimizing the weighted number of tardy jobs, M'Hallah and Bulfin [21] propose branch and bound algorithms, while Chen and Chen [22] propose hybrid metaheuristics that integrate the tabu search and variable neighborhood descent approach. In addition, Chen [23] presents several iterated hybrid metaheuristic algorithms, while Zhu and Heady [24] propose a mixed integer programming model to minimize the sum of earliness and tardiness penalties.

For the unrelated PMSP with the machine and sequence-dependent setup time, few studies have been performed. For minimizing the total tardiness, Chen [25] considers the problem with an additional strict due date constraint for some jobs. That author proposes a simulated annealing algorithm that incorporates the feasibility improvement method. In addition, Lin et al. [26] propose an iterated greedy algorithm and a simple dispatching rule, which are respectively referred to as primary customers and the shortest completion time, to generate the initial solution.

Meanwhile, Rocha et al. [27] propose a branch and bound algorithm and a GRASP metaheuristic for minimizing the makespan added to the weighted tardiness. Paula et al. [28] propose a non-delayed relax-and-cut algorithm based on a Lagrangian relaxation of a time-indexed formulation to minimize the total weighted tardiness. For minimizing the total earliness and tardiness penalties, Nogueira et al. [29] propose three different heuristics based on the GRASP metaheuristic, and Zeidi and Hosseini [30] propose a genetic algorithm with a simulated annealing method as a local search procedure to improve the solution quality.

3 Parallel machine scheduling problem for minimizing total tardiness

3.1 Problem description

The scheduling problem investigated in this study considers n independent jobs, $J = \{1, 2, \ldots, n\}$, on m unrelated or uniform parallel machines, $I = \{1, 2, \ldots, m\}$. Each machine $i \in I$ is ready at time zero and can process all jobs. Each job $j \in J$ is processed by exactly one of the machines with processing times p_{ij} ($i \in I$), is available in time zero, and it has a due date d_j. A machine and sequence-dependent setup time, s_{ijl}, is incurred between two different jobs, $j \neq l$. The machine setup can be started and completed during the idle time, as commonly assumed in the literature [31]. All the parameters are deterministic non-negative integers. A job sequence is a subset of J processed by a machine in a sequence, in which each job is non-preemptively processed only once. Each job in a sequence has a completion time, C_j, and tardiness is defined as $T_j = \max\{0, C_j - d_j\}$. The aim is to find the set of job sequences that processes all jobs and minimizes their total tardiness. In the standard three-field notation, this problem is denoted as R *or* $Q/s_{ijl}/\Sigma T_j$. It is NP-hard because it is an extension of the NP-hard $1//\Sigma T_j$ [32].

3.2 Wagner's model

Rocha et al. [27] adapted for the parallel machine scheduling problem the models based on sequence-position variables proposed by Wagner [33] and precedence variables proposed by Manne [34], both of which were originally proposed for the job shop problem. In Wagner's model, α_{ijp} is one if job j is processed in machine i in the p-th position (and zero, otherwise), β_{ijlp} is one if jobs j and l are processed by machine i at the p-th and $(p + 1)$-th positions,

respectively, (and zero, otherwise), and t_{ip} denotes the starting time in machine i in the p-th position. In this model, the position amount p is equal to the number of jobs. The model itself is the following:

$$\min \sum_{j \in J} T_j \tag{1}$$

Subject to:

$$\sum_{i \in I} \sum_{p \in J} \alpha_{ijp} = 1, \qquad \forall j \in J \tag{2}$$

$$\sum_{j \in J} \alpha_{ijp} \leq 1, \qquad \forall\, i \in I, p \in J \tag{3}$$

$$\sum_{j \in J} \alpha_{ijp} \leq \sum_{l \in J} \alpha_{il(p-1)}, \qquad \forall i \in I, p \in J, p \geq 2 \tag{4}$$

$$\beta_{ijl(p-1)} + 1 \geq \alpha_{ij(p-1)} + \alpha_{ilp}, \qquad \forall\, j, l, p \in J, j \neq l, p \geq 2, i \in I \tag{5}$$

$$t_{ip} \geq t_{i(p-1)} + \sum_{j \in J} p_{ij}\, \alpha_{ij(p-1)} + \sum_{j \in J} \sum_{\substack{l \in J \\ l \neq j}} s_{ijl} \beta_{ijl(p-1)}, \qquad \forall\, i \in I, p \in J, p \geq 2 \tag{6}$$

$$T_j \geq t_{ip} + p_{ij} - \left(1 - \alpha_{ijp}\right) M - d_j, \qquad \forall\, j, p \in J, i \in I, \tag{7}$$

$$t_{ip} \geq 0, \qquad \forall\, p \in J \tag{8}$$

$$\alpha_{ijp} \in \{0,1\}, \qquad \forall j, p \in J, i \in I, \tag{9}$$

$$\beta_{ijlp} \in \{0,1\}, \qquad \forall j, l, p \in J, j \neq l, i \in I \tag{10}$$

The objective function (1) minimizes the total tardiness of the jobs. Constraints (2) ensure that each job is assigned to only one machine and one position. Constraints (3) ensure that no more than one job is assigned to each position of a machine. Constraints (4) ensure that if a job is assigned to a position p, $p \geq 2$, another job is assigned to position $p - 1$ of the same machine. Constraints (5) determine the sequence of jobs on the machines. Constraints (6) calculate the start time of the positions on each machine. Constraints (7) calculate the tardiness of each job. Finally, the constraints (8) to (10) define the conditions of non-negativity and integrality of the variables.

3.3 Manne's model

In the Manne's model, α_{ij} is one if job j is processed in machine i (and zero, otherwise), β_{ijl} is one if the job l is processed after (not necessarily immediately after) job j in machine i (and zero, otherwise), and t_j denotes the starting time of job j. The model itself is the following:

$$\min \sum_{j \in J} T_j \tag{11}$$

Subject to:

$$\sum_{i \in I} \alpha_{ij} = 1, \qquad \forall j \in J \qquad (12)$$

$$(2 - \alpha_{ij} - \alpha_{il})M + (1 - \beta_{ilj})M + t_j \geq t_l + p_{il} + s_{ilj}, \qquad \forall j, l \in J, j \neq l, i \in I \qquad (13)$$

$$(2 - \alpha_{ij} - \alpha_{il})M + \beta_{ilj}M + t_l \geq t_j + p_{ij} + s_{ijl}, \qquad \forall j, l \in J, j \neq l, i \in I \qquad (14)$$

$$T_j \geq \left(t_j + \sum_{i \in I} p_{ij}\alpha_{ij} \right) - d_j, \qquad \forall j \in J \qquad (15)$$

$$t_j \geq 0, \qquad \forall j \in J \qquad (16)$$

$$T_j \geq 0, \qquad \forall j \in J \qquad (17)$$

$$\alpha_{ij} \in \{0,1\}, \qquad \forall j \in J, i \in I \qquad (18)$$

$$\beta_{ijl} \in \{0,1\}, \qquad \forall j \in J, l \in J, j \neq l, i \in I \qquad (19)$$

The objective function (11) minimizes the total tardiness of the jobs. Constraints (12) ensure that each job is processed by only one machine. Constraints (13) and (14) describe the precedence relationship between the jobs, i.e., for each pair of jobs, (*l*, *j*) or *j* is processed after *l*, or *l* is processed after *j*. Constraints (15) calculate the tardiness of each job. Finally, constraints (16) to (19) define the non-negativity and integrality of the variables.

3.4 Positional model

The proposed model uses the same type of positional variable proposed by Wagner [33]. Hence, it was given the "positional model" name. As Wagner's model, the number of positions per machine is equal to the number of jobs of the problem. The big-M constant is used to determine which of the available positions is occupied. In practice, only a portion of the positions is occupied by jobs. In the positional model, the positions not occupied by jobs (called "real jobs") are now occupied by a job created exclusively for this purpose, called the "dummy job." Thus, all positions are occupied by real or dummy jobs.

The dummy job is represented by zero. The real jobs are allocated to only one position of a machine. The dummy job can be allocated to no position of a machine or to no more than one. The dummy job does not affect the objective function value of the problem; thus, its parameters d_0, p_{i0}, and s_{i0j} must have values equals to zero, and parameters s_{ij0} have large values. Therefore, the dummy job is allocated in the first position, and the real jobs are allocated after the dummy job ($S_1 = \{0, j_1, j_2, ...\}$). In this case, the setup time that occurs is s_{i0j}, which is equal to zero. Consequently, it does not affect the value of the objective function. If the dummy job is allocated between real jobs ($S_2 = \{ j_1,..., 0, .., j_2, ...\}$), one of the setup times that occurs is s_{ij0}, which is a very large value. It is so large that it greatly increases the value of the objective function. Thus, the solution process is induced to place the dummy job before the real jobs and never between them.

The first innovation of the positional model in relation to the presented models is to not use the big-M constant to linearize the disjunctive constraints (or precedence constraints). Then, the model is originally linear and can therefore be quickly resolved [35]. The model has the following variables: x_{ijp} is one if job *j* is processed in machine *i* in the *p*-th position (and zero, otherwise), z_{ijlp} is one if jobs *j* and *l* are processed by machine *i* at the *p*-th and (p + 1)-th

positions, respectively, (and zero, otherwise), C_{ip} is the completion time in the *p*-th position in machine *i*, and T_{ip} is tardiness in the *p*-th position in machine *i*.

The second innovation of the positional model is to use a number of positions per machine (*k*) that is smaller than the total number of real jobs. The aim is to make the most compact model. This is possible because, in practice, the number of jobs allocated by the machine is smaller than the total number of jobs. This is because the problem has more than one machine. This restricts the search space, which can eliminate the optimal solution. Therefore, an optimality condition must be developed in this case.

Proposition 1: The optimality condition defines that, if there is at least one dummy job allocated per machine in the job sequences of all machines, the optimal solution identified when $k < n$ is equal to the optimal solution when $k = n$.

Proof: Suppose there is an optimal solution for $k = n$. Let s^k be a set of optimal sequences for the PMSP with $k < n$, and the sequence of each machine contains a dummy job. In this case, any real job could be reallocated at any position or in place of a dummy job of some other sequence. If this new solution has a lower cost we have a contradiction, because s^k is an optimal solution. Therefore, s^k is really an optimal solution for $k < n$ and also for $k = n$. Suppose now there is a sequence in s^k, for any machine, with *k* real jobs and no dummy job. In this case, increasing the number of positions from *k* to *k*+1, there may be a real job that if reallocated to this sequence would generate a lower cost solution. Therefore, the optimality of s^k is not guaranteed for $k < n$, and it is necessary to increase the value of *k* until there is a dummy job for each sequence or until $k = n$.

It is not known in advance how many jobs will be allocated per machine. Therefore, the number of positions per machine should be adequate for all the real jobs allocated, and at least one dummy job is allocated per machine. In this study, we used the empirical formula 1: $k = \lceil \bar{k} \rceil + 2$, where $\bar{k}$ is the largest amount of jobs found to any of *i* machines after run the linear relaxation of the model using the position number equal to *n*. However, if it fails, add one position and run the model again until the optimality condition be met. Then, the positional model considered *p* positions $P = \{1, 2, \ldots, k\}$. The positional model is presented as follows.

$$Min \sum_{i \in I} \sum_{p \in P} T_{ip}, \tag{20}$$

$Subject\ to$:

$$\sum_{i \in I} \sum_{p \in P} x_{ijp} = 1, \qquad \forall j \in J \tag{21}$$

$$\sum_{j \in J+\{0\}} x_{ijp} = 1, \qquad \forall i \in I, p \in P \tag{22}$$

$$\sum_{\substack{l \in J+\{0\} \\ j \neq l}} z_{ijlp} = x_{ijp}, \qquad \forall i \in I, j \in J + \{0\}, p \in P, p \leq k - 1 \tag{23}$$

$$\sum_{\substack{j \in J+\{0\} \\ j \neq l}} z_{ijlp+1} = x_{ilp}, \qquad \forall i \in I, l \in J + \{0\}, p \in P, p \geq 2 \tag{24}$$

$$C_{i1} = \sum_{j \in J+\{0\}} p_{ij} x_{ij1}, \qquad \forall i \in I \tag{25}$$

$$C_{ip} \geq C_{i,p-1} + \sum_{j \in J+\{0\}} \sum_{\substack{l \in J+\{0\} \\ j \neq l}} s_{ijl} z_{ijlp-1} + \sum_{l \in J+\{0\}} p_{il} x_{ilp}, \qquad \forall i \in I, p \in P, p \geq 2 \qquad (26)$$

$$T_{ip} \geq C_{ip} - \sum_{j \in J+\{0\}} d_j x_{ijp}, \qquad \forall i \in I, p \in P \qquad (27)$$

$$C_{ip} \geq 0, \qquad \forall i \in I, p \in P \qquad (28)$$

$$T_{ip} \geq 0, \qquad \forall i \in I, p \in P \qquad (29)$$

$$x_{ijp} \in \{0,1\}, \qquad \forall i \in I, j \in J + \{0\}, p \in P \qquad (30)$$

$$z_{ijlp} \in \{0,1\}, \qquad \forall i \in I, j, l \in J, j \neq l, p \in P, p \geq 2 \qquad (31)$$

The objective function (20) minimizes the total tardiness in all positions and, consequently, of all jobs. Constraints (21) ensure that each real job is processed by only one machine in one position. Constraints (22) ensure that all positions of all machines are occupied by only one real or dummy job. Constraints (23) and (24) describe the precedence relationships between jobs. That is, for each pair of jobs, (*l*, *j*) or *j* is processed immediately after *l*, or *l* is processed immediately after *j*. Constraints (25) calculate the completion time at the first position of each machine. Constraints (26) calculate the completion time at all positions except the first of each machine. Constraints (27) calculate the tardiness in the positions of each machine. Finally, constraints (28) to (31) define the non-negativity and integrality of the variables.

4 Computational experiments

Two instances types are generated to evaluate the models, both using the number of jobs, $n \in \{10, 20, 30\}$, and the number of machines, $m \in \{2, 3, 4\}$. The first type considers unrelated parallel machines, and the uniform distribution processing times $p_{ij} \sim U[10, 80]$. The second type considers uniform parallel machines, with $p_{ij} = p_j/v_i$, where $p_j \sim U[10, 200]$, and $v_i = i$ (machine indices). The setup times are generated for both types using an uniform distribution $s_{ijl} \sim U[20, 40]$, and are corrected to satisfy triangular inequality. The triangular inequality states that, for any three jobs *j*, *l*, *k* requiring the same resource (machine *i*), the inequality $s_{ijk} \leq s_{ijl} + p_{il} + s_{ilk}$ is ensured. In order to do that the same procedure from Rocha et al. [27] is used. The due dates are generated following the method from Rocha et al. [27], $d_j \sim U[\text{maximal processing time}, 2h/q]$. Parameter *h* is the makespan of identity solution (1, 2, …, *n*), where each job is assigned to a machine capable of finishing it first. Parameter *q* indicates the congestion level of the production system. The larger the *q*, the more congested the system will be, and the more tardy the jobs will be. The values defined for $q \in \{1, 3, 5\}$. For each combination of *n*, *m* and *q* are generated five instances randomly using different seeds. Then, the tests consist of 90 instances for each type.

The mathematical models are implemented with the C++ API for Concert Technology and are solved with IBM ILOG CPLEX 12.5. Tests are performed on a Dell Inspiron notebook, Intel Core i5-2430M 2.40-GHz processor with 4 GB of memory and the Windows 7 operating system. The maximum time allowed for running any model was 3,600 s. If the solver was unable to find the optimal solution, the best integer solution found is reported.

The methods compared are Wagner's (W), Manne's (M), positional using $k = n$ (Pn), and positional using $k \leq n$ (Pk) models. The test results are split into two groups, unrelated instances (Unr), and uniform instances (Unif). The meaning of the table headings is the

following: *n* denotes the number of jobs, *m* is the number of machines, and *q* represents the production system congestion level.

Table 1 shows the average percentage deviation between the best feasible solution (*bfs*) and the linear relaxation (*lr*) obtained by the method, which is calculated as 100 * (*bfs* − *lr*) / *lr*. Table 2 shows the number of instances that are unsolved in terms of optimality for each combination. Table 3 shows the average percentage gap, which is calculated as 100 * (*bfs* – *blb*) / *blb*, where *blb* is the lower bound obtained by the method. Table 4 shows the elapsed CPU times in seconds to solve the instances.

Table 1 Linear relaxation deviation (%) of instances solved by Wagner's (W), Manne's (M), and positional (Pn) models.

	n	m	q=1			q=3			q=5		
			W	M	Pn	W	M	Pn	W	M	Pn
Unr	10	2	100.00	46.37	100.00	100.00	91.25	14.13	100.00	91.08	2.94
	10	3	100.00	34.91	100.00	100.00	88.70	9.91	100.00	87.05	3.54
	20	3	100.00	17.44	100.00	100.00	96.43	24.65	100.00	94.55	4.61
	20	4	100.00	32.39	100.00	100.00	93.32	22.76	100.00	93.18	4.98
	30	3	100.00	28.53	100.00	100.00	98.87	31.65	100.00	97.91	5.78
	30	4	100.00	41.28	100.00	100.00	95.32	28.09	100.00	96.14	5.99
	Average		100.00	33.49	100.00	100.00	93.98	21.87	100.00	93.32	4.64
Unif	10	2	100.00	32.54	100.00	100.00	92.43	17.26	100.00	92.89	4.19
	10	3	100.00	54.21	100.00	100.00	90.32	17.81	100.00	91.43	5.16
	20	3	100.00	18.43	100.00	100.00	97.15	26.79	100.00	95.17	5.36
	20	4	100.00	34.53	100.00	100.00	94.74	25.73	100.00	94.53	4.57
	30	3	100.00	29.76	100.00	100.00	98.49	24.98	100.00	98.01	5.50
	30	4	100.00	45.78	100.00	100.00	96.71	35.19	100.00	96.78	5.41
	Average		100.00	35.88	100.00	100.00	94.97	24.63	100.00	94.80	5.03

Table 2 Unsolved number of instances using Wagner's, Manne's, and positional (Pn and Pk) models.

	n	m	q=1				q=3				q=5			
			W	M	Pn	Pk	W	M	Pn	Pk	W	M	Pn	Pk
Unr	10	2	0	0	0	0	2	0	0	0	4	2	0	0
	10	3	0	0	0	0	4	1	0	0	5	3	0	0
	20	3	3	0	0	0	5	5	0	0	5	5	0	0
	20	4	3	0	0	0	5	5	0	0	5	5	0	0
	30	3	5	3	3	2	5	5	5	1	5	5	4	0
	30	4	5	3	2	1	5	5	4	1	5	5	3	0
	Total		16	6	5	3	26	21	9	2	29	25	7	0
Unif	10	2	0	0	0	0	1	0	0	0	4	2	0	0
	10	3	0	0	0	0	4	1	0	0	4	3	0	0
	20	3	2	0	0	0	5	5	2	2	5	5	0	0
	20	4	3	0	1	1	5	5	2	0	5	5	0	0
	30	3	4	1	4	2	5	5	5	5	5	5	4	4
	30	4	5	0	5	1	5	5	5	5	5	5	5	2
	Total		14	1	10	4	25	21	14	12	28	25	9	6

Table 3 Gap (%) of instances solved by Wagner's, Manne's, and positional (Pn and Pk) models.

	n	m	q=1				q=3				q=5			
			W	M	Pn	Pk	W	M	Pn	Pk	W	M	Pn	Pk
Unr	10	2	0.00	0.00	0.00	0.00	9.21	0.00	0.00	0.00	29.01	17.01	0.00	0.00
	10	3	0.00	0.00	0.00	0.00	22.39	2.14	0.00	0.00	26.78	12.37	0.00	0.00
	20	3	31.24	0.00	0.00	0.00	99.12	97.88	0.00	0.00	96.39	95.43	0.00	0.00
	20	4	45.81	0.00	0.00	0.00	98.19	96.54	0.00	0.00	95.43	94.53	0.00	0.00
	30	3	71.51	17.23	60.00	40.00	99.98	99.12	21.45	2.34	99.80	98.76	2.10	0.00
	30	4	82.12	19.32	40.00	20.00	99.17	98.91	17.89	1.54	98.81	97.89	1.98	0.00
Average			38.45	6.09	16.67	10.00	71.34	65.77	6.56	0.65	74.37	69.33	0.68	0.00
Unif	10	2	0.00	0.00	0.00	0.00	12.45	0.00	0.00	0.00	32.39	11.70	0.00	0.00
	10	3	0.00	0.00	0.00	0.00	24.32	3.22	0.00	0.00	29.54	24.54	0.00	0.00
	20	3	38.67	0.00	0.00	0.00	94.76	92.89	8.26	4.32	98.64	92.23	0.00	0.00
	20	4	54.89	0.00	20.00	18.64	93.19	91.90	3.06	0.00	97.54	92.35	0.00	0.00
	30	3	78.67	0.37	80.00	40.00	97.43	95.94	18.88	14.68	99.12	96.50	3.41	1.70
	30	4	97.34	0.00	100.00	20.00	98.14	96.71	24.16	19.18	98.77	96.98	3.01	0.76
Average			44.93	0.06	33.33	13.11	70.05	63.44	9.06	6.36	76.00	69.05	1.07	0.41

Table 4 Runtimes of instances solved by Wagner's, Manne's, and positional (Pn and Pk) models.

	n	m	q=1				q=3				q=5			
			W	M	Pn	Pk	W	M	Pn	Pk	W	M	Pn	Pk
Unr	10	2	53.43	0.03	14.34	2.34	1984.30	609.43	1.23	0.54	3378.91	2459.51	2.43	0.76
	10	3	27.65	0.36	69.12	8.90	3456.21	803.21	1.67	0.44	3600.00	2546.23	2.14	0.89
	20	3	1987.23	1.98	212.34	21.45	3600.00	3600.00	345.98	45.67	3600.00	3600.00	31.25	3.29
	20	4	2134.28	2.34	1239.54	309.34	3600.00	3600.00	53.98	4.32	3600.00	3600.00	23.87	4.32
	30	3	3600.00	2436.90	2376.89	1437.89	3600.00	3600.00	3600.00	2189.12	3600.00	3600.00	2980.32	1587.30
	30	4	3600.00	2567.87	1984.32	1134.54	3600.00	3600.00	3412.98	1897.21	3600.00	3600.00	2371.72	1187.32
Average			1900.43	834.91	982.76	485.74	3306.75	2635.44	1235.97	689.55	3563.15	3234.29	901.96	463.98
Unif	10	2	57.89	0.33	16.93	5.05	2087.18	1127.46	2.17	2.05	3409.12	2759.03	1.06	0.51
	10	3	29.07	0.52	76.28	10.20	3309.12	1442.43	2.71	1.78	3376.91	3305.13	1.77	1.01
	20	3	2178.34	1.33	279.70	26.84	3600.00	3600.00	1918.16	1600.33	3600.00	3600.00	124.79	47.86
	20	4	2098.43	3.79	1898.19	777.83	3600.00	3600.00	2230.35	1054.74	3600.00	3600.00	82.06	34.84
	30	3	3509.32	727.67	2996.56	1853.55	3600.00	3600.00	3600.00	3600.00	3600.00	3600.00	3600.00	2922.86
	30	4	3600.00	4.35	3600.00	1447.77	3600.00	3600.00	3600.00	3600.00	3600.00	3600.00	3600.00	3027.06
Average			1912.18	123.00	1477.94	686.87	3299.38	2828.31	1892.23	1643.15	3531.01	3410.69	1234.95	1005.69

Analyzing the results of the linear relaxations of the models it can be observed that the Wagner's model obtains value equal to 0 for all unrelated or uniform instances. The Manne's model obtains tighter values than the positional model (Pn) when $q = 1$, while the positional model obtains tighter values when $q = 3$ and 5. For example, 5.03% and 94.80% at average for positional and Manne's models, respectively, at uniform instances and $q = 5$. The positional model obtains tighter values as the congestion level increases. The linear relaxation of the positional model considered $k = n$.

The positional model (Pn and Pk) obtains more optimal solutions than the other models. This only does not occurred in uniform instances and q = 1. The Pk model fails to find 27 optimal solutions considering all instances, while the Manne's model no longer obtains 99. The positional model Pk obtains more optimal solutions than the Pn model, the biggest difference is in the unrelated instances and q = 3, 30 versus 23, respectively. The number of unresolved instances until optimality is higher in the uniform type (169) than in the unrelated type (149) considering all models evaluated. The positional model obtains the optimal solution for all unrelated instances with 10 and 20 jobs. For the number of jobs equal to 30 the models have more difficulties to obtain the optimal solutions.

The smaller gaps are presented by Manne's model for q = 1, while for q = 2 and 3 are presented by the positional model. The more congested is productive system, the more the jobs are tardy, and better positional model gaps in comparison to the other models. For example, the Pk model obtained an average gap 0% versus 60.33% of Manne's model in unrelated instances and q = 3.

The positional model presents the shortest computational times between the evaluated models, except in the uniform instances and q = 1. The more congested is the productive system, faster the positional model is compared to the other models. This difference is up to 0.51 s versus 2759.03, that is, three orders of magnitude faster, in the uniform instances and q = 3, considering the Pk and Manne's models, respectively. The lower number of positions per machine in the Pk model compared to the Pn model helped to reduce computational time.

Of all the analyzed criteria, Wagner's model obtains performance well below that of the positional model and even in relation to the Manne's model. This aspect would be investigated if there are other studies with similar results. Thus, the work of Lange and Werner [36] was found to use models based on precedence and position variables on a parallel machine approach to minimize the total tardiness of a single-track train scheduling problem. In tests performed with instances of ten jobs or more, the model based on position variables was not able to obtain an optimal solution within 2 h of runtime, whereas the model based on precedence variables obtained the optimal solution in 2.02 s on average.

It was observed into the instances with q = 3 and 5 that the number of jobs allocated by the machine by linear relaxation (k'_i) and by the final solution (k_i) maintained the following relation: $k_i \leq \lceil k'_i \rceil + 2$. This is because the linear relaxations of these instances are tighter. For q = 1 this relation was not identified and the relation of formula 1 was maintained.

A brief resume comparing the positional model (Pk) and the best model from the literature (Manne) is made following. In relation to the mean percentage deviation of the linear relaxation value and the best integer solution is 42.69% versus 74.41%, this difference is greater at congestion level q = 3, 4.84% versus 94.06%, in relation to the number of optimal solutions is 153 versus 81, in relation to the mean gap was 5.09% versus 45.62%, and in relation to the average runtime was 829.17 s versus 2177.78 s, respectively. The same comparison can be made between to the Pn and Pk models to observe the impact of reducing the number of position in model performance is 153 versus 126, 5.09% versus 11.23%, and 829.16 s versus 1287.63 s, respectively.

The above results indicate that the positional model has the expected effect and that, unlike the time-indexed formulations, the positional model could be solved in a computational time (see Table 4) as reasonable as those of the models based on the formulation of type 1 (see Section 1). The positional model uses positional variables, similar to Wagner's model, and obtains the best computational performance compared to it and Manne's model. It is thus numerically proved that the proposed innovations are the contributing factors of the achieved improvement.

5 Conclusion

The computational results show that the positional model is more efficient when compared to other models from literature. This because is eliminated the use of the big-M

constant. Consequently, the model is linear and easier to solve [37]. In addition, reduction of the number of position (Proposition 1) helps to reduce the computational effort.

The use of unrelated and uniform instances has shown that the positional model obtains the best performance and formula 1 is always met even with very different processing times. Therefore, a high degree of variability for the processing times in the uniform instances was chosen with a variation up to 4 times in the speed between the machines (i = 4). In unrelated instances the number of jobs allocated per machine varies around an average. While in the uniform instances the number of jobs allocated to the faster machines (smaller index) is higher than in the slower machines. This information can be used to define a different k per machine type in order to reduce the computational time required.

The Pk model is more efficient than the Pn model though it needs to perform the linear relaxation with n positions per machine to define the value of k before running with the variables with the integrality condition. To help illustrate this, 153 versus 126 optimal solutions, 5.09% versus 11.23% mean gap, and 829.16 s versus 1287.63 s runtime, are obtained for the Pk and Pn models, respectively.

Future work will involve determining a means to eliminate the negative effect that loose due dates have on decreasing the efficiency of the positional model for q = 1. Other future work is to develop a decomposition algorithm. In addition, these approaches could be tested in other scheduling problems that use the big-M constant.

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Balanced clustering based decomposition applied to Master thesis defense timetabling problem

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Abstract Timetabling is an area of increasing interest in recent decades due to its practicality and complexity. In this paper we present an algorithm using decomposition approach based on balanced clustering to solve a real-world university exam timetabling problem in Vietnam. Essential elements in this problem are the considering of professor - theses similarity and workload balancing. The first stage in the algorithm is clustering part of the inputs (theses) while balancing the clusters' quantity. The second stage involves a metaheuristic search that attempts to find the best way to partition the rest of the inputs (professors) through solving the sub-problems in parallel and recomposing partial solutions . Test results for real-world instances are presented.

1 Introduction

Timetabling is a crucial and extremely time consuming task in many educational institutions. The general term *university timetabling* typically refer to university course and examination timetabling, both have been studied widely within the academic literature (see [1], [5]). The problem is often formalized as a combinatorial constraint optimization problem which involves finding an assignment of variables to appropriate values, evaluated by constraints and objective functions.

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The main contribution of this work consists of a balanced clustering based decomposition approach to solve a real-world examination timetabling problem with the considering of similarity matching and workload balancing, namely the Master Thesis Defense Timetabling problem (MTDT). This is an arising problem in Vietnamse university education and is still solved by human effort. The first stage in the proposed algorithm relates to clustering part of the inputs (theses) while balancing the clusters' quantity, using the K-means balanced clustering algorithm. Exploiting the result of the first stage by Bipartite Matching, the second stage involves a metaheuristic search that attempts to find the best way to partition the rest of the inputs (professors) through recomposing partial solutions of sub-problems.

The paper is organized as follows. Section 2 reviews the related work. Section 3 describes the Master defense thesis timetabling problem as it is perceived at the universities in Vietnam. This timetabling problem is quite different from most problems encountered in the literature, since it deals with similarity matching and workload balancing. We are not aware of any similar problems in literature. We discuss the proposed algorithm using Balanced-clustering heuristic and Decomposition strategy in Sections 4. Experimental results are presented in Section 5. Section 6 concludes the paper and draws future research directions.

2 Related work

Timetabling within a university context has long been recognized as a difficult problem from both theoretical and practical perspective. Decomposition is one of the popular solution methods for this kind of problem [2]. The main idea of this method is "divide and conquer": a large problem is broken into smaller sub-problems which are easier to solve optimally because the search spaces of these sub-problems are significantly diminished. The method has proved its effectiveness in many works in examination timetabling problems [2,3]. Burke and Newell (1999) investigated a decomposition approach by using sequential heuristics to assign the first set of exams which were evaluated as the most difficult ones by graph coloring heuristics. In practice, the algorithm dramatically reduced the time required and also provide high quality solutions on the real-world data [2,3]. Qu and Burke (2007) developed a new general adaptive decomposition technique that partitioned iteratively the problem into two subsets, namely the $difficult\ set$ and the $easy\ set$, by the difficulty of scheduling them in previous iterations [4]. Matias Sorensen and al [6] (2013) applied a two-stage decomposition using bipartite matching and integer programming to solve a practical timetabling problem in Denmark. Magana-Lozano and al [7] (2014) investigated an approach which decomposed a given problem into smaller sub-problems to solve and then sequentially recomposed the partial solutions into a complete solution. Ali Hmer and al [8] (2014) presented an multi-phase hybrid metaheuristics approach which decomposed the solving progress of the problem into 3 phases: pre-processing, construction and enhancement. However these algorithms are often specific and slight changes in the problem definition can raise difficulty in the adaptation of the special purpose algorithms.

Clustering, a process aiming at grouping the data such that homogeneous data fall in the same group, has long been identified and researched in various fields of data mining, i.e. machine learning, pattern recognition, image analysis, bioinformatics, computer graphics. Clustering itself is not a specific algorithm but the general task to be solved. Among the algorithms solving this task, K-means clustering algorithm is one of the most popular algorithm [12], [13] thanks to its simplicity, efficiency, flexibility and easy implementation [8]. Optimization in the K-means often relates to better initialization, avoid the local optimum,

scalability but balancing the quantity between formed clusters is also an interesting aspect to consider. In balanced clustering problem, there are two conflicting objectives to concern:

- Minimizing Mean square error (MSE)
- Balancing cluster sizes

The balanced constrained algorithms prior to balancing objective and treat the traditional minimizing MSE objective as secondary criterion. Bradley and al. (2000) [10] added the constraints to the original K-means algorithm that force each cluster should have at least a predefined number of points in each cluster, then solve the clustering assignment step as a Minimum Cost Flow (MCF) linear network optimization problem [14]. Zhu and Li (2010) [11] proposed the method that uses additional prior knowledge to constrain the size of each cluster, then presented a heuristic algorithm to transform size constrained clustering problems into integer linear programming problem. Malinen and Franti (2014) [9] introduced an algorithm that provide us a strictly balanced result. The algorithm replaces the assignment step of the traditional k-means clustering by bipartite matching between n data points and n pre-allocated cluster slots; each cluster has exact n/k data points. The new assignment step is solved by using Hungarian algorithm [15]. Although balanced clustering has the potential to solve various applications, i.e workload balancing, circuit design, image processing [8], it has not been well studied in timetabling in particular and applications in general.

Thesis defense timetabling problem is an arising problem in educational context in the world. The problem may be various due to the different policy of each country. Kochaniková et al. (2013) [21] proposed a local search solver to deal with a thesis defense timetabling problem instance in Czech Republic; their policies is quite different than ours. Battistutta et al. (2015) [19] also introduced a local search method using Stimulated Annealing to deal with the thesis defense timetabling problem in Italian universities. For vietnamsese universities, Huynh and al (2012) first addressed this problem, formulated a model for the problem and solved it using genetic algortithm [7]. Bui and al (2012) [20] modeled the problem as a bi-objective then used direction-based multi-objective evolutionary algorithm to tackle. Pham and al (2015) [8] used an updated model for the problem and applied Tabu Search metaheuristic along with Constraint-based Local Search architecture to solve. In this paper, we will present an improved model of MTDT problem in which the professors attending the defense session are organized into juries, each jury is placed in a room, covers some committees of the defense session.

3 MTDT problem

3.1 Problem description

Schedule the timetable for master thesis defense is a struggling mission that staffs must carry out in most of Vietnamese universities due to some policies. In each defense session (two or three defense sessions are open each academic year), there is a set of master students who will defense their thesis. Each student has one master thesis being scheduled in the defense session, henceforth, we use thesis-student instead of student or his thesis. The jury of each thesis-student consists of five members: two examiners, a president, a secretary, and an additional member and this jury must be scheduled in one room and a slot of the session satisfying a given set of constraints. Among five members of the jury, there must be two members who are not professors/lecturers of the university and who are invited to participate

in the jury: one is an examiner and the other is additional member. The supervisor of a thesis-student cannot be a member of the jury of that thesis-student. Two juries sharing a member must be scheduled in two different slots. The assignments of the professors/lecturers to juries should optimize some objectives, for instance, the occurrences of professors/lecturers in juries should be balanced, the theme of a thesis-student should match with the expertise of two examiners participating in the jury of that thesis-student, etc. We describe in the following section the mathematical formulation of the problem.

3.2 Problem formulation

Input

- $S = \{0, \ldots, n-1\}$: the set of thesis-students, for each thesis-student s, $sup(s)$ is the supervisor of s
- $IP = \{0, \ldots, m_1 - 1\}$: set of professors of the university
- $EP = \{m_1, \ldots, m_1 + m_2 - 1\}$: set of professors outside the university
- $P = IP \cup EP$: set of professors participating the defense schedule
- $l(p)$: represents the level of professors $p, \forall p \in P$ (i.e., 1: doctor, 2: associate professor, 3: full professor)
- $R = \{0, \ldots, r-1\}$: the set of rooms
- $T = \{0, \ldots, t-1\}$: the set of time slots
- $K = \{0, \ldots, k-1\}$: the set of specialization keywords
- $Ks(s) = [sk_1, \ldots, sk_k]$: vector represent match between thesis s and keywords, $s \in S$
- $Kp(p) = [pk_1, \ldots, pk_k]$: vector represent match between professor p and keywords, $p \in S$
- $m(s,p) = Ks(s).Kp(p)^\intercal$: the score that measures the matching of expertise of professor p and the thesis-student s

Decision variables

- $xp(s,i)$ represents the professor assigned in the i^{th} position of the committee of thesis-student s
  - $xp(s,1)$ represents the examiner 1, $xp(s,1) \in EP$
  - $xp(s,2)$ represents the examiner 2, $xp(s,2) \in IP$
  - $xp(s,3)$ represents the president, $xp(s,3) \in IP$
  - $xp(s,4)$ represents the secretary, $xp(s,4) \in IP$
  - $xp(s,5)$ represents the commissioner, $xp(s,5) \in EP$
- $xr(s)$: room of the jury of the thesis-student s

Invariants

- $o(p) = \sharp\{(i,s) \mid i \in \{1, \ldots, 5\} \wedge s \in S \wedge xp(s,i) = p\}$: number of juries that the professor p participates in
- $e(p) = \sharp\{(i,s) \mid i \in \{1,2\} \wedge s \in S \wedge xp(s,i) = p\}$: number of times professor p is scheduled as examiner
- $or(r) = \sharp\{s \mid s \in S \wedge xr(s) = r\}$: number of committees that the jury in room r covers
- $minP = \min_{p \in P}\{o(p)\}$
- $maxP = \max_{p \in P}\{o(p)\}$

Hard Constraints

- **H0. Internal-external member policy**: in a commitee, examiner 1 and commissioner come from outside of the host university; examiner 2, chairman and secretary come from the host university:
  $xp(s,1) \in EP \wedge xp(s,2) \in IP \wedge xp(s,3) \in IP \wedge xp(s,4) \in IP \wedge xp(s,5) \in EP, \forall s \in S$
- **H1. Member conflict**: all members in each committee must be different from each other:
  $xp(s,i) \neq xp(s,j), \forall 1 \leq i < j \leq 5, s \in S$
- **H2. Supervisor-member conflict**: all members in each committee must be different from the supervisor of the student:
  $xp(s,i) \neq sup(s), \forall s \in S, i \in 1..5$
- **H3. Chairman-secretary academic rank policy**: in each committee, the academic rank of chairman must be equal or higher than that of secretary:
  $l(xp(s,3)) \leq l(xp(s,4)), \forall s \in S$
- **H4. Examiner appearance restriction**: restrict the number of times that each professor is scheduled as examiner:
  $e(p) \leq \lambda, \forall p \in P$
- **H5. Room-time conflict**: restrict the number of times that each room is used - must be less or equal than the number of time slots:
  $\sharp\{s_i \in S \mid xr(s_i) = r\} \leq \sharp\{T\}, r \in R$
- **H6. Jury policy**: Each professor must attend only 1 jury, which places in a room:
  $xp(s_1,i) = xp(s_2,j) \Rightarrow xr(s_1) = xr(s_2), \forall s_1 \neq s_2 \in S, i,j \in 1..5$

Objective functions

- **F1. Professor workload balancing**: All professor should be spread over the positions of committees. The function reflects the gap between the most and the least number of appearance of professors:
  $F_1 = maxP - minP$
- **F2. Professor-thesis expertise matching**: The two examiners should match with the thesis in all committees. The function reflects the total match between examiners and theses in committees:
  $F_2 = \sum_{s \in S} m(s, xp(s,1)) + m(s, xp(s,2)$

With the above formulation, we can then use the weighted-sum method to calculate the fitness function as defined in formula (1). The goal is then to find a feasible solution that minimize the fitness function

$$F = w_1 * F_1 + w_2 * F_2 \tag{1}$$

with w_1, w_2 is the weight of objective function F_1, F_2.

The table below illustrates an example of a schedule:

Example (Valid defense session schedule)

Student-thesis: [S0, S1, S2, S3, S4, S5]
Internal Professor: [P0, P1, P2, P3, P4, P5]
External Professor: [P6, P7, P8, P9]
Room: [R0, R1]
Slot: [T0, T1, T2]

St	Sup	Ex1	Ex2	Ch	Se	Cm	Rm	Sl
S0	P4	P6	P2	P1	P5	P7	R0	T0
S1	P2	P8	P4	P0	P3	P9	R1	T0
S2	P5	P8	P4	P0	P3	P9	R1	T1
S3	P0	P6	P5	P1	P2	P7	R0	T1
S4	P4	P7	P1	P2	P5	P6	R0	T2
S5	P5	P9	P3	P0	P4	P8	R1	T2

Table 1: Example of a schedule

Table 1 gives an example of the defense with 6 students $\{S0, S1, S2, S3, S4, S5\}$, 6 internal professors $\{P0, P1, P2, P3, P4, P5\}$, 4 external professors $\{P6, P7, P8, P9\}$, 2 rooms $\{R0, R1\}$ and 3 time slots $\{T0, T1, T2\}$. Column 1 represents the student-thesis, column 2 represents the supervisor of the student. Columns 3-6 represent the members of the committees, namely the examiner 1, the examiner 2, the chairman, the secretary and the commissioner. The schedule partitions the professors into 2 juries: jury 1 with professors $\{P1, P2, P5, P6, P7\}$ is in charge of committees of students $\{S0, S3, S4\}$, jury 2 with professors $\{P0, P3, P4, P8, P9\}$ is responsible for committees of students $\{S1, S2, S5\}$. All committees of each jury will take place in only one room (jury 1 in room R0, jury 2 in room R1), guarantees that the professors don't have to change the room during the defense session, thanks to the hard constraint **H6**. This is an important constraint in practical and an improvement from the work [8]. It saves the professors from moving relentlessly between rooms during time slots and avoids the time asynchronous problem (sometimes there are overtime committees). The table is also an example for an ideal solution because all two objective functions have the best value: all professors attends the defense session equally (3 times) and all examiners are best match with the theses.

4 Proposed Algorithm

The proposed algorithm decomposes the solving of the problem into two main phases. The first phase involves in clustering the theses of students based on their specialization and equally in terms of quantity, each cluster will be assigned to a jury. The second phase of the algorithm relates to finding best way to partition the professors into juries using decomposition and recomposition partial solutions of sub-problems.

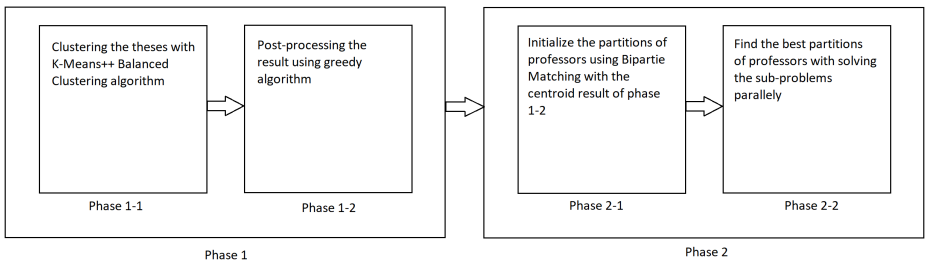

Table 2: Overview of the proposed algorithm

4.1 Balanced Clustering Techniques

In general, exam timetabling is the problem of allocating limited resources such as time slots, rooms to given set of exams. One of the valuable information that can be exploited is the relation of the exams, i.e: the common students between two exams, the condition of facilities required (room capacity, equipment). This relation affects how the resources should be assigned to the exams; for example exams that shares the same students, same condition required should not be scheduled in the same time slot. Analyzing the structure of feasible high quality solutions can help us to recognize the extra features of these solutions and using them in the search strategy by decomposing the original problem into smaller problems to solve. Note that this method cannot be applied to all problems because two reasons: early assignments may lead to later infeasibility and some soft constraints / objective functions cannot be evaluated when the problems are decomposed [1], but if we succeed to overcome these drawbacks, we can save the great amount of computation because the search spaces of the sub-problems are significantly smaller than that of the original problem [15].

In the MTDT problem, we first balanced clustering the thesis-students based on their specializations, due to some reasons below:

- In a feasible solution, the thesis-students are splited into clusters, each cluster is taken by a jury, due to constraint H_6. Therefore, the theses in a cluster should relevant to each other and match with the specializations of the members in the jury in order to optimize the objective F_1.
- Balancing the number of thesis-students in each clusters is the best way to balancing the appearance of the professors - objective F_2, because each professor is fixed in a jury and therefore the workload of a professor has close connection to the number of thesis-students taken by his jury, in many cases they are equal when the number of professor in a jury is limited.
- Balancing the number of thesis-students in each cluster helps to always guarantee the constraint H_5 (Room-time conflict) because when the thesis-students are balanced clustered, there are no cluster having too much number of student-thesis that may lead to the exceeding usage of time slot.

The sub-problem of balanced clustering the thesis-students by their specialization (phase 1-1) can be defined as follow:

Phase 1-1: Clustering the objects $(x_0, x_1, ..., x_{n-1})$ which are the set of vectors representing the match between theses and the specialization keywords; into k clusters $C = C_1, C_2, ..., C_k$ with k is the number of juries, such as each cluster C_k has n/k objects and optimizing the WCSS function:

$$\underset{C}{argmin} \sum_{i=1}^{k} \sum_{x \in C_i} ||x - \lambda_i||^2 \tag{2}$$

with λ_i is the centroid of cluster C_i

To tackle the problem defined above, we applied the Balanced K-means clustering algorithm, which is introduced in [9]. Because the number of objects in the real instances often not so large, we used the K-means++ in the centroid initialization step rather than choosing centroid randomly as the original algorithm. We named the enhanced algorithm "Balanced K-means++ clustering" whose pseudo-code is as below:

Algorithm 1 Balanced K-means++ clustering (BKPC)

1: **procedure** BALANCED K-MEANS++ CLUSTERING($\{x_1, x_2, .., x_n\}$, K)
2: $\{s_1, s_2, .., s_n\} \leftarrow$ *K-means++*($\{x_1, x_2, .., x_n\}, K$)
3: **for** $k \leftarrow 1$ to K **do** $\mu_k^0 \leftarrow s_k$
4: **while** (stopping condition not met) **do**
5: **for** $k \leftarrow 1$ to K **do**
6: $\omega_k^{t+1} \leftarrow \{\}$
7: **for** $n \leftarrow 1$ to N **do**
8: $j \leftarrow bipartiteMatching(x_n, \{cl_1^t, cl_2^t, .., cl_n^t\})$
9: $\omega_j \leftarrow \omega_j \cup \{x_n\}$ *(reassignment of vectors)*
10: **for** $k \leftarrow 1$ to K **do**
11: $\mu_k \leftarrow \frac{1}{|\omega_k|} \sum_{x \in \omega_k} x$ *(recomputation of centroids)*
12: **return** $\{\mu_1, \mu_2, .., \mu_n\}$

The key of the Balance Clustering algorithm is the replacing of assigning the objects to the cluster which has nearest centroid in the original K-mean algorithm by solving the Bipartite Matching problem, which is shown in line 8. Because the goal is to cluster the objects equally in terms of quantity, we can pre-allocate n cluster slots for n objects, each cluster will have the same size with n/k objects (assuming that n is divisible by k, in general case there will be (n mod k) clusters of size $\lceil n/k \rceil$, and k - $(n \, mod \, k)$ clusters of size $\lfloor n/k \rfloor$. Then, the assignment problem between objects and clusters will become the Bipartite Matching problem between n objects and n cluster slots that minimize the Mean Square Error, using the Hungarian algorithm [16]. The elements of weight matrix W will be calculated by:

$$W(i,j) = d(x_i, \mu_{j \, mod \, k}) \tag{3}$$

with

- $W(i,j)$: element of row i, column j of weight matrix
- x_i: specialization vector of student-thesis i
- $\mu_{j \, mod \, k}$: the centroid $c(j)$ of cluster slot j, determined by the formula: $c(j) = j \, mod \, k$
- $d(x_i, \mu_{j \, mod \, k})$: distance from the vector x_i to the centroid of the cluster slot sl_j

The following example will illustrates the Bipartite Matching step in BKPC with an real instance of MTDT problem:

Example

- No of thesis-students = 6
- No of rooms(cluster) = 2
- Cluster slots 0,2,4 ∈ cluster 0, cluster 1,3,5 ∈ cluster 1

Students \ Cluster Slots	CIS 0	CIS 1	CIS 2	CIS 3	CIS 4	CIS 5
Student 0(S0)	0.49	1.92	0.49	1.92	0.49	1.92
Student 1(S1)	1.77	0.37	1.77	0.37	1.77	0.37
Student 2(S2)	1.82	0.83	1.82	0.83	1.82	0.83
Student 3(S3)	0.11	1.69	0.11	1.69	0.11	1.69
Student 4(S4)	0.05	1.86	0.05	1.86	0.05	1.86
Student 5(S5)	0.8	0.09	0.8	0.09	0.8	0.09

- Result: Cluster 0 = [J0, J3, J4]; Cluster 1 = [J1, J2, J5];

Table 3: Example of the Bipartite Matching step in BKPC

There are 6 students in this instance, divided into two clusters. The weight matrix W has the size of 6x6. Applying the Hungrian algorithm we get the assignment result (S_0, Cl_0), (S_1, Cl_3), (S_2, Cl_1), (S_3, Cl_2), (S_4, Cl_4), (S_5, Cl_5). Eventually, we get two clusters $C_0 = \{S_0, S_3, S_4\}$ and $C_1 = \{S_1, S_2, S_5\}$.

After the phase 1-1, the output of the algorithms are the clusters of thesis-students which balancing the cluster sizes and optimizing the similarity. However, as mentioned before, one of the most common problem when using decomposition technique is that early assignments may lead to later infeasibility. In this particular case, the later infeasibility can happen when there is a professor that is the supervisor of at least 1 student in every cluster.

Example

Student	Sup	UnSat Clustering
S0	P4	C0
S1	P2	C1
S2	P5	C1
S3	P0	C0
S4	P4	C1
S5	P5	C0

Table 4: Example of later infeasibility

In this example, the 6 students $S_0, S_1, ..., S_5$ are clustered into two clusters: cluster C_0 with students $\{S_0, S_3, S_5\}$ and cluster C_1 with students $\{S_1, S_2, S_4\}$ after the phase 1-1. However, the professors P4 and P5 appear as the supervisor of a student in each cluster and therefore in phase 2, they cannot be assigned to any jury due to the hard constraint H2 - Supervisor-member conflict. Although this situation is quite rare in real-world instances because theses of the same supervisor often has quite similar topic and should not be spread into all clusters, the following greedy algorithm is used to overcome it:

Post-processing algorithm (Phase 1-2)

- **Step 1**: Get the $P_{vio} = \{p \in P \,|\, \exists s_i \in C_i, sup(s_i) = p\,, \forall\, i \in R\}$ is the set of professor that appear as supervisor of at least 1 student in each cluster
- **Step 2**: If $P_{vio} = \emptyset$ then stop. Else, choose $p_1 \in P_{vio}$, $C_1 \in C$ such as $o(p_1, C_1) = \sharp\{sup(s) = p_1, s \in C_1\}$ is minimal, choose $s_1 \in C_1 \,|\, sup(s_1) = p_1$
- **Step 3**: If p_1 is the only element in the set P_{vio}, choose $(s_2, C_2) \,|\, C_2 \neq C_1, sup(s_1) \neq sup(s_2)$ such as swapping s_1 and s_2 that maintains best the quality of clusters (the change of the WCSS function when swapping is minimized). Else, choose $p_2 \in P_{vio}, s_2 \in C_2$ with $o(p_2, C_2) = min\{o(p_i, C_j) \,|\, p_i \neq p_1, C_j \neq C_1\}$
- **Step 4**: Swapping the student s_1 and s_2 then back to step 1

The goal of this greedy algorithm is trying to get rid of the situation of professors appearing as supervisor in every cluster by making as minimum swapping move of thesis-student between clusters as possible.

To evaluate the efficiency of the algorithm using in phase 1, we conduct the experiment comparing two programs:

- Program CP (Constraint Programming): using Constraint Programming (CP), modeling with Minizinc and solving with solver Gecode).
- Program BCCP (Balanced Clustering Constraint Programming): using heuristic Balanced clustering in phase 1; in phase 2 using CP (Minizinc model language and solver Gecode). The program is run in ten times and the average result is considered.

Instance	CP Time	BCCP Time	CP Quality	BCCP Quality
1 (6 stu, 10 prof)	6.566s	0.255s	F1 = 0, F2 = 17	F1 = 0.0, F2 = 16.7
2 (7 stu, 11 prof)	139s	1.118s	F1 = 2, F2 = 24	F1 = 2.0, F2 = 23.2
3 (8 stu, 12 prof)	328s	2.139s	F1 = 2, F2 = 22	F1 = 2.0, F2 = 21.0
4 (9 stu, 13 prof)	11m10s	7.752s	F1 = 1, F2 = 29	F1 = 1.0, F2 = 28.7
5 (11 stu, 15 prof)	57m13s	20.3s	F1 = 1, F2 = 35	F1 = 1.0, F2 = 34.7
6 (12 stu, 16 prof)	2h04m19s	11m34s	F1 = 2, F2 = 27	F1 = 2.0, F2 = 26.1
7 (13 stu, 16 prof)	10h22m4s	39m48s	F1 = 1, F2 = 39	F1 = 1.0, F2 = 38.3
8 (14 stu, 17 prof)	54h17m34s	1h32m5s	F1 = 1, F2 = 33	F1 = 1.0, F2 = 33.0
9 (15 stu, 18 prof)	timeout(60h)	4h30m16.785s	F1 = 2, F2 = 40	F1 = 2.0, F2 = 41.2

Result 1: Evaluating the balanced-clustering method

From the result table, we can see that:

- Balanced clustering heuristic in BBCP help us to reduce the solving time significantly comparing to exact method CP.
- There is a trade-off between the time used and the stability of the quality of solution, but this trade-off is acceptable because the quality is reduced slightly while the time consumed reduced significantly. Moreover, when it comes to larger instance with the time restriction, the exact method fails to give the better solution than the heuristic one.
- Although the computation time of BBCP is much more superior to that of CP, BBCP still seems to struggle to deal with large problem.

4.2 Decomposition and Recomposition Strategies

After phase 1, we have already achieved a way to cluster the theses balancedly in quantity and optimally in specialization similarity between theses in each cluster. An natural idea is to assign the members to the juries based on the theses clusters they are in charge of, because of these following reasons:

- After assigning the members to the juries, the problem of input size (n students, m juries) will be broken into m sub-problems of input size $[\frac{n}{m}]$ students, 1 jury.
- The match between both theses and professors with specialization keywords are represented by the same way: a k-dimension vector with k is the total number of specialization keywords. Therefore, we can exploit the result of theses clustering through Bipartite Matching the vectors of professor specializations and the centroids of the theses clusters.

Jury members initialization algorithm (Phase 2-1)

- **Step 1**: Calculate the matrix W with size of n x n with n is the number of professor in the defense session:

$$W(i,j) = d(x_i, \mu_{j \bmod k}) \tag{4}$$

  with:
  - $W(i,j)$: element of row i, column j of weight matrix
  - x_i: specialization vector of professor i
  - $\mu_{j \bmod k}$: the centroid $c(j)$ of cluster slot j, determined by the formula: $c(j) = j \bmod k$
  - $d(x_i, \mu_{j \bmod k})$: distance from the vector x_i to the centroid of the cluster slot sl_j

- **Step 2**: Applying the Hungarian algorithm to solve the bipartite matching problem between professors and student-thesis clusters

With each way of pre-assign professors to the jury, the original problem of (n students, m juries) are broken into m sub-problems of $[\frac{n}{m}]$ students, 1 jury. The sub-problem not only has significantly reduced input size but also has significantly less complicate model, because the room decision variable x_r is no longer needed and the hard constraint H6 (Jury policy) is now guaranteed, because there is only 1 jury now.

On the other hand, from m feasible solutions to m sub-problems, we always be able to get a feasible solution for the initial problem, because:

- All hard constraints H0 - Internal-external member policy, H1 - Member conflict, H2 - Supervisor-member conflict, H3 - Chairman-secretary academic rank policy are depends only on the schedule for each students. Therefore, if the schedule for each student in the sub-problems don't violate these constraints, the recomposed solution are feasible for them.
- Hard constraints H4 - Examiner appearance restriction and H6 - Jury Policy are guaranteed to not be violated because each professor attends only 1 jury.
- Hard constraint H5 - Room-time conflict is guaranteed to not being violated because the quantity of students in each jury is balanced, therefore there will be no room with too much students.

The connections between the objective functions of sub-problems and the original problem are as following:

- **F1. Professor workload balancing**:

$$F_1 = max(F_1^i \mid i \in 1..m) \tag{5}$$

  with F_1^i is the value of objective function 1 of sub-problem i

- **F2. Professor-thesis expertise matching**: The two examiners should match with the thesis in all committees. The function reflects the total match between examiners and theses in committees:

$$F_2 = \sum_{i=1}^{m} F_2^i \tag{6}$$

  with F_2^i is the value of objective function 2 of sub-problem i

As we can see from the above formulas:

- Minimizing the objective function F_1 of original problem is equivalent to minimizing the objective funcions F_1^i of the sub-problems
- Maximizing the objective function F_2 of original problem is equivalent to maximizing the objective function F_2^i of the sub-problems

Therefore, with each given partition way of professors, our mission is to find the feasible and optimized solutions for the sub-problems. To evaluate the efficiency of the method, we conduct the experiment comparing two programs:

- Program BCCP (Balanced Clustering Constraint Programming): using heuristic Balanced clustering in phase 1; in phase 2 using CP (Minizinc model language and solver Gecode) which is an exact method.
- Program BCIPS (Balanced Clustering Initial Partial Solution): using using heuristic Balanced clustering in phase 1; in phase 2 using the initial professor partition in Phase 2-1 and then solving the sub-problems in parallel with CP (Minizinc model language and solver Gecode).

The experiments are conducted with the configuration: processor core i7 4710HQ, ram 8gb in ten times and get the average result, each time using the same student-theses clustering result from phase 1.

Instance	BCCP Time	BCIPS Time	BCCP Quality	BCIPS Quality
1 (6 stu, 10 prof)	0.255s	0.191s	Q1 = 0.0, Q2 = 16.7	Q1 = 0.0, Q2 = 16.2
2 (7 stu, 11 prof)	1.118s	0.226s	Q1 = 2.0, Q2 = 23.2	Q1 = 2.0, Q2 = 21.1
3 (8 stu, 12 prof)	2.139s	0.236s	Q1 = 2.0, Q2 = 21.0	Q1 = 2.0, Q2 = 19.4
4 (9 stu, 13 prof)	7.752s	0.254s	Q1 = 1.0, Q2 = 28.7	Q1 = 1.0, Q2 = 26.3
5 (11 stu, 15 prof)	20.3s	0.306s	Q1 = 1.0, Q2 = 35.0	Q1 = 1.0, Q2 = 32.9
6 (12 stu, 16 prof)	11m34s	0.331s	Q1 = 2.0, Q2 = 26.1	Q1 = 2.0, Q2 = 24.8
7 (13 stu, 16 prof)	39m48s	0.342s	Q1 = 1.0, Q2 = 38.3	Q1 = 1.0, Q2 = 34.9
8 (14 stu, 17 prof)	1h32m5s	0.264s	Q1 = 1.0, Q2 = 33.0	Q1 = 1.0, Q2 = 31.4
9 (15 stu, 18 prof)	4h30m16.785s	0.306s	Q1 = 2.0, Q2 = 41.2	Q1 = 2.0, Q2 = 41.0

Result 2: Evaluating the decomposition method with initial professor partition

As we can see from the experimental results, the BCIPS takes much less time than the BCCP, it is because the sub-problems have less complicated model and much smaller input size, especially in large instance with more juries, because the sizes of the sub-problemss are the number of student in a jury. On the other hand, the time consumed by BCCP program increases exponentially. However, the quality of the BCIPS program is not as good as BCCP program. This is because we only need to maximize the similarity between the student-thesis and the two examiners, not all the 5 members as the initialization metho0d in phase 2-1. Therefore, we have to find a better alternative professor partition than the initial one. With the solving time with each given professor partition is relatively small when compared to the exact method, while the size of the set of all professora partition is enormous

$$spaceSize = \prod_{i=0}^{m-1} C_{n-i}^{p/m} \tag{7}$$

we choose the local search method to tackle the problem, with the metaheuristic Tabu Search to overcome the local minima problem. The local move is defined as swapping two professors from two partitions:

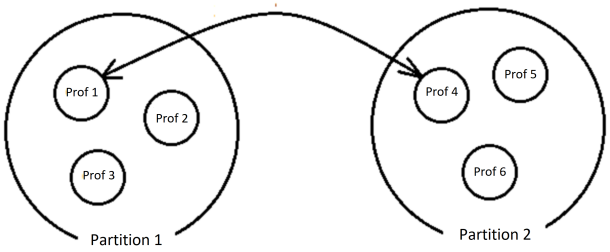

Figure 1: Local move of the Tabu Search in phase 2-2

The optimize function for the search is the weighted-sum function F of the two objective functions F_1 and F_2. In our implementation, the memory structure storing the previous moves' features (Tabu list) is an two-dimension array:

$$tabu[1..p][1..m]$$

with p and m is the number of professors and juries. In iteration $iter$, the local swapping move professor p_i (currently in jury m_i) with professor p_j (currently in jury m_j) can be considered as two assignment moves: assign p_i to m_j and p_j to m_i, therefore the feature of the move can be stored by assigning $tabu[p_i][m_j]$ and $tabu[p_j][m_i]$ to $iter$.

5 Experimental Results

To evaluate the proposed algorithm (we name it Balanced Clustering Parallel Tabu Search), we conduct the experiment comparing the 3 programs:

- BCPTS (Balanced Clustering Parallel Tabu Search): using proposed algorithm, which is a heuristic method.
- CP (Constraint Programming): using Constraint Programming (CP), modeling with Minizinc and solving with solver Gecode with first-fail strategy and default parameters), which is an exact method.
- CBLS-TS (Constraint-based local search - Tabu Search): using the constraint-based local search library OpenCBLS [18] to model the problem and solving by the default generic Tabu Search, which is a heuristic method.

The three programs are run on the machine with an Intel Core I7 4710HQ, 8gb ram, running Ubuntu 14.04. They are tested with real instances from the defense session in April and October 2016 from School of Information and Communication Technology, Hanoi University of Science and Technology. For the two approximate algorithm BCPTS and CBLS-TS, each program is run ten times for each instance, the average result and the lower and upper bound of the objective functions are considered:

Instance	BCPTS Time	CP Time	CBLS-TS Time	BCPTS Quality	BBCP Quality	CBLS-TS Quality
1 (6 stu, 10 prof)	19.49s	6.566s	17.40s	F1 = 0.0, F2 = 16.7(16-17)	F1 = 0, F2 = 17	F1 = 0.4(0-1), F2 = 13.3(11-16)
2 (7 stu, 11 prof)	22.94s	139s	26.89s	F1 = 2.0, F2 = 23.2(22-24)	F1 = 2, F2 = 24	F1 = 2.3(2-3), F2 = 16.3(14-19)
3 (8 stu, 12 prof)	20.28s	328s	30.03s	F1 = 2.0, F2 = 21.0(20-22)	F1 = 2, F2 = 22	F1 = 2.4(2-3), F2 = 15.6(13-20)
4 (9 stu, 13 prof)	22.54s	11m10s	32.71s	F1 = 1.0, F2 = 28.7(28-29)	F1 = 1, F2 = 29	F1 = 2.5(2-3), F2 = 17.8(14-23)
5 (11 stu, 15 prof)	24.67s	57m13s	33.31s	F1 = 1.0, F2 = 35.0(35-35)	F1 = 1, F2 = 35	F1 = 1.7(1-2), F2 = 23.1(18-29)
6 (12 stu, 16 prof)	28.73s	2h04m19s	34.11s	F1 = 2.0, F2 = 26.1(24-27)	F1 = 2, F2 = 27	F1 = 3.1(2-4), F2 = 16.5(13-20)
7 (13 stu, 16 prof)	28.11s	10h22m4s	36.45s	F1 = 1.0, F2 = 38.3(38-39)	F1 = 1, F2 = 39	F1 = 1.3(1-2), F2 = 25.7(21-32)
8 (14 stu, 17 prof)	30.46s	54h17m34s	37.5s	F1 = 1.0, F2 = 33.0(33-33)	F1 = 1, F2 = 33	F1 = 1.2(1-2), Q2 = 19.1(16-25)
9 (15 stu, 18 prof)	52.55s	timeout(60h)	57.37s	F1 = 2.0, F2 = 41.2(40-42)	F1 = 2, F2 = 40	F1 = 2.8(2-3), F2 = 32.9(25-36)
10 (16 stu, 20 prof)	67.53s	timeout(60h)	71.43s	F1 = 0.0, F2 = 45.4(45-46)	F1 = 0, F2 = 43	F1 = 2.7(2-3), F2 = 36.5(32-41)
11 (17 stu, 21 prof)	75.87s	timeout(60h)	80.67s	F1 = 1.0, F2 = 47.5(46-48)	F1 = 1, F2 = 45	F1 = 2.5(2-3), F2 = 38.2(33-43)
12 (18 stu, 22 prof)	84.07s	timeout(60h)	88.61s	F1 = 1.0, F2 = 50.42(49-51)	F1 = 1, F2 = 48	F1 = 2.3(2-3), F2 = 41.6(37-45)

Result 3: Experimental Result

From the experimental result, the proposed algorithm BCPTS:

- When comparing to the exact algorithm CP:

  + The CP program is superior to the BCPTS in both term of speed and quality when it comes the small instance. However, the computation time of the CP increase exponentially when the input size become larger and exceed the timeout (60h) when there are more than 15 students, while the BCPTS time is feasible for all the instances.

  + There is the trade-off between speed and quality as the solutions given by BCPTS is not reach the best quality for the objective function F2. However, the quality of these solutions are quite near to the optimized ones, while the time is reduced extremely, especially for the large instances.

  + For the large instances, the exact method CP can not reach the best solution within the timeout period and gives solutions with less quality than the BCPTS.

– When comparing to the approximate algorithm CBLS-TS:
  + Both programs are able to generate a feasible solution in short time for all instances
  + However, the quality of the solutions given by BCPTS is much better than that of the CBLS-TS. This is because the MTDT problem is a combinatorial problem with high complexity, many variables and constraints and enormous solution space. Therefore, applying directly the metaheuristic Tabu Search to the problem is less effective than using more sophisticated techniques as the proposed algorithm.

Therefore, we can confirm that the proposed algorithm BCPTS provide the good trade-off between speed and quality, which has high practical value for the real-world usage.

6 Conclusion

In this paper, a heuristic algorithm using decomposition approach based on balanced clustering for a real world Examination Timetabling problem has been shown. This splits the solving of the original problem into two phases, which reduces the input size and complexity significantly. Computational results shows that the proposed algorithm provides a good trade-off between speed and quality and has high practical value. The good result for the problem prove the potential of decomposition approach in general and especially the balanced clustering heuristic to the solving of combinatorial problems. It is true that special structure is required for applying the decomposition, which limits the set of applicable problems, but if we can attempt to use the method successfully, the advantages gained cannot be underestimate. In future works, we will consider applying the balanced clustering to more CSP problem which has workload balancing objective - a popular type of objective in the real world.

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A Selective-Discrete Particle Swarm Optimization Algorithm for Solving a Class of Orienteering Problems

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Abstract This study addresses a class of NP-hard problem called the Orienteering Problem (OP), which belongs to a well-known class of vehicle routing problems. In the OP, a set of nodes that associated with a location and a score is given. The time required to travel between each pair of nodes is known in advance. The total travel time is limited by a predetermined time budget. The objective is to select a subset of nodes to be visited that maximizes the total collected score within a path. The Team OP (TOP) is an extension of OP that incorporates multiple paths. Another widely studied OP extension is the Team OP with Time Windows (TOPTW) that adds the time windows constraint. We introduce a discrete version of Particle Swarm Optimization (PSO), namely Selective-Discrete PSO (S-DPSO) to solve TOP and TOPTW. S-DPSO has a different movement compared with other DPSO algorithms reported in the literature. S-DPSO considers four different movement schemes: (a) following its own position, (b) moving towards its personal best position, (c) moving towards the global best position, and (d) moving towards the combination of three above-mentioned schemes. The best movement scheme is selected in order to determine the next position of the particle. The S-DPSO algorithm is tested on the benchmark instances. The experiment results show that

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S-DPSO performs well in solving benchmark instances. S-DPSO is promising and comparable to the state-of-the-art algorithms.

1 Introduction

The OP was first introduced by Tsiligirides [33]. In OP, a set of nodes that associated with a location and a score is given. The time required to travel between each pair of nodes is known in advance. The total travel time is limited by a predetermined time budget. The objective is to select a subset of nodes to be visited that maximizes the total collected score in a path. An OP extension is the Team OP (TOP) that incorporates multiple paths. TOP is considered as a class of the Vehicle Routing Problem (VRP) with profits and multiple vehicles. Each vehicle is represented as a path with the aim of selecting customers so as to maximize the collected profits and subject to a travel time restriction [1,9].

Another widely studied OP extension is the Team OP with Time Windows (TOPTW). The TOPTW adds the time windows constraint. A visit to a particular node has to be made within its time window. A comprehensive survey about the OP can be found in Vansteenwegen et al. [34]. Recently, Gunawan et al. [11] extend the survey by including latest variants of the OP, including the proposed solution approaches and the most recent applications of the OP, such logistics, trip planner and other areas. Only limited population-based algorithms have been introduced to solve the OP and its variants [11].

We introduce a discrete version of Particle Swarm Optimization (PSO), namely **Selective-DPSO (S-DPSO)**. PSO is a population-based metaheuristic algorithm that originates from studies of synchronous bird flocking, fish schooling, and bees buzzing [14]. It evolves a population or swarm of individuals called particles. The main characteristic of PSO is the capability to solve a problem by moving each particle in the search space based on its velocity, its personal best position, and the particle swarm's global best position.

PSO is designed for solving continuous optimization problems; therefore, it is not suitable for the combinatorial optimization problems. The main reason is that it is not possible for particles to continuously "fly" through a discrete-valued space [13]. However, its fast convergence and easy implementation have driven researchers to extend the continuous PSO to discrete problems. In order to solve discrete optimization problems, some researchers use a discrete particle representation and create a discrete position procedure by following an analogous structure of the classical PSO equations, known as Discrete PSO (DPSO) [21,19]. Several variants of PSO that are used to solve discrete optimization problems can be seen in various problem domains, such as the traveling salesman problem [29], vehicle routing problem [30,31,10], scheduling problem [16] and orienteering problem [21, 25,8].

Instead of directly following the current position of a particular particle, the personal best solution of a particular particle (personal best position) and the global best solution taken from all particles (particle swarm's global best position), which are commonly used in the DPSO, S-DPSO is designed to allow each particle to evaluate one of the position choices generated by the proposed movement schemes. Four different movement schemes that follow the analogy of the PSO position updating rule are proposed. The four movement schemes are: (a)

following its own position, (b) moving towards its personal best position, (c) moving towards the global best position, and (d) moving towards the combination of three above-mentioned schemes. By doing so, each particle has more chances to move to a better position. S-DPSO is implemented for solving two variants of the Orienteering Problem (OP), namely TOP and TOPTW. The results are very promising and comparable to the ones of the state-of-the-art algorithms.

The rest of this paper is organized as follows. Section 2 summarizes a literature overview related to the PSO that has been applied in various domains, including the OP. In Section 3, we describe the Selective Discrete-PSO (S-DPSO) algorithm. We also briefly describe TOP and TOPTW. The benchmark instances and experimental results are presented in Section 4. Finally, our conclusion and future work are summarized in Section 5.

2 Related Work

This section briefly summarizes several applications of the DPSO algorithm in various combinatorial optimization problems, such as flowshop scheduling, machine scheduling and vehicle routing problems. The most recent applications of PSO in order to solve variants of the OP are also included.

Liao et al. [16] propose DPSO in order to solve flowshop scheduling problems. The particle and the velocity are redefined. An efficient approach is developed to move a particle to a new sequence. Furthermore, they incorporate a local search scheme into the proposed algorithm, called PSO-LS. Comparisons with a continuous PSO algorithm and two genetic algorithms are conducted in order to verify the proposed DPSO algorithm. Computational results show that the proposed DPSO algorithm is very competitive.

Pan et al. [23] introduce DPSO to solve the no-wait flowshop scheduling problem. The main contribution of this work is due to the fact that particles are represented as discrete job permutations and a new position update method is developed based on the discrete domain. In addition, the DPSO algorithm is hybridized with the Variable Neighborhood Descent (VND) algorithm to further improve the solution quality.

Tseng and Liao [32] propose a DPSO algorithm by incorporating the Net Benefit of Movement (NBM) algorithm to solve the lot-streaming flowshop scheduling problem. This DPSO improves the existing DPSO [16] by introducing an inheritance scheme, inspired by a genetic algorithm, into particles construction. Kashan and Karimi [13] present a DPSO algorithm for scheduling parallel machines. The proposed DPSO uses a discrete combinatorial solution representation and equations analogous to those of the PSO equations. Marinakis and Marinaki [18] propose a hybrid algorithmic nature inspired methodology, namely the hybrid genetic-particle swarm optimization algorithm to solve the vehicle routing problem. By introducing an intermediate phase between the two generations, namely the phase of evolution of the population, the proposed algorithm gives more efficient individuals and, thus, it improves the effectiveness of the algorithm.

Marinakis et al. [19] introduce another hybrid algorithm, namely the hybrid particle swarm optimization (HybPSO), for the vehicle routing problem. It combines PSO, the Multiple Phase Neighborhood Search-Greedy Randomized Adaptive Search Procedure (MPNS-GRASP) algorithm, the Expanding Neighborhood

Search strategy and a Path Relinking strategy. This algorithm is suitable for solving very large-scale vehicle routing problems within short computational times.

Chen et al. [4] introduce an interactive self-learning PSO algorithm for the routing pickup and delivery of multiple products with material handling in multiple cross-docks problem. The concept of self-learning is incorporated so as to attain the optimal solution within reasonable computational time, and decrease the chance of being trapped in a local optimum.

Dang et al. [7] propose an effective Particle Swarm Optimization-based Memetic Algorithm (PSOMA) for TOP. Most MA designs incorporate various local search techniques into a global search scheme, e.g. a genetic algorithm. MA and PSO are both based on social evolution or behavior rather than biological ones, and there are benefits to be gained from combining techniques into a single form for solving combinatorial optimization problems.

Muthuswamy and Lam [21] introduce DPSO for solving the TOP. This DPSO is a modification of PSO that applies a discrete or qualitative distinction between variables. The RVNS and 2-Opt are used as the local search operators. The insert and exchange neighborhoods in the RVNS technique are employed randomly in order to update the next generation particles. Dang et al. [8] introduce an effective PSO-inspired Algorithm (PSOiA) for TOP. This method is based on the preliminary study of a PSO-based memetic algorithm (PSOMA). Moreover, a new fast evaluation process based on an interval graph model is introduced. This process enables more iterations for the PSO without increasing the global computational time.

3 Selective-Discrete Particle Swarm Optimization

3.1 Particle Swarm Optimization (PSO)

The standard PSO algorithm is initially designed for solving continuous optimization problems. In PSO, a solution, which is represented as a particle, moves through the search space in order to reach the global optimum. During a particular movement, each particle adjusts its position based on its own experience and its neighboring particles. All particles share their information so that they would be directed towards the best position in the search space.

The formal definitions of PSO are as follows. Let K be a set of particles, we define the best solution of particle $k \in K$ at iteration t (the personal best position) and the global best solution taken from all particles at iteration t (the particle swarm's global best position) as p^t_{k-best} and g^t_{best}, respectively. At iteration t, particle k updates its next position x^{t+1}_k by using the following equations [21]:

$$v^{t+1}_k = wv^t_k + c_1 r_1 (p^t_{k-best} - x^t_k) + c_2 r_2 (g^t_{best} - x^t_k) \quad (1)$$

$$x^{t+1}_k = x^t_k + v^{t+1}_k \quad (2)$$

v^t_k and x^t_k represent the velocity and the position of particle k at iteration t, respectively. w is a constant value for controlling the impact of the previous velocity, c_1 is the cognitive parameter for remembering the best particle position that has been reached so far, c_2 is the social parameter controlling the communication

among particles in order to converge towards the global best position, r_1 and r_2 are uniformly distributed random variables between [0,1].

In order to solve combinatorial optimization problems which require a discrete solution space, some transformation techniques are required, such as the integer value [28], the binary coding scheme [22] and a permutation based solution representation [36]. We extend the DPSO by introducing the Selective - DPSO (S-DPSO) algorithm that considers several movement schemes with the purpose of improving the movement of the particles, instead of directly following its current position, the personal best position and the global best position, which are commonly used in DPSO.

In the following subsections, we briefly describe two variants of the OP, TOP and TOPTW, as our case studies, followed by the explanation of how to present the solution and construct the initial population of S-DPSO . We then explain the proposed movement schemes for updating the particles' positions.

3.2 TOP and TOPTW

The TOP and TOPTW are defined as follows. Consider an undirected network graph $G = (V, A)$ where $V = \{1, 2, \ldots, |V|\}$ is a set of nodes that would be visited at most once and $A = \{(i, j) : i \neq j \in V\}$ refers to the set of arcs connecting two different nodes i and j. The non-negative travel time between nodes i and j is represented by c_{ij}. Each node $i \in V$ has a positive score u_i that is collected when node i is visited. In the context of the TOPTW, we include a time window $[e_i, l_i]$ where e_i and l_i refer to the earliest and latest times allowed for starting the visit at node i. In case of an early arrival, a visit will only start when the time window opens.

Let $M = \{1, 2, \ldots, |M|\}$ be a set of paths. If $|M| = 1$. In the context of VRP, each path represents one vehicle. Each path $m \in M$ is constrained within the time budget T^{max}. The objective is to maximize the total collected score by visiting nodes in $|M|$ paths. The mathematical models can be found in works of Vansteenwegen et al. [34] and Gunawan et al. [11].

3.3 Initial Population Construction

The initial population is created by a set of independent particles K. Each particle $k \in K$ consists of a set of paths M where each path $m \in M$ contains a set of visited nodes. Each node only appears at most once in all paths. Pseudo-code for generating the initial population is outlined in Algorithm 1.

In this paper, both start and end nodes are assumed to be the same node, which is node 0. Let T be the number of iterations. We define the following strings for representing our solutions in the proposed S-DPSO:

- N_k^t is a string of numbers that consists of $|V|$ nodes and $(|M|-1)$ dummy nodes at iteration t for particle $k \in K$. The dummy nodes are represented by node 0 as well. They are used for separating paths, as proposed by Lin and Yu [17].
- $X(N)_k^t$ is a string of numbers that consists of **visited nodes**, $(|M|-1)$ dummy nodes, start and end nodes of particle $k \in K$ at iteration t. The length of $X(N)_k^t$

Algorithm 1 Initial Population Construction (V, M)

begin
for all $k \in K$ **do**
  Generate N_k^0
  Initialize $X(N)_k^0 = \emptyset$
  Initialize $X(N)_k^0 \leftarrow$ node 0 (start node)
  $pos := 1$
  while (the end node or the dummy node is not reached yet) **do**
    if the pos-th node of N_k^0 is feasible **then**
      mark the node at the pos-th position
    end if
    $pos := +1$
  end while
  $X(N)_k^0 \leftarrow X_k^0 \cup$ {marked nodes}
  $p_{k-best}^0 \leftarrow N_k^0$
  $X(p)_k^0 \leftarrow X(N)_k^0$
end for
Select the best $X(p)_k^0$ and its respective $p_{k-best}^0 (\forall k \in K)$, denoted as $X(g)^0$ and g_{best}^0.
return $N_k^0, X_k^0, p_{k-best}^0, X(p)_k^0 (\forall k \in K)$ **and** $g_{best}^0, X(g)^0$
end

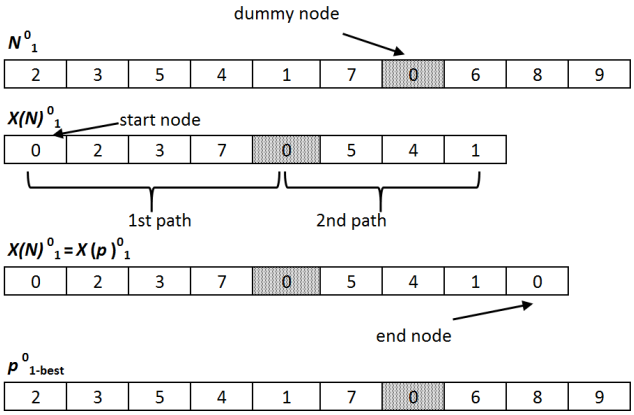

Fig. 1: Initial Population Construction

can be shorter than the one of N_k^t since it only includes visited nodes, not the entire V.

- p_{k-best}^t is a string of numbers that consists of $|V|$ nodes and $(|M|-1)$ dummy nodes for particle $k \in K$ at iteration t. It keeps one $N_k^{t'} (t' \in \{0, \ldots, t\})$ that generates the best $X(N)_k^{t'}$ (e.g. with the highest objective function value).
- $X(p)_k^t$ is a string of numbers that consists of **visited nodes**, $(|M|-1)$ dummy nodes, start and end nodes of particle $k \in K$ at iteration t. This string keeps the feasible solution of p_{k-best}^t.
- g_{best}^t is a string of numbers that consists of $|V|$ nodes and $(|M|-1)$ dummy nodes. It keeps the best p_{k-best}^t at iteration t that generates the best $X(p)_k^t (\forall k \in K)$.
- $X(g)^t$ is a string of numbers that consists of **visited nodes**, $(|M|-1)$ dummy nodes, start and end nodes of particle $k \in K$ at iteration t. This string keeps the feasible solution of g_{best}^t.

In the initial population construction ($t = 0$), we generate N_k^0 for each particle k. We randomly generate a permutation of $|V|$ nodes and followed by inserting $(|M| - 1)$ dummy nodes (nodes 0). Figure 1 illustrates N_1^0 with $n = 10$ and one dummy node for particle 1 with $|M| = 2$. The process of generating the initial feasible solution for the first path of the first particle is started by selecting the first node (e.g. node 2) of N_1^0. If the selected node is feasible (no constraint violation), we then mark this node as the selected node. Other nodes would be added to the path one by one from left to right to represent the sequence in which they are visited, by ensuring the feasibility of these allocations. The process of allocating nodes in a particular path is terminated if one of the following conditions is met: 1) the last node of N_1^0 is reached or 2) one of dummy nodes is reached.

We then update $X(N)_1^0$ accordingly. It is started by allocating the start node, node 0. All marked nodes would be allocated in $X(N)_1^0$. For this example, nodes 2, 3, 7 and 0 (dummy node) are allocated. The process for the second path is started by considering the first non-dummy node from N_1^0. We start from node 5 and repeat the same procedure. Here, we assume that we can only allocate nodes 5, 4 and 1. Node 7 cannot be allocated due to the feasibility issue and we also reach the dummy node. Finally, we add the end node to X_1^0.

There is a special condition if all nodes before the dummy node cannot be allocated, we jump to the subsequent nodes after the dummy node. p_{1-best}^0 and $X(p)_1^0$ are then generated which is taken from N_1^0 and $X(N)_1^0$, respectively. The entire process is repeated for all particles. Among all particles, we select the best $X(p)_{k-best}^0$ including its p_{k-best}^0 to generate $X(g)^0$ and g_{best}^0, respectively.

3.4 Particle Updating Procedure

In both PSO and DPSO, the position of a particular particle is updated by using equations 1 and 2. In our proposed S-DPSO, at each iteration t, we allow each particle k to evaluate three different strings: N_k^t, p_{k-best}^t and g_{best}^t. By doing so, a particle has more opportunities to select a better position to move.

At iteration t, the new position N_k^{t+1} is updated by considering four different movement schemes (or the particle velocities), as shown in the following equations.

$$v(1)_k^{t+1} = (w \otimes N_k^t) \oplus R_{k'}^t \tag{3}$$

$R_{k'}^t$ represents another $N_{k'}^t$ which is randomly selected $(k' \neq k)$.

$$v(2)_k^{t+1} = (c_1 \otimes p_{k-best}^t) \oplus N_k^t \tag{4}$$

$$v(3)_k^{t+1} = (c_2 \otimes g_{best}^t) \oplus N_k^t \tag{5}$$

$$v(4)_k^{t+1} = (c_2 \otimes g_{best}^t) \oplus ((c_1 \otimes p_{k-best}^t) \oplus ((w \otimes N_k^t) \oplus R_k^t)) \tag{6}$$

Each equation represents a different movement scheme. Equation 1 represents the updated velocity at iteration $(t+1)$ which depends on the current N_k^t and the acceleration of w $(0 < w < 1)$. Equations 2 and 3 formulate the updated velocity based on the personal best position p_{k-best}^t (with the acceleration of c_1 $(0 < c_1 < 1)$) and the global best position g_{best}^t (with the acceleration of c_2 $(0 < c_2 < 1)$),

respectively. Equation 4 updates the velocity based on the combination of N_k^t, p_{kbest}^t and g_{best}^t.

The **multiplication operator** $\otimes$ is used to randomly select a set of nodes from N_k^t, p_{k-best}^t and g_{best}^t (refer to equations (3)-(6)) that would be removed. A number of selected nodes are represented as s_w, s_1 and s_2, respectively. Equations (7)-(9) calculate their values. After removing the nodes from the current N_k^t, p_{k-best}^t and g_{best}^t, the **addition operator** $\oplus$ is used to add nodes to them.

$$s_w = \lceil (|V| + |M| - 1) \times w \rceil \tag{7}$$

$$s_1 = \lceil (|V| + |M| - 1) \times c_1 \rceil \tag{8}$$

$$s_2 = \lceil (|V| + |M| - 1) \times c_2 \rceil \tag{9}$$

where $\lceil a \rceil$ denotes the smallest integer that is larger than or equal to a.

We continue with generating four different feasible solutions, denoted as $x(1)_k^{t+1}$, $x(2)_k^{t+1}$, $x(3)_k^{t+1}$ and $x(4)_k^{t+1}$ with respect to $v(1)_k^{t+1}$, $v(2)_k^{t+1}$, $v(3)_k^{t+1}$ and $v(4)_k^{t+1}$. They follow the same idea of constructing the initial population (Section 3.3). Their objective function values f are calculated. The best movement scheme that provides the best objective function value is selected, as shown in Equation 10. N_k^{t+1} is also updated accordingly.

$$X(N)_k^{t+1} = \arg\max_x f(x) \quad x \in \{x(1)_k^t, x(2)_k^t, x(3)_k^t, x(4)_k^t\} \tag{10}$$

Take note that if the best objective function value improves the current objective function value of the feasible solution generated by p_{k-best}^t, p_{k-best}^{t+1} and $X(p)_k^{t+1}$ would be updated. The same idea applies to g_{best}^{t+1} and $X(g)^{t+1}$. This updating procedure is applied to all particles in K.

Figure 2 illustrates an example of generating N_1^1. The same approach is applied to $v(2)_1^1$, $v(3)_1^1$ and $v(4)_1^1$ as well. For example, in order to generate $v(1)_1^1$, we calculate $s_w = \lceil (9 + 2 - 1) \times 0.3 \rceil = 3$ (with $w = 0.3$). It means that three nodes are removed randomly from N_1^0 (e.g. nodes 3, 1 and 6), as shown in Figure 2. We then pick one $N_{k'}^0 (k' \in K \setminus \{1\})$ randomly, denoted as R_1^0. We find the positions of removed nodes in R_1^0 and inserted them back by referring to their sequence in R_1^0. The new generated string is denoted as $v(1)_1^1$. We apply the same idea explained in Section 3.3 to generate $x(1)_1^1$ and calculate the objective function value. $v(1)_1^1$ replaces N_1^1 by assuming $v(1)_1^1$ provides the best solution. The S-DPSO algorithm is illustrated in Algorithm 2.

4 Computational Experiments

We provide a short description of the benchmark instances and the state-of-the-art algorithms for a comparison purpose. All benchmark instances can be downloaded at `http://www.mech.kuleuven.be/en/cib/op`. The details of the characteristics of benchmark instances can be referred to the survey papers [11,34]. We also describe the experimental setup and the computational results of our experiments.

Algorithm 2 S-DPSO $(V,\ M)$

begin
$t \leftarrow 0$;
INITIAL POPULATION CONSTRUCTION $(V,\ M)$
while $(t < T)$ **do**
  for all $k \in K$ **do**
    Generate $v(1)_k^{t+1}, v(2)_k^{t+1}, v(3)_k^{t+1}, v(4)_k^{t+1}$
    Generate $x(1)_k^{t+1}, x(2)_k^{t+1}, x(3)_k^{t+1}, x(4)_k^{t+1}$ and calculate their objective function values
    Select the best objective function value
    Update N_k^{t+1} and $X(N)_k^{t+1}$ accordingly
    if $f(X(N)_k^{t+1}) > f(X(p)_k^t)$ **then**
      $p_{k-best}^{t+1} \leftarrow N_k^{t+1}$
      $X(p)_k^{t+1} \leftarrow X(N)_k^{t+1}$
    else
      $p_{k-best}^{t+1} \leftarrow p_{k-best}^t$
      $X(p)_k^{t+1} \leftarrow X(p)_k^t$
    end if
    if $f(X(N)_k^{t+1}) > f(X(g)^t)$ **then**
      $g_{best}^{t+1} \leftarrow N_k^{t+1}$
      $X(g)^{t+1} \leftarrow X(N)_k^{t+1}$
    else
      $g_{best}^{t+1} \leftarrow g_{best}^t$
      $X(g)^{t+1} \leftarrow X(g)^t$
    end if
  end for
  $t \leftarrow t + 1$;
end while
return $X(g)^t$
end

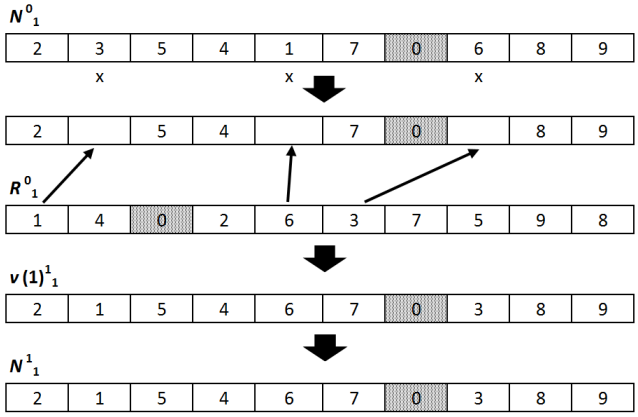

Fig. 2: Particle Updating Procedure

4.1 Benchmark Instances and Approach Comparison

Chao et al. [3] introduce the benchmark TOP instances which are classified into 7 instance sets. The number of nodes varies between 21 to 102 nodes with 2 to 4 paths. Dang et al. [8] propose the state-of-the-art PSO algorithm, namely PSO-inspired algorithm (PSOiA). PSOiA focuses on sets 4, 5, 6 and 7 of instances and compare PSOiA with two other algorithms: Memetic Algorithm (MA10) [2] and

PSO-based MA (PSOMA) [7]. PSOiA, MA10 and PSOMA are executed 10 times for each instance.

Righini and Salani [24] design the OPTW instances by modifying 29 Solomon's instances [26] and 10 Cordeau's instances [5]. Montemanni and Gambardella [20] add another set of 37 instances for the OPTW. The TOPTW instances are generated by extending the OPTW instances with different values of number of paths: 2 to 4 paths. The performance of the S-DPSO algorithm is compared with two state-of-the-art algorithms for the TOPTW: Iterative Three-Component Heuristic (I3CH) [12] and Artificial Bee Colony (ABC) [6]. I3CH is claimed to be superior than other algorithms, such as Iterated Local Search (ILS) [35], Slow Simulated Annealing (SSA) [17] and Granular Variable Neighborhood Search (GVNS) [15]. ABC is selected since ABC is the latest population-based algorithm that has been proposed for solving the TOPTW. It outperforms the earlier population-based algorithm, Ant Colony System (ACS) [20].

4.2 Algorithm Setup and Parameter Setting

The proposed S-DPSO algorithm is coded in C++. All experiments are performed on a PC with a 3.4 GHz processor and 20GB of RAM, under the Windows 7 Operating System. Each experiment is performed for 10 runs, for which the best and average results are presented. Take note that our algorithm uses the number of iterations as the stopping criterion.

In our S-DPSO, there are five parameters that are used, namely the number of particles $|K|$, the number of iterations T, three different acceleration values, w, c_1 and c_2. Since parameter setting may affect the performance of the algorithm, five instances were randomly selected from TOPTW and TOP instances for the parameter analysis. We set initial parameter values as follows: $|K| = \{15, 30, 60\}$, $T = \{800, 1000, 1500\}$, $w = \{0.3, 0.6, 0.9\}$, $c_1 = \{0.3, 0.6, 0.9\}$ and $c_2 = \{0.3, 0.6, 0.9\}$. After running preliminary experiments, the parameter values that provide the best performance are: $|K| = 30, T = 800, w = 0.9, c_1 = 0.6$ and $c_2 = 0.6$. This set of values would be used for solving the entire benchmark instances.

4.3 Computational Results

Table 1 compares S-DPSO with MA10 [2], PSOMA [7] and PSOiA [8]. The $Numb$ column provides the number of instances in a particular set of instances. For each algorithm, we calculate the percentage deviation of the best and average results of 10 runs with the best known solutions (BKs) for each instance. We then calculate the average of both for each instance set, denoted as P_{best} and P_{avg}, respectively. The CPU column represents the average CPU times (in seconds) used by a particular algorithm for solving a particular instance set. Additionally, we also calculate the number of instances in which the best known solutions are obtained, denoted as $NBest$.

Table 1: Overall Comparison of S-DPSO to MA10, PSOMA and PSOiA on TOP instances

Instance Set	$Numb$	MA10			PSOMA			PSOiA			S-DPSO		
		P_{best}	P_{avg}	CPU	P_{best}	P_{avg}	CPU	P_{best}	P_{avg}	CPU	P_{best}	P_{avg}	CPU
Set 4	54	0.030	0.207	189.119	0.026	0.285	86.994	0.002	0.110	226.673	0.016	0.029	191.108
Set 5	45	0.061	0.095	51.110	0.015	0.090	23.990	0.000	0.034	73.922	0.015	0.023	212.080
Set 6	15	0.000	0.017	28.387	0.000	0.000	9.771	0.000	0.000	37.263	0.000	0.004	131.804
Set 7	43	0.013	0.106	149.656	0.021	0.179	66.177	0.000	0.030	37.263	0.018	0.027	353.097
Grand Average		0.031	0.129	123.397	0.019	0.173	55.856	0.001	0.056	112.918	0.015	0.024	235.820
$NBest$			146			146			156			148	

Table 2: Overall Comparison of S-DPSO to ABC and I3CH on TOPTW instances

Instance Set	$Numb$	ABC				S-DPSO				I3CH		
		P_{best}	P_{avg}	CPU	$NBest$	P_{best}	P_{avg}	CPU	$NBest$	P	CPU	$NBest$
$m = 1$												
Solomon 100	29	0.277	0.455	4.421	22	0.023	0.077	44.593	28	0.688	26.714	21
Solomon 200	27	0.930	1.323	21.352	7	0.723	0.868	63.872	11	1.340	132.137	7
Cordeau 1-10	10	1.212	1.489	78.540	5	0.838	1.122	115.293	6	1.072	109.010	3
Cordeau 11-20	10	3.144	3.522	103.670	1	2.035	2.821	100.871	6	4.277	130.230	3
Average		1.009	1.303	33.247	-	0.644	0.857	68.150	-	1.442	88.616	-
Total		-	-	-	35	-	-	-	51	-	-	34
$m = 2$												
Solomon 100	29	0.265	0.428	13.000	21	0.296	0.336	72.955	19	0.490	69.314	17
Solomon 200	27	0.841	1.158	30.648	3	0.577	0.690	84.134	6	0.470	463.800	10
Cordeau 1-10	10	2.499	3.034	149.690	2	1.473	1.686	196.691	2	1.108	247.060	3
Cordeau 11-20	10	3.229	3.634	221.840	1	2.081	2.478	194.459	3	2.704	304.610	1
Average		1.153	1.452	64.734	-	0.785	0.921	109.195	-	0.856	263.808	-
Total		-	-	-	27	-	-	-	30	-	-	31
$m = 3$												
Solomon 100	29	0.376	0.629	22.548	14	0.105	0.312	104.564	22	0.198	135.852	19
Solomon 200	27	0.248	0.347	12.493	22	0.492	0.829	106.448	20	0.016	89.244	25
Cordeau 1-10	10	2.451	2.904	228.930	1	0.752	1.254	334.943	3	0.360	424.000	3
Cordeau 11-20	10	3.551	4.231	247.640	1	1.795	2.075	329.641	3	1.111	496.950	4
Average		1.021	1.302	75.749	-	0.550	0.851	165.162	-	0.275	204.721	-
Total		-	-	-	38	-	-	-	48	-	-	51
$m = 4$												
Solomon 100	29	0.741	1.045	29.545	8	0.144	0.333	155.884	23	0.162	199.545	21
Solomon 200	27	0.000	0.000	0.530	27	0.000	0.033	160.036	27	0.000	0.170	27
Cordeau 1-10	10	3.083	3.489	233.280	1	1.614	1.782	514.943	2	0.365	566.510	5
Cordeau 11-20	10	3.428	3.834	267.580	2	1.479	2.622	569.641	6	0.452	728.620	7
Average		1.140	1.362	77.364	-	0.462	0.718	259.045	-	0.169	246.614	-
Total		-	-	-	38	-	-	-	58	-	-	60
Grand Average		1.081	1.355	62.774	-	0.610	0.837	150.388	-	0.686	200.940	-
Grand Total		-	-	-	138	-	-	-	187	-	-	176

We observe that S-DPSO outperforms MA10 and PSOMA in terms of the values of P_{avg} for each instance set except set 6 where PSOMA and PSOiA perform best. In terms of P_{best} values, S-DPSO obtains the best values for sets 4 and 5. All algorithms are able to obtain the best known solutions for set 6 instances. On average (referring to "Grand Average" values), S-DPSO is able to produce better P_{best} and P_{avg} values compared against those of MA10 and PSOMA. Its P_{avg} value is only 0.024% while MA10 and PSOMA values are 0.129% and 0.173%, respectively.

S-DPSO is comparable to PSOiA. In terms of P_{avg}, S-DPSO outperforms PSOiA in all instance sets, except set 6. On the other hand, PSOiA performs better than S-DPSO in most instance sets with respect to P_{best} values. Note that both perform well in solving set 6 ($P_{max} = 0.000\%$). Finally, in terms of "Grand Average" values, we can draw a similar conclusion. S-DPSO produces a better value of P_{avg} than the one of PSOiA. On the other hand, PSOiA performs best in terms of P_{best}.

Since we are not able to obtain the source codes of the state-of-the-art algorithms, we report their original CPU times. Dang et al. [8] mention that machine performances of PSOiA, PSOMA and MA10 are almost the same. It is concluded that PSOMA is faster than MA10 but less robust than PSOMA since the value of P_{avg} is lower. Computational times of PSOiA and MA10 are almost the same but PSOiA performs better than MA10. Based on this remark, it is worthy to mention that our S-DPSO is more robust than other algorithms but with the cost of computational time. Take note that our computer is 1.5 $\times$ slower than the one of PSOiA. In terms of $NBest$ values, S-DPSO is able to obtain 148 best known solutions while MA10 and PSOMA only obtain 146 solutions. PSOiA performs best by reaching 156 solutions.

Table 2 compares the performance of S-DPSO to those of ABC [6] and I3CH [12]. Since I3CH is only run once and we treat its results as the best results of I3CH, we only calculate P for I3CH. S-DPSO outperforms ABC in terms of the values of P_{avg} and P_{best} for all values of m. However, S-DPSO requires more computational time. The values are from 0.550% to 0.921% while ABC values are always more than 1.000%. The overall performances are represented by Grand Average values of P_{best} and P_{avg}. S-DPSO is able to obtain 0.610% and 0.837%, respectively. On the other hand, ABC obtains 1.081% for P_{best} and 1.355% for P_{avg}.

We compare the values of our P_{avg} with P values of I3CH. The average results P_{avg} represents the average expected quality of the multiple-run algorithm when it is executed only once [27]. S-DPSO is dominant in solving instances with $m = 1$, but I3CH outperforms S-DPSO for other m values. In terms of computer spec, I3CH is around 2 times faster than ours. I3CH also run in parallel machines due to the characteristics of the algorithm, while our S-DPSO is run in a sequential basis. We can conclude that S-DPSO spends less computational time than I3CH does. In conclusion, S-DPSO performs well within reasonable computational times.

S-DPSO obtains 187 best known solutions while I3CH and ABC obtain best known solutions for 176 and 138 instances, respectively. S-DPSO again outperforms ABC in terms of $NBest$. S-DPSO requires more experiments (e.g multiple runs) compared with I3CH does in order to obtain more best known solutions.

5 Conclusion

We propose a discrete version of Particle Swarm Optimization (PSO), namely Selective-Discrete PSO (S-DPSO). We improve DPSO by introducing four different movement schemes: (a) following its own position, (b) moving towards its personal best position, (c) moving towards the global best position, and (d) moving towards the combination of three above-mentioned schemes. The best movement scheme is selected in order to determine the next position of each particle. The proposed S-DPSO algorithm is tested on the benchmark instances of the Team OP and the Team OP with Time Windows.

The results show that S-DPSO is capable of producing high-quality solutions. On four instance sets of TOP instances, an average run has a gap of only 0.024% with the best known solutions. It is able to find 148 (94.3%) best known solutions. For the TOPTW instances, the average gap is around 0.837% from the best known solutions. The best known solutions of 70.8% of instances are found. In general, S-DPSO is comparable to the state-of-the-art algorithms.

Future research areas include incorporating local search mechanism in the design of the algorithm and the concept of accepting worse solutions (e.g. Simulated Annealing) that may lead us to better solutions. It would be more beneficial to run the algorithm in a parallel version due to the characteristics of the algorithm. The results are encouraging for the application of S-DPSO to solve other variants of the OP, such as Time Dependent OP and Multi-Objective OP. Finally, it would be interesting to apply the S-DPSO algorithm to solve other combinatorial optimization problems with similar characteristics, such as the vehicle routing problem and machine scheduling problems.

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Home Health Care Delivery Problem

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Abstract We address the Home Health Care Delivery Problem (HHCDP), which is concerned with staff scheduling in the home health care industry. The goal is to schedule health care providers to serve patients at their homes that maximizes the total collected preference scores from visited patients subject to several constraints, such as workload of the health care providers, time budget for each provider and so on. The complexity lies in the possibility of cancellation of patient bookings dynamically, and the generated schedule should attempt to patients' preferred time windows. To cater to these requirements, we model the preference score as a step function which depends on the arrival time of the visit and the resulting problem as the Team Orienteering Problem (TOP) with soft Time Windows and Variable Profits. We propose a fast algorithm, Iterated Local Search (ILS), which has been widely used to solve other variants of the Orienteering Problem (OP). We first solve the modified benchmark Team OP with Time Window instances followed by randomly generated instances. We conclude that ILS is able to provide good solutions within reasonable computational times.

1 Introduction

The demand for home health care (HHC) services has increased substantially due to population aging [20]. HHC provides a wide range of services, including nursing care, medical, paramedical and social services, that can be provided to patients at home [14,16]. Due to aging populations, the demand for HHC is rapidly increasing. For example, in 2011, more than 4 million patients received HHC services in the

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U.S. [6]. Ministry of Health (MOH) Singapore introduces Intermediate and Long-Term Care (ILTC) services for patients who require further care and treatment after being discharged from an acute hospital as well as community-dwelling senior residents who may be frail and need supervision or assistance with their activities of daily living. In 2013, MOH developed a set of home care guidelines.

This study addresses a particular application problem of the staff scheduling in the home health care industry, namely the Home Health Care Delivery Problem (HHCDP), on a daily basis. In the context of the classical scheduling problem, HHCDP is considered as a workforce scheduling and routing problem. Workforce scheduling and routing problem refers to those scenarios involving the mobilization of personnel in order to perform work related activities at different locations [2]. Staffs are mostly required to travel from one location to other locations in order to perform the work since the number of works across different locations is usually larger than the available number of employees. Several real-world requirements, such as time windows, transportation modality, start-end locations, skills and qualifications, increase the complexity of the problem. For more details about workforce scheduling and routing problems, the reader can refer to [2]. The HHCDP is considered as a combination of staff rostering and VRP with time windows in [25].

In our context, instead of considering as a VRP (which is in essence a demand perspective), we view this problem from the supply perspective as well. While we try to satisfy as many patients as possible, the number of requests may exceed the service capacity and some of them may be cancelled after the schedule has been generated. Since we want to maximize the patients' satisfaction, measured in terms of scores, our problem can be modelled as a variant of the Orienteering Problem (OP).

In the standard OP, a set of agents are scheduled to serve a set of customers (e.g. patients). Each agent is limited by the time budget and time windows. Each customer can only be visited at most once. For simplicity, we assume that all agents start and end at the same location (e.g. the hospital). The problem incorporates other real-world requirements related to the health care industry, such as the continuity of care, workload fairness and demand uncertainty (due to request cancellations). We allow the agent to arrive late with a certain penalty value rather than not visiting the patient. In the OP term, the collected score is affected by the penalty value, if any. As some requests from patients may be cancelled due to unforeseen factors after the schedule has been generated, we express such uncertainty as a probability of occurrence which is assumed to be known beforehand.

Hence, this paper considers HHCDP from both provider and patient perspectives - while we maximize the workload utilization rate of providers without violating their time budgets, we also maximize the satisfaction level of patients with respect to the number of patients to be visited by allowing late arrivals. We term our problem as the Team OP with soft Time Windows and Variable Profits (TOPsTWVP). For a comprehensive review of the OP, the reader can refer to the two surveys by Vansteenwegen et al. [23] and Gunawan et al. [10].

We explore the potential of Iterated Local Search (ILS) to solve HHCDP. ILS is a simple but effective metaheuristic [15] and has been applied successfully to solve different variants of the OP, such as works by Vansteenwegen et al. [24] and Gunawan et al. [8, 11]. We adopt a similar algorithm [11] with several modifications,

such as tackling variable scores/profits, and soft time window constraints. Here, we name it as Enhanced ILS (EnILS).

The main contributions of this paper are listed below:

- We introduce a new variant of the Team Orienteering Problem with soft Time Windows and Variable Profits (TOPsTWVP). To the best of our knowledge, this is the first study dealing with both soft Time Windows and Variable Profits. In this problem, late service is allowed with an appropriate penalty that affects the score/profit. By relaxing the time windows, the number of visited patients will increase without affecting patients' satisfaction significantly.
- Most of interesting applications of the OP are in logistics, tourism and defense. We extend the application of the OP to solve the Home Health Care Scheduling Problem (HHCSP).
- We adopt and implement a fast Iterated Local Search algorithm that has been used for solving other variants of the OP [11]. Note that some obtained results are also feasible for the original TOPTW problem. They are comparable to the state-of-the-art algorithms.

The remainder of this paper is organized as follows. Section 2 summarizes the relevant literature. The description of the HHCDP problem including the mathematical formulation, is presented in Section 3. The Iterated Local Search is explained in Section 4. Section 5 presents computational experiments. Finally, Section 6 describes the conclusions, limitations and possible future works of our research.

2 Related Work

Since our problem is an extension of the TOPsTWVP model, we start by reviewing the literature on the OP and its related variants briefly, followed by the related research on the HHCDP. The OP has been extensively studied in various applications, such as the mobile crowdsourcing problem, the Tourist Trip Design Problem (TTDP), the logistic problem and others [10].

Erdoğan and Laporte [5] introduced the OP with Variable Profits (OPVP). The score for each node is associated with a parameter that determines the percentage of score collected, either as a discrete or continuous function of the time spent. One example of the OPVP application arises in the fishing sectors. Longer stays at certain locations may increase the amount of fish caught. In their work, a branch-and-cut algorithm is proposed to solve some modified TSP instances. There is still room for improvement since the algorithm requires large computational times and can only solve small instances.

Mota et al. [18] modeled the operating room scheduling problem in terms of choice elective patients, aiming for throughput maximization, as a new variant of the TOPTW, namely the TOP double Time Windows (TOPdTW). Both paths and nodes have a time window to be fulfilled. The number of paths equals to the number of operating rooms times the number of shifts times the number of days while the number of nodes refers to the number of operating rooms. A genetic algorithm is proposed to solve benchmark TOPTW instances and randomly generated TOPdTW instances. The computational results are promising although they are still preliminary.

The Home Health Care problem aims to provide the care and support needed to patients in their own homes [1]. It covers different supports, such as elderly people, people with physical disabilities. Cisse et al. [4] classified the Home Health Care Routing and Scheduling Problem (HHCRSP) process into three different levels: strategic, tactical and operational levels, and mapped the into related OR problems. They extended the earlier review [6] which only covers articles before 2016. The details of relevant features, constraints, objectives and methods in the existing HHCRSP studies are also presented.

Rasmussen et al. [21] looked at the daily home care crew scheduling problem as a generalization of the Vehicle Routing Problem with Time Windows (VRPTW). The problem is formulated as a set partitioning problem and solved by an exact branch-and-price algorithm. Visit clustering schemes are also developed in order to reduce computational times significantly, with the cost of the quality of the solutions. The schemes are able to find solutions of larger instances, which cannot be solved optimally. Akjiratikarl et al. [1] also considered the home care worker scheduling problem as the VRPTW.

Yuan and Fügenschuh [25] presented a case study on the problem of scheduling nurses on a weekly basis with the objective of minimizing the total cost as well as the total working time, without compromising the service quality. The problem is treated as a combination of the staff rostering problem and the VRPTW. The proposed algorithm which is based on local search approaches can produce an estimation of cost reduction up to 10% in solving a real-world instance.

Lin et al. [14] addressed a particular problem of the Home Health Care that provides therapy services, namely the Therapist Assignment Problem (TAP). The problem is described from patient and therapist perspectives and modeled as a mixed-integer programming model. The model is validated by using an instance extracted from a rehabilitation service provide in Hong Kong and some randomly generated instances.

Chen et al. [3] introduced a multi-period Home Health Care Scheduling Problem under stochastic service and travel times. The chance constraints are introduced into the formulation in order to cope with uncertainty in durations. The effectiveness of the proposed approaches is tested on synthetic instances for both deterministic and stochastic scenarios.

Nguyen and Montemanni [19] addressed the nurse home services problem and proposed two mixed integer linear programming models based on Big-M method and arc timing method, respectively. Both models cater soft and hard time windows. Certain penalties would be imposed if the service starts between certain periods. However, hard time windows are also imposed to avoid unnecessary overtime. Experiments are conducted on a set of randomly generated instances.

3 Home Health Care Delivery Problem (HHCDP)

We formulate the HHCDP as an Integer Linear Programming (ILP) model. The HHCDP is defined as the following tuple: $\langle N, T \rangle$. Let N be a set of locations, $N = N_t \cup N_s$, where N_t and N_s represent a set of patients' locations and health care providers' start-end locations, respectively. Here, we assume start and end locations for all providers are at the same location, location 0 ($N_s = \{0\}$). T is a

symmetric pairwise travel time matrix and $t_{ij} \in T$ denotes the travel time between two different locations i and j. Let M be a set of health care providers.

Each patient's location $i \in N_t$ has a positive dependent reward u_{im} that would be collected when he/she is visited by provider m. In most cases, patients prefer to be visited by their primary provider. This is reflected by a higher reward in our case. Each visit requires a service time T_i and it should be started within a particular time window $[e_i, l_i]$. e_i and l_i refer to the earliest and latest times allowed for starting the visit at location i. We allow a late arrival with the cost of penalty although this is undesirable. On the other hand, if the provider arrives before e_i, the waiting time occurs.

Since we assume location 0 is the start and end locations, therefore $u_{0m} = T_0 = 0$. Each provider $m \in M$ is constrained within the time limit $[e_0, l_0]$. We have $e_0 = 0$ and $l_0 = T_{max}$, where T^{max} is the time budget or the maximum duration to complete a duty day. The objective is to maximize the expected total collected score from visiting patients by all providers. We include the penalty in the objective function value due to a late visit. The penalty is calculated by multiplying a certain percentage of reduction to a particular score $\{(R_1, R_2, \ldots, R_n) \in [0, 1]\}$. For example, if the arrival at location i is late and less than δ_i, the adjusted score would be $R_1 \times u_{im}$. The details can be referred below:

$$\hat{u}_{im} = \begin{cases} 0, & \text{if } (S_{im} - l_i) \leq 0; \\ R_1 \times u_{im}, & \text{if } 0 < (S_{im} - l_i) \leq \delta_i; \\ R_2 \times u_{im}, & \text{if } \delta_i < (S_{im} - l_i) \leq 2 \times \delta_i; \\ \vdots & \\ R_n \times u_{im}, & \text{if } (n-1) \times \delta_i < (S_{im} - l_i) \leq n \times \delta_i. \end{cases}$$

The following decision variables are used in the mathematical model:

- $X_{ijm} = 1$ if a visit to patient i is followed by a visit to patient j by provider m, 0 otherwise.
- $Y_{im} = 1$ if a visit to patient i by provider m, 0 otherwise.
- $\hat{S}_{im}$ = the start time of service at patient i by provider m.

The HHCDP mathematical formulation is adopted from the work of [11] with several modifications:

$$Maximize \sum_{m \in M} \sum_{i \in N \setminus \{0\}} \pi_i Y_{im} \hat{u}_{im} \tag{1}$$

$$\sum_{j \in N \setminus \{0\}} X_{0jm} = 1 \, , \forall m \in M \tag{2}$$

$$\sum_{i \in N \setminus \{0\}} X_{i0m} = 1 \, , \forall m \in M \tag{3}$$

$$\sum_{i \in N \setminus \{0\}} X_{ikm} = Y_{km} \, , \forall k \in N \setminus \{0\}, m \in M \tag{4}$$

$$\sum_{j \in N \setminus \{0\}} X_{kjm} = Y_{km} \, , \forall k \in N \setminus \{0\}, m \in M \tag{5}$$

$$\sum_{m \in M} Y_{im} \leq 1 \ , \forall i \in N \setminus \{0\} \tag{6}$$

$$\hat{S}_{im} \geq e_i \ , \forall m \in M, i \in N \tag{7}$$

$$\hat{S}_{im} + T_i + t_{ij} - \hat{S}_{jm} \leq \hat{L}(1 - X_{ijm}) \ , \forall i, j \in N, m \in M \tag{8}$$

$$\sum_{i \in N \setminus \{0\}} (T_i Y_{im} + \sum_{j \in N \setminus \{0\}, j \neq i} t_{ij} X_{ijm}) \leq T_{max} \ , \forall m \in M \tag{9}$$

$$\hat{S}_{im} \geq 0 \ , \forall i \in N, m \in M \tag{10}$$

$$X_{ijm}, Y_{im} \in \{0, 1\} \ , \forall i, j \in N, m \in M \tag{11}$$

The objective function 1 is to maximize the expected total collected score from visited patients' locations from all providers. Each location i has a probability of occurrence π_i on a particular day. Each patient has a chance to cancel the appointment. Constraints 2 ensure that each provider starts and ends at location 0. Constraints 4 and 5 determine the connectivity of each provider m. Constraints 6 guarantee that each location i, except location 0, is visited at most once.

Constraints 7 ensure that the start time at location i of provider m is after e_i. Constraints 8 imply that if locations i and j are visited consecutively, then the start time at location j has to be greater than or equal to the start time at location i plus the service time at location i and the travel time from locations i to j. They ensure the timeline of each provider m. Note that $\hat{L}$ is a very large constant value. Constraints 9 limit the time budget for each provider m by T_{max}. Constraints 10 are the non-negativity condition for $\hat{S}_{im}$. Finally, the binary conditions for X_{ijm} and Y_{im} are constrained by equations 11.

4 Solution Approach

In this section, we describe the Iterated Local Search (ILS) algorithm, namely Enhanced ILS (EnILS), which is adopted from the one proposed by Gunawan et al. [11]. ILS has been successfully used to solve various variants of the OP, such as OPTW [8], TDOP [9] and TOPTW [11]. We extend the applicability of the algorithm in solving the HHCDP. EnILS consists of two phases, constructive and improvement phases. We only briefly explain the algorithm especially the parts which are different from the original one. For more details of the original ILS, readers can refer to the work of Gunawan et al. [11].

4.1 Construction Phase

An initial solution is built by a construction heuristic. The idea is to generate a set of all feasible candidate requests F that can be inserted. Each element of F, denoted as $\langle n, p, m \rangle$, represents a feasible insertion of request n in position p of provider m. This set can be very large; therefore, we only consider a subset of possible insertions $F_s \subset F$. Those feasible insertions are ranked according to their $ratio_{n,p,m}$ values. The ratio value for each insertion is calculated based on equation 12. $Diff_{n,p,m}$ represents the difference between the total time spent before and after the insertion of location n in position p of provider m.

$$ratio_{n,p,m} = \left(\frac{\pi_i \times \hat{u}_{nm}^2}{Diff_{n,p,m}} \right) \tag{12}$$

In order to select which insertion to be picked from F_s, we apply the idea of the Roulette-Wheel selection concept [7]. The main idea is that the probability of an element being selected is proportional to its $ratio_{n,p,m}$ value. The element with a higher probability has a higher chance to be selected. F and F_s are updated iteratively. This constructive heuristic is applied until $F = \emptyset$.

4.2 Improvement Phase

In this phase, we implement a metaheuristic based on Iterated Local Search (ILS) in order to further improve the quality of the initial solution S_0 at a particular iteration. We denote S^* as the best found solution so far at a particular iteration, respectively. For the first iteration of this improvement phase, S^* equals to S_0.

The main idea of ILS is to explore the solution space by generating and evaluating the neighbors of S_0. We apply LocalSearch in order to generate the best neighborhood. In LocalSearch, we run six different operators consecutively, as shown in Table 1. The first four operators focus on rearranging the visited locations of providers in order to provide more times to allocate more locations which is done by the last two operators. The first improving neighbor replaces S_0. If a stagnation condition is met, a perturbation strategy on S_0 is then applied. The outline of the ILS algorithm is presented in Algorithm 1.

Table 1: Local Search operations

Operations	Descriptions
SWAP1	Exchange two locations within one provider
SWAP2	Exchange two locations within two providers
2-OPT	Reverse the sequence of certain locations within one provider
MOVE	Move one location from one provider to another provider
INSERT	Insert locations into a provider
REPLACE	Replace one scheduled location with one unscheduled location

The list of operators are identical with the one of [11]. The major difference lies in the checking process when the operator is accepted or not. For example, we may allow swapping two locations (SWAP1 or SWAP2) although the objective function value maybe worse due to late arrivals; however, we may be able to insert more locations in a particular provider later. This arrangement corresponds to

Algorithm 1 ILS (N, M)

```
S_0 ← Construction(N, M)
S_0 ← LocalSearch(S_0, N*, N', M)
S* ← S_0
NoImpr ← 0
while TimeLimit has not been reached do
   S_0 ← Perturbation(S_0, N*, N', M)
   S_0 ← LocalSearch(S_0, N*, N', M)
   if S_0 better than S* then
      S* ← S_0
      NoImpr ← 0
   else
      NoImpr ← NoImpr + 1
   end if
   if (NoImpr+1) Mod Threshold = 0 then
      S_0 ← S*
   end if
end while
return S*
```

the purpose of HHCDP where we allow providers to reach their destinations late although this is undesirable. Some penalties would be imposed due to lateness.

After applying LocalSearch, we implement the perturbation strategy, Perturbation in order to escape from local optima [11]. If the current solution S_0 is better than S^*, we update the best found solution so far S^*. This part is related to the AcceptanceCriterion component of ILS. If S^* is not updated for a certain number of iterations, ((NoImpr+1) Mod Threshold = 0), we restart the search from the best found solution, S^*. Threshold is a parameter that need to be set. Finally, the entire algorithm will be run within the computational budget, TimeLimit.

In Perturbation, we apply two different steps: ExchangePath and Shake. After a certain number of iterations without improvement, we apply ExchangePath; otherwise, Shake is selected. The efficiency of our algorithm depends on both steps. The strategy of selecting two different providers in ExchangePath are based on generating permutations by adjacent transposition method [13]. This step does not change the objective function value directly since we only swap all locations from two different providers. However, in subsequent iterations especially when we apply LocalSearch, more opportunities for operators that have to be applied from the first provider to the last one. The other step, Shake, is based on the one proposed by Vansteenwegen et al. [24]. The focus is to remove certain nodes from each provider, depends on the starting location and subsequent locations need to be removed.

5 Experiments

A comprehensive analysis of the results is reported in this section. We first describe the experiment setup and instances used. We then summarize the performance of the proposed algorithm, EnILS.

5.1 Experiment Setup and Instances

The algorithm is implemented in Java which is executed on a personal computer with Intel(R) Core(TM) i5-6500 with 3.2 GHz CPU, 16 GB RAM. Each instance is run five times for which the average results are presented. We adopt the same parameter values in the earlier work [11]. The parameter tuning is grounded on the Design of Experiment (DOE) methodology.

We use two different groups of instances in our experiments. The first group of instances is taken from the benchmark TOPTW instances [17,22]. The size of instances varies from 48 to 228 locations with the number of providers up to four providers. All benchmark instances can be downloaded from `http://www.mech.kuleuven.be/en/cib/op`.

Since there are no benchmark TOPsTWVP instances, we modify the TOPTW instances by 1) assuming the probability of occurrence of node i, π_i, is set to one, 2) setting $R_1, R_2, \ldots, R_n$ values for all nodes and providers. The second group of instances is randomly generated with varying the two above-mentioned points.

5.2 Experiment Results

5.2.1 Modified benchmark TOPTW instances

The most recent comparison of the state-of-the-art algorithms for TOPTW is conducted by Gunawan et al. [11]. Their proposed algorithms are able to find 50 best known solution values on the available benchmark instances. The experiments were compared with other algorithms by using the *SuperPi* [12] in order to ensure the fairness. Basically, the computational time is adjusted to the speed of the computers used in other approaches. We could not directly compare our results with two state-of-the-art algorithms: I3CH [12] and SAILS [11] since we allow soft time windows in HHCDP. Comparisons of objective function values would have no significance.

However, we observe that several results of EnILS are also feasible to the original TOPTW problem. In other words, there is no time window constraint violation. This could happen since providers also prefer not to delay the service to patients unless they can visit more patients with the possibility of getting lower objective function values. The feasible results are summarized in Table 2. We compare with the results of I3CH and SAILS, referring to I3CH computational times.

Table 2: Overall Comparison of EnILS to I3CH and SAILS

Instance Set	*Numb*	I3CH			SAILS		
		$<$	$=$	$>$	$<$	$=$	$>$
$m = 1$	76	21	25	1	22	25	0
$m = 2$	76	17	18	1	18	18	0
$m = 3$	76	10	8	2	10	10	0
$m = 4$	76	18	7	2	18	9	0
Total	304	66	58	6	68	62	0

Each instance set consists of 76 instances ($Numb$). We count how many feasible solutions which are smaller (<), equal (=) and greater (>) than those of I3CH and SAILS for each instance set. For example, feasible solutions of EnILS which are better than the ones of I3CH are 6 instances, in total.

With regards to I3CH results, EnILS is able to obtain 42.8% of feasible instances (130 out of 304 instances). There are 6 instances with better objective function values while 58 instances with the same objective function values with the ones of I3CH. For SAILS results, we also obtain the same amount of feasible instances. Sixty two instances have the same objective function values with the ones of SAILS.

We also calculate the number of visited locations. The results show that the number of visited locations is increased since we relax the time window constraints. The number of visited locations is increased between 6% to 20% from the results of the TOPTW. On the other hand, the total profit collected is decreased due to some penalties. From the provider's perspective in the context of the HHCDP, this is acceptable since more patients are visited.

5.2.2 Randomly generated instances

We extend the experiments by adding two randomly generated instances where each has a set of different m values. We set the number of locations up to 100 with m = four providers. We assume that certain locations have lower probability of occurrence values (π_i). Instance 1 has 50 locations with a probability of occurrence = 0.5 while Instance 2 has a probability of 0.25.

Table 3 summarizes the results of different scenarios. We emphasize on identifying how many locations with high probability values would be visited. From both instances, we observe that the proposed algorithm, EnILS, is able to visit locations with higher probability values. The percentage of locations with lower probability values is only up to 20.9%. When the probability of occurrence is much lower (e.g. 0.25), the results show that we should not visit any patients since they may cancel their appointments on that particular day. In other words, they are not the first priority of the visit. From the provider perspective, it is an indication that resources need to be increased in order to satisfy all patients although they are lower priorities.

Table 3: Random Instances Results

Instance	m	Number of locations with		Number of visited locations with		Total
		$\pi_i = 1$	$\pi_i = 0.5$	$\pi_i = 1$	$\pi_i = 0.5$	
Instance 1	1	50	50	9 (81.8%)	2 (18.2%)	11
	2	50	50	19 (86.4%)	3 (13.6%)	22
	3	50	50	29 (85.3%)	5 (14.7%)	34
	4	50	50	34 (79.1%)	9 (20.9%)	43
		$\pi_i = 1$	$\pi_i = 0.25$	$\pi_i = 1$	$\pi_i = 0.25$	Total
Instance 2	1	50	50	11 (100.0%)	0 (0.0%)	11
	2	50	50	22 (100.0%)	0 (0.0%)	22
	3	50	50	34 (97.1%)	1 (2.9%)	35
	4	50	50	39 (88.6%)	5 (11.4%)	44

6 Conclusion

In this paper, we address the Home Health Care Delivery Problem (HHCDP) and model it as a variant of the Orienteering Problem (OP), namely the Team OP with soft Time Windows and Variable Profit (TOPsTWVP). In HHCDP, we allow a late visit to the patients. The problem is solved by a fast algorithm, Iterated Local Search. The ILS algorithm is able to provide good solutions within reasonable computational times for two different sets of instances: modified benchmark TOPTW instances and randomly generated instances.

We summarize some directions in which future work on this problem can be explored. The optimal solutions or best known solutions for modified instances and randomly generated instances are still unknown. Therefore, we consider to develop an exact algorithm in order to provide us with the optimal solutions. In order to test the robustness of the proposed algorithm, we will apply the sampling based approach in order to simulate the probability of occurrence of demands in certain locations and analyze the performance of the proposed algorithm. Certain locations may have higher probabilities and it is expected that the proposed algorithm select those locations with higher probabilities. More randomly generated instances would be generated in order to provide and capture real world scenarios.

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Hyper-heuristics using Reinforcement Learning for the Examination Timetabling Problem

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Abstract Selection Hyper-heuristics as general-purpose search methods controlling a set of low level heuristics have been successfully applied to various problem domains. A key to designing an effective selection Hyper-heuristic is to find the right combination of heuristic selection and move acceptance methods which are invoked successively under an iterative single-point-based search framework. The examination timetabling problem is a well-known challenging real world problem faced recurrently by many educational institutions across the world. In this study, we investigate various reinforcement learning techniques for heuristic selection embedded into a selection Hyper-heuristic using simulated annealing with reheating for examination timetabling. Reinforcement learning maintains a utility score for each low level heuristic. At each iteration, a heuristic is selected based on those adaptively updated utility scores and applied to the solution at hand with the goal of improvement. All selection Hyper-heuristics using different reinforcement learning schemes are tested on the examination timetabling benchmark of ITC 2007. The results show that ε-decay-Greedy reinforcement learning which chooses a low level heuristic with the maximum utility score with a decaying probability rate, otherwise choosing a random low level heuristic performs the best. The proposed tuned approach although does not perform as good as the state-of-the-art, it delivers a better performance than some existing Hyper-heuristics.

Keyword: reinforcement learning, Hyper-heuristic, exam timetabling

1 Introduction

The research involved in the search for suitable solutions for Examination timetabling problems requires a combination of practical and experimental-based techniques [1]. The main focus of current Hyper-heuristic research is towards the design and adaptation of heuristic methods in automating optimisation for hard computational search problems. Over the last few decades a lot of research has been conducted on the application of Hyper-heuristic techniques in solving examination timetabling problems. A number of review papers highlighting these efforts have been published [2], [3]. Experimental results generated by various different techniques in the literature have been reported, focussing on success measures including generality of the solver and the time cost [2].

For a given examination timetabling problem, whether a technique can generate a feasible and workable solution within a practical time limit is an important factor in assessing the success level of the technique employed. The solving process of the examination timetabling problem normally commonly consists of two main stages; the initial construction stage and the subsequent improvement stage. In this first stage, a feasible solution is constructed. Along with feasibility, some focus is given to the relative quality of the solution in terms of satisfying the problem, to benefit the improvement stage which is more computationally time-consuming. Then, in the second stage, the solution undergoes a series of improvement transformations until the process terminates. In this case this will occur when the time limit is reached. The improvement process outlined in this paper describes a Hyper-heuristic search methodology used to search for higher quality solutions when given specific objectives [4].

In the literature, research on the application of Hyper-heuristic techniques to solve examination timetabling problems has proved the potential of their effectiveness in this area. Certain research has focussed on combining reinforcement learning with Hyper-heuristics, yielding competitive results which suggest that reinforcement learning can be applied to improve the performance of Hyper-heuristics in terms of solution quality. However, the work in the

literature mainly focus on bringing reinforcement learning theory into the hyper-heuristic instead of actual reinforcement learning techniques.

This works aims at investigating how the various Reinforcement Learning algorithms can be used to improve the performance of Hyper-heuristics. In this work, rather than simply applying reinforcement learning trial-and-error schemes within the Hyper-heuristic as in the literature, actual algorithms and techniques from Reinforcement Learning are designed and integrated into the Hyper-heuristic to solve the examination timetabling problem. In the approach described here, the initial feasible solution is created using Squeaky Wheel construction, with a Hyper-heuristic employed for the optimization and improvement phase. The Hyper-heuristic methods consists of two levels; one is the higher heuristic search from the low-level heuristics pool, and the other is the low-level heuristic search process within the problem domain. For the higher-level heuristics, a Simulated Annealing (SA) method is combined with different reinforcement learning methods to enable intelligence-based selection of the low level heuristics. In each iteration information is gathered to establish and record the current total performance of low level heuristics. The selection methods inspired by the reinforcement learning are three different greedy search methods and the softmax method, which are compared with a simple random method to analysis performance difference. For the low level heuristics, we designed six small-move low-level heuristics, three large-move low-level heuristics and two directed low-level heuristics. This can also be considered as an interesting approach to explore how the generality of the solution process can be improved using reinforcement learning. An assessment can be made on how the combination of reinforcement learning algorithms with Hyper-heuristics may widen the application of solvers to other types of resource allocation problems.

The remainder of the paper is as follows: Section 2 briefly describes the background of this paper, the examination timetabling problem and the Hyper-heuristic method. Section 3 describes the Squeaky Wheel constructor and our RL-SA Hyper-heuristic method (**Algorithm 1**). Section 4 describes the experimental environment and time parameters used during experimentation. Section 5 presents and discusses the results and section 6 concludes the paper with a brief discussion on the effectiveness of the technique and potential future research areas.

2 Background

2.1 The Examination Timetabling Problem

Examination timetabling is one of the academic timetabling problems and has been proven to be NP-hard [5]. Solving an examination timetabling problem requires allocating the given exams with suitable timeslots and rooms under both hard and soft constraints. Among these constraints, all hard constraints must be satisfied in order to achieve a feasible solution, with the satisfaction of the soft constraints used to determine the quality of a given solution. Therefore, the solving process consists of two main parts: first, the generation of a feasible solution that meets all the hard constraints; second, minimization of violations to the soft constraints of the current solution while maintaining feasibility. What's more, the quality of (feasible) timetables can be calculated numerically based on the satisfaction level of the soft constraints which can be used to help in the evaluation of the corresponding algorithms. In the examination timetabling problems, before the scheduling process all the student enrolment data is generally known, which allows an accurate use of the available resources during the examination period.

In real-world examination timetabling problems, the type and parameter values of the hard and soft constraints varies between institutions. In particular, the soft constraints usually reflect an institutions' different preferences. As interest has increased on the application of research to examination timetabling problems, benchmarks which identify the common soft and hard constraints from real world institutions using various standard defined measures have emerged. These are used to advance the field of research in solving Examination Timetabling problems in providing a way to allow meaningful scientific comparison and the public exchange of research achievements and expertise in the domain.

2.2 Hyper-heuristic

The use of Hyper-heuristics involves combining a set of approaches that share the common goal of automating the design and adaptation of heuristic methods in order to solve hard computational search problems [21]. A heuristic refers to a whole search algorithm or used to refer to a particular decision process sitting within some repetitive control structure, while meta-heuristic refers to an overall control strategy exploring a solution search-space [21]. Unlike meta-heuristics, Hyper-heuristics search in a space of low level heuristics [22].

Specifically, a Hyper-heuristic can be seen as a process which, when given a particular problem instance and a number of low-level heuristics, manages the selection of which low level heuristic to apply at any given time, until a stopping condition is met [21]. The Hyper-heuristic operates at a higher level of abstraction without knowledge of the domain in which it operates. It only has access to a set of low level heuristics [22], simple local search operators or domain dependent heuristics [22], while the higher level strategy (High Heuristics, for selecting or generating heuristics) can either be a heuristic, a sophisticated knowledge-based technique, or a learning mechanism [20].

2.3 Related work

The research on solving examination timetabling problems involves a variety of different Hyper-heuristic methodologies. Graph colouring Hyper-heuristics were investigated as a new constructive Hyper-heuristic method to solve the examination timetabling problems in [24]. The author utilized the hierarchical hybridizations of four low level graph colouring heuristics which including largest degree, saturation degree, largest coloured degree and largest enrolment to produce four ordered lists. The sequence of exams selection for scheduling is based on the generated lists. Furthermore, a roulette wheel selection mechanism is included in the algorithm to improve the effectiveness of timeslot selection, aiming at probabilistically selecting an appropriate timeslot for the chosen exam. The proposed approach was tested upon the Carter benchmarks as well as the ITC2007 benchmarks. The experimental results proved that the graph colouring constructive Hyper-heuristic is capable of producing good results and outperforming other approaches on various benchmark instances.

Inspired by the musical improvisation process, the Harmony Search Algorithm (HSA) is a relatively new metaheuristic algorithm and is used within the Harmony Search-based Hyper-heuristic (HSHH) for solving examination timetabling problems. The Harmony Search algorithm will operate at a high level of abstraction which intelligently evolves a series of low-level heuristics in the improvement process. Each low-level heuristic equates to a move and swap strategy. The authors tested the proposed method using ITC-2007 benchmark datasets, with 12 different datasets of various complexity and size. The proposed method produced strong competitively comparable results.

Monte Carlo based hyper heuristics are designed and implemented to solve the examination problems in [6]. Simulated annealing involving a reheating scheme was introduced to this Monte Carlo based hyper heuristic. However, this method compared poorly as compared to other Hyper-heuristic methods. The author believes that the reason for this is due to fundamental weaknesses such as the choice of appropriate rewarding mechanism and the evaluation of the utility values used for heuristic selection. The conclusions suggested that the use of a choice function rather than incorporating Simulated Annealing might improve the method itself.

Burke et al. brings forward the concept of the reinforcement learning process to hyper heuristics in [7]. In this paper, Hyper-heuristic methodologies were identified to search the heuristic selection space and use selected low level heuristics to search the problem domain. According to the general definition, their proposed method is an iterative Hyper-heuristic framework formed of a single candidate solution and multiple perturbative low level heuristics. Basically, two parts including the heuristic selection and the move acceptance with certain termination criteria formed this algorithm. Inspired by related work in this area, one of the main focuses of this study is to analyse the working processes of learning heuristic selection within the automatic search for examination timetabling solutions..

3 RL-SA based Hyper-heuristic Algorithm

The Hyper-heuristic algorithm solving process for this research consists of two stages; first an initial construction phase which generates a feasible solution, secondly a further improvement phase to ultimately achieve a better solution. In the construction stage, we use Squeaky Wheel construction [8] & [14] while the optimization stage is based on the Simulated Annealing process, combining reinforcement learning with the use of Hyper-heuristics.

3.1 RL-SA based Hyper-heuristic Optimization

The Hyper-heuristic consists of three main parts; the move acceptance Simulated Annealing algorithm [10], [11], [12] & [13], heuristic selection method Reinforcement Learning algorithms and a set of various low level heuristics. The selection method will select from the low-level heuristics pool to optimize the current solution. The selected low heuristic will propose a move to optimize the current solution. The move acceptance is the reference for the algorithm

in deciding whether to accept this move or not. For each low heuristic, a utility value is used to represent its performance and is updated dynamically. The utility update methods are also based on reinforcement learning algorithms.

Algorithm 1: RL-SA ALGORITHM

```
Input –u: array holding utility value for each heuristic, totalTime, T0: initial temperature
1. // initialisation
2. Generate a random complete solution Scurrent ;
3. Initialise utility values;
4. fbest =fcurrent = f0 = quality( Scurrent );Sbest=Scurrent; //Sbest holds the best solution
5. startTime = t = time();
6. Temp=T0;sinceImp=0; numNonImprovingReheat=0;maxReheatTimes,=5;reheatFrequency=1000;
7. bool bImprovementFound=false;
8. // main loop executes until total running time allowed is exceeded
9. while ( t < totalTime ) {//while running time still enough
10.     // heuristic selection
11.     i = selectHeuristic( u ); // select a heuristic using the utility values
12.     Stemp = applyHeuristic( i, Scurrent);//update temperare solution using selected low heuristic i
13.     ftemp = quality( Stemp );//update temperare evaluate value
14.     t = time() – startTime;//update remaining time
15.     // move acceptance
16.     Δ = ftemp − fcurrent;
17.     if (Δ < 0){// improving move
18.         u[i] = reward( u[i] );Scurrent = Stemp;}
19.     else u[i] = punish( u[i] ); // worsening move
20.     r ← random ∈ [0,1];
21.     if ( Δ < 0 ||r < P(Δ, Temp) ){// accept if non-worsening or with the Boltzmann probability of P
22.         if (Δ < 0) {Scurrent = Stemp;fcurrent=ftemp; bImprovementFound=true;}
23.         if (ftemp<fbest) Sbest = Stemp; else sinceImp++;}
24.     if (sinceImp==reheatFrequency) {//when reach non improvement limit
25.         sinceImp = 0; //reset since last improvement count to 0
26.         if(!bImprovementFound) numNonImprovingReheat++;
27.         else bImprovementFound=false;
28.         if(numNonImprovingReheat>maxReheatTimes) {//when reach max reheating limit
29.             numNonImprovingReheat = 0;
30.             Temp = (T0 - Temp) / 2.0;
31.             fbest = (long)Math.Round(fbest * 1.1);}
32.         else Temp = Temp / 0.85;//increase temperature}
33.     else Temp=Temp*0.9;// decrease temperature}
34. return Sbest;
```

3.2 Reinforcement Learning

Reinforcement Learning (RL) is a general term for a set of widely used approaches which provide a learning mechanism in determining how to behave against certain outcomes, or “how to map situations to actions" through trial-and-error interactions [9]. The proposed heuristic selection method involved the design of four different selection methods as well as four different low heuristic utility reward/punish methods inspired by reinforcement learning algorithms. The detailed design description of different Reinforcement Learning inspired heuristic selection methods as well as utility update methods are given in the rest of this section. The experimental comparisons are presented in chapters 5.3 and 5.4 respectively.

3.2.1 Selection methods

(0) Equal Sequence

In this selection method, each low heuristic will be selected in sequence to ensure each will be applied in equal measure. This is inspired by the Maximize Explore theory used within reinforcement learning.

(1) Greedy

In this selection method, the process checks whether all the low-level heuristics have been selected at least once. If not, a random low heuristic is selected, with this process continuing until all low-level heuristics have been selected. From this point the low heuristic with the highest current utility value will be selected. Where there are more than one low heuristic selected, one is chosen randomly from the contenders.

(2) ε-Greedy

In this selection method, random selection is employed until all low heuristic has been selected at least once. After that, if the random probability is smaller than greedy probability ε, low-level heuristics are selected at random. Otherwise, the low heuristic with the highest utility value is selected. In determining the most effective value for the greedy probability ε, experimental tests were performed with ε set to 0.1 as well as ε set to 0.01.

(3) ε-Decay-Greedy

Similar to the previous method, although the value of the greedy probability ε is dynamic, decreasing in value as the optimization process continues. The rational for this is based on the principle that as the Hyper-heuristic algorithm learns, the random low heuristic will become less necessary. The algorithm for this selection method is outlined in detail below.

Algorithm 2: e-decay-greedy

```
input: iteration: current iteration times;llhList: arraylist of all the low-level heuristics
1. int index;//index of low heuristic
2. double edecay = 1 / (System.Math.Sqrt(iteration));
3. Random randDouble = new Random();
4. if (randDouble.NextDouble() < edecay || !checkSelected(llhList)){
5.    //under edecay probablity, do random select
6.    Random randInt = new Random();
7.    index = randInt.Next(0, llhList.Count);}
8.  else{
9.     index = llhMaxUtilityIndex(llhList);
10.    index = randomDuplicateSelectionSum(index, llhList);}
11. //select one of the highest utility heuristic
12.  return index;}
```

(4) softmax selection

Unlike the previously described selection methods, this calculates the softmax probability value for each low heuristic and uses this probability value for selecting among the low-level heuristics.

3.2.2 Utility update methods

Within the process of updating the utility values, a positive "reward" (increment) is given to a low heuristic if it was effective in generating a better solution, otherwise a "punish" value is applied. For all of the utility update methods, the lower bound of the utility value is set to 0, upper bound set to 40 and initial value of 20. These values were reached as a result of trial experimentation.

(5) Sum utility

In this method the reward value is +1 and the punish value is -1.

(6) Average utility value/Monte Carlo

In this method the reward value is +1 and the punish value is -1. The sum utility is then divided by the iteration count to set as an average.

(7) Discount sum

This method is similar to the first utility update method, although the positive reward and negative punish values are dynamic. The positive reward value is calculated as (+1)*(0.9^ (iteration times)), with the negative punish value calculated as (-1)*(0.9^ (iteration times)).

Algorithm 3: Discount Sum Utility method

```
Input: bool flag: reward or punish, int index, llhList: heuristics arrylist, SelectedTimes, discountFactor=0.9.
1. for (int i = 0; i < llhList.Count; i++){
2. if (llhList[i].LlhId == index){
3.     if(flag) llhList[i].utility+= (+1) * discountFactorR^selectedTimes;
4.     else llhList[i].utility+= (-1) * discountFactorR^selectedTimes;}}
```

(8) Temporal difference

The theory of this utility update method is that it will estimate the future possible reward based the current performance of each low-level heuristics. First, this method will calculate the sum utility with the reward and punish values (again set as +1 and -1 respectively). Secondly, an estimate reward value is calculated. The selected probability of the current low heuristic in the next step is 1.0 / (number of low-level heuristics) and the reward Probability (the probability it might make an improvement in the next iteration if selected) is 0.5 (50%). Therefore the estimate reward is (selected Probability)*(reward Probability). This estimate reward is then added to the current calculated sum utility as the low heuristic's final utility value. The step size value has been experimentally tested between 1 and 5 to determine the most effective setting.

3.3 Low-level heuristics design

For this work, 19 low-level heuristics were implemented, grouped into four main types of heuristic and described below. Most of the above low-level heuristics are either 1-opt or 2-opt and there is also a mixture of some randomness and deterministic selection of exams and slots. We purposely test low-level heuristics with simple moves rather than low-level heuristic with intelligence and complex moves because we want to make sure that the Hyper-heuristic can recognise good moves and make an intelligent decision based on these simple moves. Furthermore, we want to make the problem-domain knowledge heuristics easy to implement and the Hyper-heuristic more generalised.

3.3.1 Small perturbative moves

Table 1: small heuristics

H1 (random period assignment):
Select a Random Exam in Period P1 with Room assignment R1. Assign Exam to Random Period P2 (if feasible), do not change Room assignment.
H2 (random room assignment):
Select a Random Exam in Period P1 with Room assignment R1. Move the Exam to a feasible Random Room R2 – do not change the Period.
H3 (random assignment):
Select a Random Exam in Period P1 with Room assignment R1. Assign Exam to Random Period P2 (if feasible) and Random Room R2 which can accommodate the exam (if feasible – exclusive constraint).
H4 (random period swap):
Select 2 Random Exams, one in Period P1 with Room assignment R1 and the other in Period P2 (≠P1) with Room assignment R2. Swap them both to the others Periods (if both feasible) – no swapping of Rooms.
H5 (random room swap):
Select 1 Random Exam with Room assignment R1 (ignore the period – ensure not exclusive). Select another Random Exam with similar capacity requirement assigned to Room R2 (≠R1). Swap them both to the others Room (if both feasible, e.g. both not exclusive) – no swapping of Periods.
H6 (random exam swap):
Select 1 Random Exam with Room assignment R1 in Period P1. Select another Random Exam Room assignment R2 in Period P2, Swap them both to the others Room (if both feasible, e.g. both not exclusive) and swapping their Periods.

3.3.2 Large perturbative moves

Table 2: large heuristics

H7 (random group of exams swap based on period):
Select two Random Periods P1 and P2 and swap ALL exams between them.
H8 (random group of exams move based on period):
Select one Period P1 and move each exam assigned to that period to a new feasible period, one by one (APPLY H1).
H7_2 (random group of exams swap based on room):
Select two Random Room R1 and R2 and swap ALL exams between them.
H8_2 (random group of exams move based on period):
Select one Room R1 and move each exam assigned to that room to a new feasible room, one by one (APPLY H2).
H7_3 (random group of exams swap based on timeslot):
Select two Random Timeslot T1 and T2 and swap ALL exams between them.
H8_3 (random group of exams move based on period):
Select one Timeslot T1 and move each exam assigned to that timeslot to a new feasible timeslot, one by one (APPLY H3).

Note: Timeslot in the above table refers to the combination of period and room.

3.3.3 Very large moves

Table 3: shuffle heuristics

H9 (random group of exams shuffle based on period):
Shuffle all periods at random. Loop over all Periods *i=P_1..P_K; H6(i, Random(i+1,K))*.
H9_2 (random group of exams shuffle based on room):
Shuffle all rooms at random. Loop over all Rooms *i=R_1..R_K; H6(i, Random(i+1,K))*.
H9_3 (random group of exams shuffle based on timeslot):
Shuffle all timeslots at random. Loop over all Timeslots *i=T_1..T_K; H6(i, Random(i+1,K))*.

3.3.4 Directed moves

Table 4: directed heuristics

H10 (ranked exams group move - consider both hard and soft constraint violations):
Generate a weighted-List based on all hard and soft constraint violations for exams. For the top 10 exams in the list, re-allocate them to a random new timeslot. The new timeslot is selected from the suitabletimeslot-List. Update weighted-List whenever a move is made.
H10_2 (ranked exams group swap - consider both hard and soft constraint violations):
Generate a weighted-List based on all hard and soft constraint violations for exams. From the top 10 exams in the list, select two random exams, swap their timeslot. Update weighted-List whenever a move is made
H11 (ranked exams group move - consider only soft constraint violations):

Generate a softconstraint-List based on all soft violations for exams. For the top 10 exams in the list, re-allocate them to a random new timeslot. The new timeslot is selected from the suitabletimeslot-List. Update softconstraint-List whenever a move is made.
H11_2 (ranked exams group swap - consider only soft constraint violations):
Generate a softconstraint-List based on all soft constraint violations for exams. From the top 10 exams in the list, select two random exams, swap their timeslot. Update softconstraint-List whenever a move is made.

4 Experimental Environment

The algorithm was implemented and tested on a PC with a 2.9 GHz Intel Core i7processor, 8GB RAM and macOS 10.12.05. The program was coded in C# targeting the .NET Framework 4.5. For each problem set, the program was executed for ten iterations, with a 270 second time limit per iteration determined by a benchmarking application released by the competition organizers. During initial experimentation it was found that allowing adaptive construction to execute for approximately 10% of the total execution time provided the best results.

5 Results and Analysis

As described above, four different selection methods and four different utility update methods were designed and implemented. Note that for the exam timetabling solution, the lower the score achieved the better the solution quality as it represents the total violations of hard and soft constraints. The least amount and penalty of violations, the higher quality the resultant solution.

5.1 Benchmark data sets

The algorithm was tested with the 2007 International Timetabling Competition (ITC2007) benchmark data sets. These were chosen due to the fact they are based on real examination data problem sets with a richer set of constraint considerations, to a certain extent helping to determine how this approach may assist in real-world similar problems.

Table 5: benchmark attributes

Datasets	Exams	Students	Periods	Rooms	Conflict Density	Period Hard Constraints	Room Hard Constraints
Exam 1	607	7891	54	7	5.05%	12	0
Exam 2	870	12743	40	49	1.17%	12	2
Exam 3	934	16439	36	48	2.62%	170	15
Exam 4	273	5045	21	1	15.00%	40	0
Exam 5	1018	9253	42	3	0.87%	27	0
Exam 6	242	7909	16	8	6.16%	23	0
Exam 7	1096	14676	80	15	1.93%	28	0
Exam 8	598	7718	80	8	4.55%	20	1
Exam 9	169	655	25	3	7.84%	10	0
Exam 10	214	1577	32	48	4.97%	58	0
Exam 11	934	16439	26	40	2.62%	170	15
Exam 12	78	1653	12	50	18.45%	9	7

Table 5 lists the main features for each of the examination datasets of ITC2007. The smallest could be argued to be either Exam 9 or Exam 12. The largest exam datasets include Exam 3/Exam 11 or Exam 7. Exam 3 and Exam 11 are almost identical, apart from the fact that Exam 11 has a much smaller period size. The following tests focus on Exam

7, 9, 11 and 12 as they include the biggest and smallest data size as well as conflict density and provide a reasonable variation for testing purposes. The tests on the four datasets were used to guide the design of the final Hyper-heuristic reheating scheme, selection method and the utility value update method.

5.2 Simulated Annealing operator tests

For the simulated annealing process, four groups of experiments were carried out to define the best settings for reheating frequency and to determine whether to combine the reheating scheme with the reset of the low-level heuristics utility value on reaching the maximum reheating count (which is 5).

5.2.1 Reheating frequency

In the literature, the Simulated Annealing value is usually suggested as 10000 [10] & [11]. However in this approach, to help avoid getting caught within local optima, the reheating times are reduced to 1000, verified as effective via experimentation. The reheating frequency is tested at values 1000 and 10000 on Exam 7, 9, 11 and 12 with ten runs for each dataset. Here, a simple random selection method and sum utility update method is used, and the utility value is not reset when the maximum reheating count is reached.

Table 6: reheating frequency tests

Datasets	**reheat frequency = 1000**		**reheat frequency = 10000**	
	AVG	BEST	AVG	BEST
Exam 7	8631	**8497**	8858	8805
Exam 9	1347	**1278**	1352	1300
Exam 11	35782	**35266**	35504	35611
Exam 12	5985	5965	6034	**5911**

The results presented in table 6 above suggest that the reheat frequency of 1000 is generally better. Therefore, in the following test the reheat frequency is set as 1000.

5.2.2 Reset low-level heuristics value

The utility values of the low-level heuristics can become similar in value after a certain amount of iterations, which can adversely affect the selection methods and render the process ineffective. We test the approaches of both resetting and not resetting the utility value of the low-level heuristics when the maximum reheating point has been reached. This uses a simple ε-greedy selection method and sum utility update method.

Table 7: reset low-level heuristics utility value tests

Datasets	reset		No reset	
	AVG	BEST	AVG	BEST
Exam 7	7530	**6721**	7890	7528
Exam 9	1340	**1320**	1520	1506
Exam 11	35733	**35519**	35673	35375
Exam 12	5995	**5955**	6015	6165

The results presented in table 7 above suggest that generally the performance is improved by resetting the utility value of all low-level heuristics on reaching the maximum reheating iteration count. Therefore, this approach is adopted in the selection method tests outlined in the next section.

5.3 Selection Method test

As described above, four different selection methods were designed, some with differing operators. Tests were performed on all the selection methods with different operator values on Exam 7, 9, 11 and 12 with ten runs per dataset.

These tests use the sum utility update method. The **best** results for each selection method are presented in the table below.

Table 8: different selection methods comparison

Datasets	**equal sequence**	**greedy**	**e-greedy, e=0.01**	**e-greedy, e=0.1**	**Decay greedy**	**Softmax, Temperature=0.1**	**Softmax, Temperature=1**
	BEST	BEST	BEST	BEST	BEST	BEST	BEST
Exam 7	8644	7792	8381	7953	**6501**	8856	8912
Exam 9	1372	1303	1343	1306	1288	**1276**	1396
Exam 11	35840	35295	35038	35281	34054	35832	35441
Exam 12	6065	5965	5965	5965	**5965**	6065	5965

In this test, for the ε-greedy two values, 0.1 and 0.01 are tested, with the setting 0.1 beating 0.01 in most cases. For the softmax selection method, the setting of temperature controls the balance of exploration and exploitation. The test temperature values of 0.1 and 1 are chosen based on the literature, with the results suggesting setting this value to 0.1 on average performs better, although the advantage is not great. Overall, the decay greedy method has the lowest best result over the chosen datasets.

5.4 Utility update method

This test uses the decay greedy selection method combined with several different utility update methods. For this test, the temporal difference utility update method uses an estimation step-size setting value of 1 in order to establishing effectiveness of this approach.

Table 9: different utility value update method comparison

Datasets	**Sum**	**Average**	**Discount**	**Temporal Difference**
	Best	Best	Best	Best
Exam 7	6501	8824	**6721**	9233
Exam 9	1288	1370	**1258**	1423
Exam 11	34954	35529	**34390**	34399
Exam 12	6065	6065	**5883**	5916

It is clear from the results that the discount sum utility update method performs best over the chosen datasets.

5.5 Comparison with the published literature

Having chosen the best selection method (decay greedy) and the best utility update method (discount sum utility), experimentation is carried out over all 12 datasets, with a best evaluation value recorded over 10 times for each. A comparison over the current best techniques in the literature is presented in table 10 below.

The results obtained show that the proposed method is reasonably competitive, and in fact has achieved a best result for Exam 10, and is approaching the best for several others. The approach taken can therefore be argued as promising and worth further research and experimentation. Although for the rest of the datasets, the proposed approach didn't generate any further best results, it should be noted that the proposed approach did manage to solve all ITC2007 datasets despite their complexity. Meanwhile, the proposed approach is capable of delivering relatively good solutions compared to other techniques in general. Examination of the penalty values for the generated results, it is clear that the proposed approach performs better for the datasets with smaller size (such as Exam 10, Exam 6, Exam 9) than for the datasets with larger size (such as Exam 7 and Exam 3). It is reasonable to suggest that this is due to the smaller datasets having been allowed a longer learning time than the larger datasets.

Table 10: Best result comparison to the literature

Datasets	RL-SA-HH	Other Techniques				
	BEST	**DSO [14]**	**Muller ITC2007 [15]**	**Adaptive Liner Combination [16]**	**Graph Coloring [17]**	**Multistage algorithm [18]**
Exam 1	6059	5186	**4370**	5231	6234	5814
Exam 2	863	405	400	433	**395**	1062
Exam 3	14027	9399	10049	**9265**	13302	14179
Exam 4	20031	19031	18141	**17787**	17940	20207
Exam 5	3637	3117	**2988**	3083	3900	3986
Exam 6	26910	**26055**	26585	26060	27000	27755
Exam 7	6572	**3997**	4213	10712	6214	6885
Exam 8	10485	**7303**	7742	12713	8552	10449
Exam 9	1267	1048	**1030**	1111	N/A	N/A
Exam 10	**14357**	14789	16682	14825	N/A	N/A
Exam 11	34054	30311	34129	**28891**	N/A	N/A
Exam 12	5509	**5369**	5535	6181	N/A	N/A

5.6 Comparison with other Hyper-heuristic methods

It is interesting to examine the experimental results of other Hyper-heuristic techniques which have been applied to solving the examination timetabling problem using the ITC 2007 benchmarks. A comparison with the approach undertaken in this work is useful to determine whether it improves on the current work using Hyper-heuristic.

Table 11: Comparison to other Hyper-heuristic methods in the literature

Datasets	RL-SA-HH	Other Techniques			
	BEST	**HSHH [25]**	**GCCHH [19]**	**EAHH [19]**	**AIH [23]**
Exam 1	**6059**	11823	6234	8559	6235
Exam 2	863	976	**395**	830	2974
Exam 3	14027	26670	13302	**11576**	15832
Exam 4	20031	////	**17940**	21901	35106
Exam 5	**3637**	6772	3900	3969	4873
Exam 6	**26910**	30980	27000	28340	31756
Exam 7	6572	11762	**6214**	8167	11562
Exam 8	10485	16286	**8552**	12658	20994
Exam 9	**1267**	-	-	-	-
Exam 10	**14357**	-	-	-	-
Exam 11	**34054**	-	-	-	-
Exam 12	**5509**	-	-	-	-

The results in table 11 above, show that this approach has achieved best results for three of the data sets as compared to the other hyper heuristics in solving the examination timetabling problem for the benchmark ITC2007. Indeed, for the other datasets the results are relatively good (above the average), suggesting that this work makes a positive contribution to improving research within the area of Hyper-heuristics.

6 Conclusion

This paper outlined the implementation of a combination of various algorithms of Reinforcement Learning and Hyper-heuristics to solve the examination timetabling problems. The construction stage uses an adaptive Squeaky Wheel algorithm to generate a feasible and relatively good initial solution. The improvement phase focusses on combining reinforcement learning algorithms into a Hyper-heuristic, with the examination timetabling problems as

the target application area. It uses four selection methods and four utility value update methods inspired by reinforcement learning, with a series of experimentation performed to test the effectiveness of the approach.

The main aim of this work is in identifying a general approach to utilizing reinforcement learning algorithms in solving complicated resource allocation problems. The examination timetabling problems were selected due to their high complexity and the fact this complexity means they are not suitable to be solved directly using reinforcement learning alone. The work and results presented show that combining reinforcement learning with a Hyper-heuristic forms a workable way to solve complicated problems. This also contributes to the automatic scheduling research area using a creative hyper heuristic. Multiple selection methods and utility values methods inspired by reinforcement learning algorithms fits the learning theory of the hyper heuristic and are designed and implemented in this work. Experimental comparisons focussing on the performance of these different selection methods and utility value methods are also presented in this paper All the designed selection method as well as utility value methods are capable of solving the ITC2007 dataset examination timetabling problems. Among the various selection methods and utility value methods, those with the best performance are selected to generate the final best result. The experimental best results generated by the proposed approach are compared to the other techniques and proves the potential of bringing Reinforcement Learning Algorithms to the area of Hyper-heuristics. The problem area of examination timetabling, and in particular the chosen benchmarks used are closely reflective of real world scheduling problems, bringing theory to real world problem solving. In addition, the tests carried out also cover the investigation of how reheating influences the simulated annealing searching process.

Future work is planned to extend the reinforcement learning Hyper-heuristic to other resource allocation optimization problems as well as combining further reinforcement learning algorithms into this area. Another strand of research from this work will be involved in the discovery of how the unsupervised learning algorithms can better fit into the optimization research area. The ultimate aim is to prove this approach works in general for a wide variety of optimisation problems, with encouraging results already obtained in progressing towards this goal.

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The Application of Nondominated Sorting Genetic Algorithm (NSGA-III) for Scientific-workflow Scheduling on Cloud

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Abstract Virtual machines on cloud are promising environments for deploying scientific workflows due to its unlimited on-demand and scalable machines. The pay-per-use model and a variety of the processing capacity, virtual machines allow large-scale workflows to be executed with reasonable cost and competitive performance. Efficient allocation of a set of dependent tasks on such dynamic resources is advantageous as it leads to well utilization resources. This becomes a multi-objective optimization of task scheduling and has attracted much interest in recent years. This study is the first attempt to determine solutions for a multi-objective optimization of scheduling on cloud where Cost, Makespan and VM utilization are the objectives. Four Scientific Workflows are selected for the study, these are CyberShake, Epigenomics, LIGO, and Montage. The Nondominated Sorting Genetic Algorithm (NSGA-III) is selected as the tool. Chromosome representation based on the topological order of the workflow under consideration and a workflow scheduling algorithm is suggested. The solutions for the multi-objective optimization are presented and compared with a single objective solution. Cost and Makespan are the common two objectives for scheduling of task on cloud, where VM utilization is usually neglected. The study affirms the role of multi-objective optimization, especially where VM utilization is included.

Keywords : CyberShake, Epigenomics, LIGO, Makespan, Montage, Multi-objective optimization, Nondominated Sorting Genetic Algorithm (NSGA-III), Scientific workflow, Task scheduling, VM utilization

1 Introduction

In scientific community, large-scale scientific applications such as bioinformatics, astronomy, and physics usually imply large volume of data and computation intensive [1]. A common method to improve the computation efficiency is to split the whole process into multiple tasks and process them over distributed systems such as cluster, grid and cloud computing. These multiple tasks are arranged to work together as a workflow and can be represented by a *Directed Acyclic Graph* (DAG) model [2]. Tasks are denoted by nodes while their dependency and order of processing are denoted by directed edges. Having an efficient scheduling would ensure that workflow is efficiently executed over distributed platforms. However, it is commonly known that determination of such efficient schedule is an *NP*-hard problem [3, 4]. Implementing scheduling algorithms for scientific workflow on cluster and grid computing has become a research field in itself [5].

Over the years, cloud computing is gaining popularity over conventional distributed systems because of it offers self-management ability to users. Several cloud services are available such as *Infrastructure-as-a-Service* (IaaS), *Platform-as-a-Service* (PaaS), and *Software-as-a-Service* (SaaS). IaaS is probably the most suitable service in order to run scientific workflow because it provides flexible, scalable virtual machines (VMs) that users can deploy and run workflow on-demand. It is also possible for users to launch both unlimited and different types of VM. Users are usually charged by considering their used VM time, pay-per-use model. This allows users to select services according to their budget.

Several scheduling objectives have been studied from different perspectives, including cloud provider, broker, and user perspective [6, 7]. From the cloud user perspective, *Cost* and *Makespan* (the maximum completion time) are the two common objectives where multi-objective research in this area is most prolific [6] as they are obvious conflicting objectives. Nevertheless, other objectives are also significantly important, some of these are respond time, energy consumption, communication cost and overhead and VM utilization [6-10]. VM utilization is advantageous in IaaS cloud since it represents how well a scientific workflow is executed on leased VM. It has an impact to both *Cost* and *Makespan* as well. So, scheduling with VM utilization awareness can help balancing both *Cost* and *Makespan*.

Due to its *NP*-hard nature, evolutionary algorithms (EA) has an application in scheduling the execution of scientific workflows on cloud, especially when the required schedule is multi-objective optimization. Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO) and Nondominated Sorting Genetic Algorithm-II (NSGA-II) are some of the popular algorithms for solving the multi-objective optimization [6]. NSGA-II [11] attracts some attention and has been improved to a new version known as NSGA-III [12]. Nevertheless, it has not been applied to scheduling problem.

This paper presents an application of NSGA-III in multi-objective optimization in scheduling scientific workflows on IaaS cloud. The multi-objective in this study is to minimize *Cost*, *Makespan* and maximize *VMutilization*. Four well-known scientific workflows [1], CyberShake, Epigenomics, LIGO, and Montage, are chosen for illustrating the performance of NSGA-III in solving this multi-objective optimization.

The organization of the paper is as follows. It begins with the description of workflow scheduling in Section 2. Related work is discussed in Section 3 and Section 4 describes the overview of NSGA-III. The detail of proposed implementation is elaborated in Section 5. Solutions from NSGA-III are shown and discussed in Section 6. The paper is concluded in Section 7 where future work is also suggested.

2 Workflow Scheduling

Workflow scheduling has been continuously studied and researched in multifarious perspectives. This includes fields in parallel and distributed system, cluster and grid computing. Recently, there has been much attention in scheduling in cloud computing. IaaS

cloud provides heterogeneous machines (i.e. a range of flexible VM) in order to provide users with multiple choices. Also Service Level Agreement (SLA) implies that efficient execution of workflow tasks is crucial in dynamic resources environment. Understanding the exact requirements is the first step in coming up with satisfactory scheduling. It has been known that determining the proper VM types, assigning tasks to machines, and running the workflow to satisfy given constraints for scheduling in cloud resources is an *NP*-hard problem.

2.1 Workflow Modeling

In general, a workflow can be modeled as a DAG. While a node represents a task and a directed edge typifies dependency between the two connected tasks. The dependency of tasks denotes that a child task can be executed once all of its parents are finished while it is possible for other independent tasks can run parallel together. Formally, a workflow W can be seen as a set of (T, E) where T is the set of tasks $\{T_1,\ldots,$ and E represents dependencies between tasks in T. A task T_i is the parent of task T_j when there is a dependency edge E_{ij} in E. A task T_k is possible to run in parallel with T_j if an edge E_{jk} and an edge E_{kj} do not exist. Note that, in scientific workflow, task dependency can also be data dependency. Typically, an output of parent task acts as an input to the child task.

While many scenarios of workflow are possible, this work focuses on IaaS model on cloud resources. Take the Amazon Web Service (AWS) provider as an example, suppose that $V = \{VM1,\ldots,$ is the set of heterogeneous VMs on IaaS cloud, each VM type indicates the capacity of virtual processing unit and instance storage. In pay-per-use model, users can launch, pause, resume and terminate VMs anytime as they wish. For example, Figure 1 depicts a DAG of five tasks (T1 to T5) and a possible allocation of these five tasks. It depicts a possible solution for this scheduling which depends very much on user objectives based on certain constraints. This work assumes that all tasks in workflows are non-preemption tasks, as this is the usual default mode, which means that no VM reservation is done in advance.

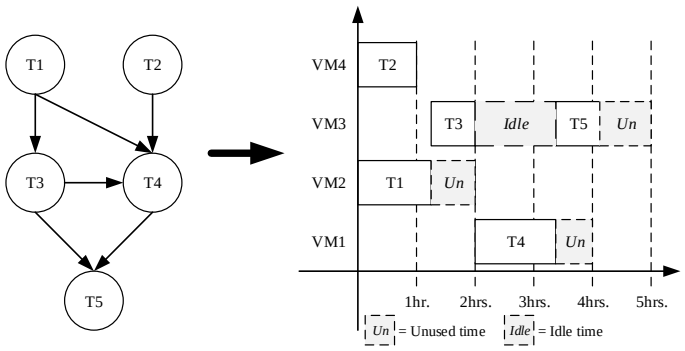


Figure 1. An example task scheduling among four VMs for five dependent tasks

2.2 Scheduling Objectives

The prime objective of allocating tasks and optimizing cloud resources is to satisfy the user requirements. As mentioned in earlier, these requirements become objectives of scheduling and can be considered from several perspectives (i.e. from the cloud provider, the cloud broker to the cloud user). Earning highest possible revenue from minimal costs is the first priority from the cloud provider's perspective. Also having a good policy for dealing with planning resource capacity, utilizing network bandwidth, reducing energy consumption, and ensuring the security of cloud data center are also critical objectives [7]. As a cloud broker, availability of services, competitive price, and meeting user requirements are the fundamental objectives

[6]. On the other hand, the performance measurements such as makespan, response time are major concerns from a cloud user's perspective [6]. Scheduling becomes much more complex in a large and complex scientific workflow. This entails a list of many possible objectives [6], Table 1 summarizes some of these objectives from different perspectives, and it is not by any means an exhaustive one.

Table 1. Cloud Resource Management Objectives

Perspective	**Objectives**
Environment	Energy, Peak Power, Thermal, CO_2 Emissions
Financial	Price, Income, Cost
Performance	Respond Time, Uptime, Throughput, Makespan
Resources	Energy Source, VM utilization, Data Locality
Others	Reliability, Security, Data Center Location

Among the above objectives, Cost and Makespan are two most prolific where optimization has been carried out. One of the most important objective which should not be neglected is VM utilization. Nevertheless, there has been no previous work in multi-objective scheduling of cloud computing has included VM utilization to date. Therefore, this work is the first attempt in multi-objective scheduling where Cost, Makespan and VM utilization are considered.

2.2.1 Cost

In IaaS cloud, a user requests to launch VM by selecting types which are provided in a catalog. VM types indicate the upper bound capacity of virtual processor and their cost. In most cases, the cost of leased VM is measured in per hour interval where partial VM-hour consumed is usually billed as a full hour. For most IaaS cloud providers such as AWS, Google App Engine, and Windows Azure, the communication cost is hidden and almost negligible from user's perspective. Therefore, the 'Cost' referred to in this work is the VM leased cost which is usually calculated in number of hours used by VMs. Referring to Figure 1, *VM1*, *VM2* are charged for two hours, while *VM3* and *VM4* are charged for four and one hours respectively.

In general, the cost can be determined by equation (1) as below :

$$Total\ Cost = \sum_{i=1}^{n} \lceil Hours\ of\ VM_i \rceil \times Cost_{per\ hour\ of\ VM_i} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

where

$\lceil Hours\ of\ VM_i \rceil$ is the ceiling of *VM* running hours

$Cost_{per\ hour\ of\ VM_i}$ is the unit cost per hour

This work adopts the same VM types and cost as in [5] which is based on AWS model as shown in Table 2. *Processing Capacity* is represented in MFLOPS (Million Floating Point Operations Per Second) unit and the *Slowdown Ratio* is the ratio of performance degradation comparing to m3.doubleXlarge which is the fastest in term of MFLOPS.

Table 2. List of VM types based on AWS model [5]

VM Type	EC2 Units	Processing Capacity (MFLOPS)	Cost/Hour	Slowdown Ratio
m1.small	1	4,400	$0.06	26.00
m1.medium	2	8,800	$0.12	13.00
m1.large	4	17,600	$0.24	6.50
m1.xLarge	8	35,200	$0.48	3.25
m3.xLarge	13	57,200	$0.50	2.00
m3.doubleXlarge	26	114,400	$1.00	1.00

2.2.2 Makespan

In execution of a batch of tasks, Makespan, the standard objective, is the maximum completion time (C_{max}) representing the time when the last task in workflow is done [6]. Referring to Figure 1, Makespan is the time when task *T5* is completed. In general, C_{max} can be determined by the following equations :

$$C_{max} = max_{j \in T} C_j \quad \text{.... (2)}$$

$$C_j = Tran_j + P_j \quad \text{.... (3)}$$

$$Tran_j = \sum_{i=1}^{n} Tran_{ij} \quad \text{.... (4)}$$

$$Tran_{ij} = InputSize_i \div DTR_{ij} \quad \text{.... (5)}$$

where

C_j is the completion time of T_j.
$Tran_j$ is the transfer time
P_j is the processing time.
n is the number of input files generated by all parent tasks of T_j.
$Tran_{ij}$ is the transfer time from T_i to T_j.
DTR_{ij} is the data transfer rate (e.g. 1 Gbps, between T_i and T_j).
$InputSize_i$ is the input size generated by *Ti.*

The assumption is that each task in workflow provides the size of all input files and transferring of multiple input files is sequential. Values stated in Table 2 is also assumed in this work, for example, if the processing time P_j of task T_j is 10 seconds in m3.doubleXlarge type, it will take 20 seconds in m3.xlarge with 2.00 slowdown ratio.

2.2.3 VM Utilization

In a conventional machine, CPU or processor utilization is a significant issue in term of cost effectiveness and performance. The idle time is the time running processor without any productivity. Similarly, in a cloud environment, the idle time of running VM leads to reduction cost effectiveness and performance. Furthermore, in a per hour period, partial VM-hour consumed leads to higher cost without utilization. Referring to Figure 1, both idle and unused

time occur in *VM3*, while *VM4* has the best VM utilization. If *V* is the set of VMs, the *VMutilization* in this work is defined as shown in Equation 6 :

$$VMutilization = average_{i \in V}(RunningTime\ of\ VM_i / BilledTime\ of\ VM_i) \quad (6)$$

Hence, the *VMutilization* is the ratio of VM running time, without idle time, divided by VM billed time. This value ranges from 0 to 1. For example, if a VM is used for 90 minutes and the user is charged for 120 minutes, then *VMutilization* is 0.75.

2.3 Scheduling Model

Section 2.2 describes the three objectives in this work. Therefore, the multi-objective scheduling is a solution where *Cost* is minimized, *Makespan (i.e. C_{max})* and *VMutilization* are maximized in processing a workflow on cloud. The scheduling model can then be formally represented using the standard *three-field notation* [4, 13] denoted below :

$$VM_j | Prec, P_{ij} | Cost, C_{max}, VMutilization \quad \ldots\ldots\ldots (7)$$

where

VM_j is the first field which stands for *j* parallel VM machines with different speeds.

$Prec, P_{ij}$ is the second field where *Prec* notates the precedence constraint between tasks and P_{ij} represents a vector of processing time (in second) for task *i* processed by VM_j.

Cost, C_{max}, *VMutilization* is the third field where the list of objectives is stated.

3 Related Work

Efficiency of task scheduling plays a significant role in utilizing resources in distributed systems in order to achieve satisfactory performance. Literature in optimization of these resources is quite prolific, especially with the application of evolutionary algorithms (EA). This leads to a field of its own commonly known as Multi-Objective Evolutionary Algorithms (MOEA) or Multi-Objective Genetic Algorithms (MOGA) [14]. Solutions to this multi-objective optimization can be considered from mainly three perspectives, provider, broker and user perspectives. Brokers can also be seen as users in some scenarios. As most workflow scheduling problems refer to the use of cloud resources, the two main objectives are *Cost* (i.e. the cost of leased VM is measured in per hour interval) and *Makespan* (i.e. makespan and execution time). These two objectives are understood as conflicting objectives.

The problem of minimization of both conflicting objectives has been usually focused on scientific workflows. The work in [15] proposed an auction-based Biobjective Scheduling Strategy (BOSS) comparing to NSGA-II and Strength Pareto EA-II (SPEA-II) in order to optimize a bi-objective problem (cost and makespan). Cloud Workflow Scheduling Algorithm (CWSA) comparing to three well-known algorithms, First Come First Served (FCFS), Easy Backfilling, and Minimum Completion Time (MCT) was proposed in [16]. Multi-objective scheduling based on the Vector Ordinal Optimization (VOO) was carried out in [17], while [18] a scheduling algorithm based on task clustering for scientific workflows in order to minimize cost and execution time. Scheduling of general DAG was carried out in [19] and [20]. Improved Differential EA (IDEA) [19] was implemented to handling tasks and their subtasks in DAG workflows on multiple cloud resources, while [20] adopted two-hierarchical scheduling strategy, service level, and task level scheduling, for minimizing cost and execution time. SPEA-II [21] was applied in order to minimize cost and execution time to a particular workflow known as BPEL (Business Process Execution Language) workflow.

Besides the most two common objectives, *Cost* and *Makespan*, energy consumption, data locality, and communication overhead were other objectives that had been studied in the task scheduling on cloud. This includes Case Library and Pareto Solution based hybrid GA (CLPS-

GA) was proposed in [8], Cellular GA (CGA) [9]. The work in [10] compared three other EAs, NSGA-II, MoCell (Multi-objective Cellular), and IBEA (Indicator-based Evolutionary Algorithm), for workflow scheduling in order to minimize energy consumption and makespan. Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D) was introduced in [22]) for energy and locality-aware multi-job scheduling on cloud. Comparison between NSGA-II and PSO for execution of scientific workflows satisfying multi-objective optimization (cost, makespan and communication overhead) on cloud was carried out in [7].

The idea of provisioning a VM as late as possible can help to reduce the leased VM time and may lead to minimize the cost. To date, no work had considered on VM utilization, with exception of a few that focused on task scheduling with the deadline of VM provisioning. PSO based algorithm [5] was introduced to optimize cost and execution time and satisfy deadline constraints, while [23] presented an adaptive, resource provisioning and scheduling algorithm in order to minimize cost and meet a user-defined deadline. Both works were carried out on scientific workflows on cloud. In the cloud resource management, several EA techniques have been applied in order to deal with the multi-objective optimization problems. A survey of 36 research works [6] was carried out on family of EA techniques, GA, PSO, ACO and NSGA-II. This affirms that a variety of EA has successfully addressed the multi-objective optimization in order to optimize cloud resources.

In the multi-objective optimization of task scheduling problems, most of the previous works had focused on the two most common objectives, *Cost* and *Makespan*, due to the need of a trade-off between them. Nevertheless, as described in Section 2.3, *VMutilization* is another objective that ought not be overlooked, especially in the pay-per-use model of IaaS cloud. Increasing of unused time and idle time (i.e. low *VMutilization*) may lead to increasing *Cost* and *Makespan* too. Therefore, this study considers *VMutilization* as another important objective in optimization of task scheduling in scientific workflows. It is also the first work which attempts the multi-objective optimization in scheduling on cloud which includes *Cost*, *Makespan* and *VMutilization*.

4 NSGA-III

As stated in the previous Section, several multi-objective optimization algorithms exist. This work selects the state-of-the-art evolutionary algorithm known as *Nondominated Sorting Genetic Algorithm-III* (NSGA-III). It is specifically designed for multi-objective optimization and is an improved version of the previous NSGA-II. Its improvement lies in the proposal of a novel selection operation to acquire a set of well-spread Pareto-optimal solutions.

As a multi-objective optimization usually produces a set of feasible solutions, the concept of nondominated solutions, called Pareto-optimal solutions, is the set of solutions which are not dominated by other solutions. In order to maintain the uniform distribution and diversity of Pareto set in NSGA-II, crowding distance was proposed (i.e. the distance between a solution and its neighbors) where a solution with larger distance was preferred. NSGA-III improves this by using a different strategy by determining the relative distance between solutions and reference points.

In the initial step, beside randomizing population of size N, NSGA-III also generates the well-distributed reference points H on an (M-1)-dimensional hyperplane for M-objective problem. The positions of reference points can be determined by using Normal-Boundary Intersection [24] and the population size N should be the smallest number, which is equal or greater than H, and is the multiple of four. The expectation is at least one individual is belonging to every reference point. Prior to the selection step in each generation, the parent generates their offspring by using crossover and mutation operation. Suppose that $U_t = P_t \, U \, Q_t$ where P_t is the parent population at t^{th} generation, Q_t is the offspring population and U_t, the combined population, usually of size $2N$. Then U_t will be sorted based on nondominated sorting and divided into multiple nondominated levels. F_1 is individual, which is completely nondominated by others, and F_2 is individual dominated by only F_1 members and so on.

In the selection process, F_1 members will be firstly chosen for P_{t+1} generation before considering next nondominated level until the size of P_{t+1} is equal to N. In order to preserve diversity, suppose that F_{last} is the last nondominated level that is chosen for P_{t+1}, Niche-Preservation Operation is applied in order to select F_{last} members. First, objective values of each member in P_{t+1} and F_{last} are normalized to correspond with the reference points. Then all of the members in P_{t+1} and F_{last} are associated to the reference points by measuring the distances between their positions and reference lines, the logical lines laid down from the ideal point to reference points. Finally, Niche-Preservation Operation iteratively operates on each reference point. For the j^{th} reference point, if there is no associated P_{t+1} member to reference point j, the associated F_{last} member, having the shortest distance, is being added to P_{t+1} otherwise the j^{th} point is not considered for the current generation. In the case of an already existing member in P_{t+1} associated to the j^{th} point and an associated F_{last} member to point j is found, a member is randomly selected and added to P_{t+1}. The procedure is repeated until the size of P_{t+1} is equal to N.

Ensuring a variety of population and preserving identical characteristic of parents are the key concepts of NSGA-III. It uses the association of population with widely-distributed reference points to represent both concepts. This strategy is capable of generating a set of well-spread Pareto-optimal solutions. The detailed operations of NSGA-III can be found in [12].

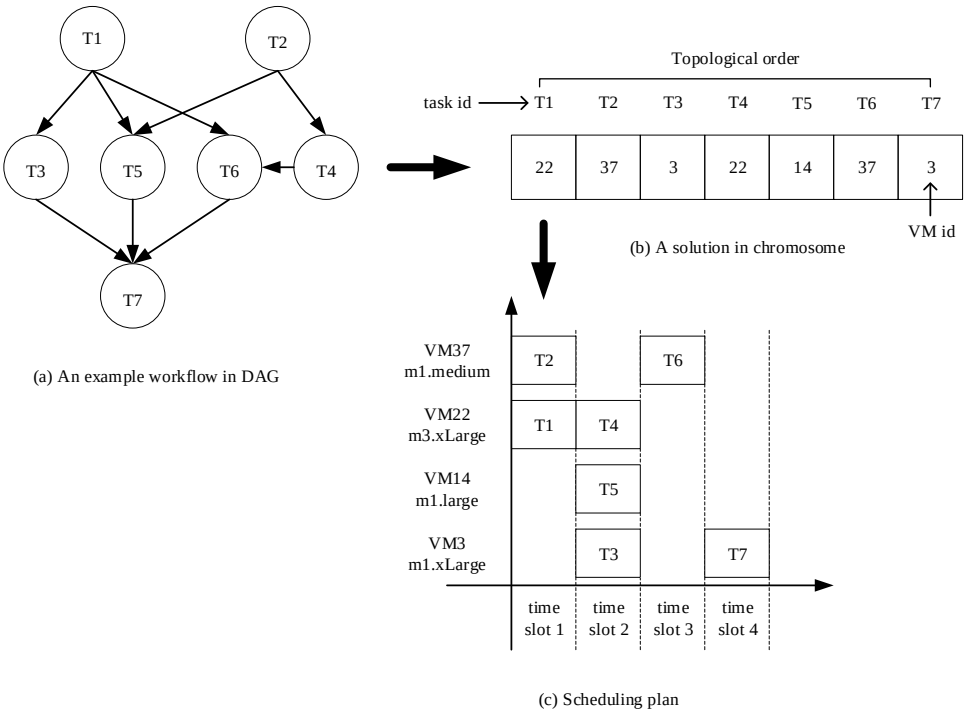
5 Multi-Objective Scheduling using NSGA-III

The challenge of Multi-Objective Genetic Algorithm (MOGA) not only limit to difficult optimization problem with increasing objectives but also in its application to real NP-hard problems. Cross-training performance of nurse scheduling is one of such example of where NSGA-II and PSO had been applied [25]. This section discusses the proposed solution of NSGA-III in the optimizing scheduling problem as stated in Section 2.

5.1 Representation of Scheduling in NSGA-III

The major objective in task allocation among VMs is to efficiently assign a particular task to a suitable VM. For ease of description, the following scenario is assumed :

No. of tasks : 7 (T1, T2, ….., T7)
No. of VM Types : 6 (m1.small, …., m3.doubleXlarge as stated in Table 2)
DAG Workflow : 7 tasks configured as in Figure 2(a)



(a) An example workflow in DAG

(b) A solution in chromosome

(c) Scheduling plan

Figure 2. An example of a DAG workflow, Chromosome representation and Solution

The execution of all tasks can be represented as a topological order of the DAG workflow. Hence, the initial process is to determine a topological order of the workflow represented by the DAG. Knowing a topological order is only a small part of the process as assigning suitable VMs to all tasks, from a possible list of VM types according to their dependency, presents a NP-hard problem. This is where Evolutionary Computation, and NSGA-III in particular, is a possible tool in searching for an efficient schedule. The chromosome in NSGA-III in this work reflects a topological order (i.e. a possible solution). Therefore, the length of chromosome is equal to the number of tasks to be scheduled (i.e. 7 tasks in this example). Referring to this scenario, as there are 7 tasks and 6 VM types, therefore, intuitively there ought to be 42 possible values (i.e. no. of tasks × no. of VM types) for each slot in the chromosome. Hence, the range from 0 to 41 is chosen to represent a VM id for a particular task. Referring to Figure 2(b), the value in each slot can be decoded by the operation : ('No. in the slot' MOD 'No. of VM types'). For instance, the value of the first slot is '22', this indicates that 'm3.xLarge' ought to be assigned to execute T1 (i.e. '22 MOD 6' is 4 which corresponds to VM id 'm3.xLarge' in Table 2). Similarly, as the value of the second slot in the chromosome is '37', this indicates that VM id 'm1.medium' ought to be assigned to execute T2.

5.2 Workflow Scheduling Algorithm

As a chromosome represents a possible scheduling solution, it has to be converted to a scheduling plan. A task slot in a chromosome represents VM id and its order of execution. To generate the scheduling plan of workflow, VM queue and parent tasks are considered together. A task is added to VM queue according to the topological order and its assigned VM id. The number of precedent tasks in VM queue determines the time slot of tasks in the queue, so a task can be allowed to start after the last time slot of its parents are processed. It is worth noting that during assessing time slot of each parent task, the topological order guarantees that all parent tasks of a task are in the scheduling plan.

Referring to Figure 2(c), the task *T6* is assigned to *VM id 37* (i.e. m1.medium VM type) having *T2* in the queue at time slot 1. Assuming that *T4* requires a large time slot, as the precedent task of *T6* is *T4*, time slot 2 is then assigned to *T4*. Therefore *T6* is assigned to m1.medium VM type at time slot 3. In fact, a possible scheduling of a scientific workflow can be represented by 2-dimensional array, where number of rows is equal to number of VM types and number of column is equal to number of tasks (i.e. 6 VM types by 7 tasks in the example in Figure 2). So an efficient scheduling of workflow is the process of assigning number of tasks and their possible VM types in this 2-dimensional array where objectives are optimized. Figure 3 describes the Workflow Scheduling Algorithm.

Algorithm: Scheduling Decoding

Input:
$C \leftarrow$ Array of chromosome with VM id having c elements
$T \leftarrow$ Array of tasks with topological sorting having t elements
$V \leftarrow$ Array of unique VM id in chromosome
with ascending sort having v elements
$P \leftarrow$ Array of parent tasks having p elements

Output:
$S \leftarrow$ A scheduling plan as 2-dimensional array with $v \times t+1$ size
the first column represents VM id
the rest columns represent task id in each time slot

```
initial S
    for i = 1 to v
        S(i,1) = V(i)
        i = i+1
    end

for j = 1 to t
    task = T(j)
    vm_id = C(j)
    row = find_row_index_in_S_by_VM_id(S, vm_id)
    col = find_first_empty_columm_in_S_by row(S, row)

    if has_parent(task) then
        P = get_list_of_parent_by_task(task)
        for k = 1 to p
            col_p = 0
            col_p = find_column_index_in_S_by_P(S, P(k))

            if col_p ≥ col then
                col = col_p + 1
            end
        end
    end

    S(row,col) = task

end
```

Figure 3. The proposed Workflow Scheduling Algorithm

5.3 Fitness Functions

Once a possible schedule is generated, its efficiency must be evaluated, this is done by means of assessing the fitness value of its chromosome. The fitness of a possible schedule comprises three aspects, *Cost*, *Makespan* and *VMutilization* as stated in Section 2. Hence, fitness functions are the equations 1, 2 and 6 stated in Sections 2.2. These fitness functions are used to assess the scheduling model as stated in Section 2.3. Note that as the scheduling model is multi-objective, hence a chromosome (i.e. a solution) with the best fitness values for all 3 objectives may not exist.

5.4 Selection, Crossover and Mutation

As in any Genetic Algorithm, in NSGA-III, a set of initial population is first generated. A percentage of the initial population is selected for *Crossover*. Although Simulated Binary Crossover (SBX) [26] was suggested for NSGA-III, this was intended for real number. *One-point Crossover* is selected for this work and it was found more suitable and efficient for integer values. Once the *Crossover* is completed, a percentage of the initial population is selected for *Mutation*. In this work, *Mutation* adopts the *Gaussian Mutation*, where 2% of the positions in the selected chromosomes are mutated, as used by the Yarpiz project [27]. The whole population is then assessed for their fitness values as described in the previous Section, this is followed by nondominated sorting. Elitism is used for the *Selection* to represent a new generation of the population.

6 Scientific Workflows used

Four scientific workflows from different scientific fields were selected for this study, these are of CyberShake, Epigenomics, LIGO, and Montage. These workflows are known applications and have been used as test beds for scientific workflows. The number of tasks for all four scientific workflows, used for this study, was set to be 100. Their brief descriptions are as follows:

CyberShake

CyberShake is a seismology application that calculates probabilistic seismic hazard curves for geographic sites in the Southern California region [1, 28]. It consists of five task types. Three of them are multiple-task type including the root tasks. Figure 4(a) depicts the structure of CyberShake workflow in DAG model.

Epigenomics

Epigenomics workflow created by the USC Epigenome Center and the Pegasus research team, it is used to automate various operations in genome sequence processing [1, 29]. There are eight task types in workflow. Four of them are singleton-task type. Figure 4(b) depicts the structure of Epigenomics workflow in DAG model.

LIGO

Laser Interferometer Gravitational Wave Observatory (LIGO) workflow is used to search for gravitational wave signatures in data collected by large-scale interferometers [1, 30]. It consists of four task types and three of them are multiple-task type as shown in Figure 4(c).

Montage

Montage is an astronomy application created by NASA/IPAC, it is used to construct large image mosaics of the sky [1, 31]. It comprises nine task types that six of them are singleton-task type as shown in Figure 4(d).

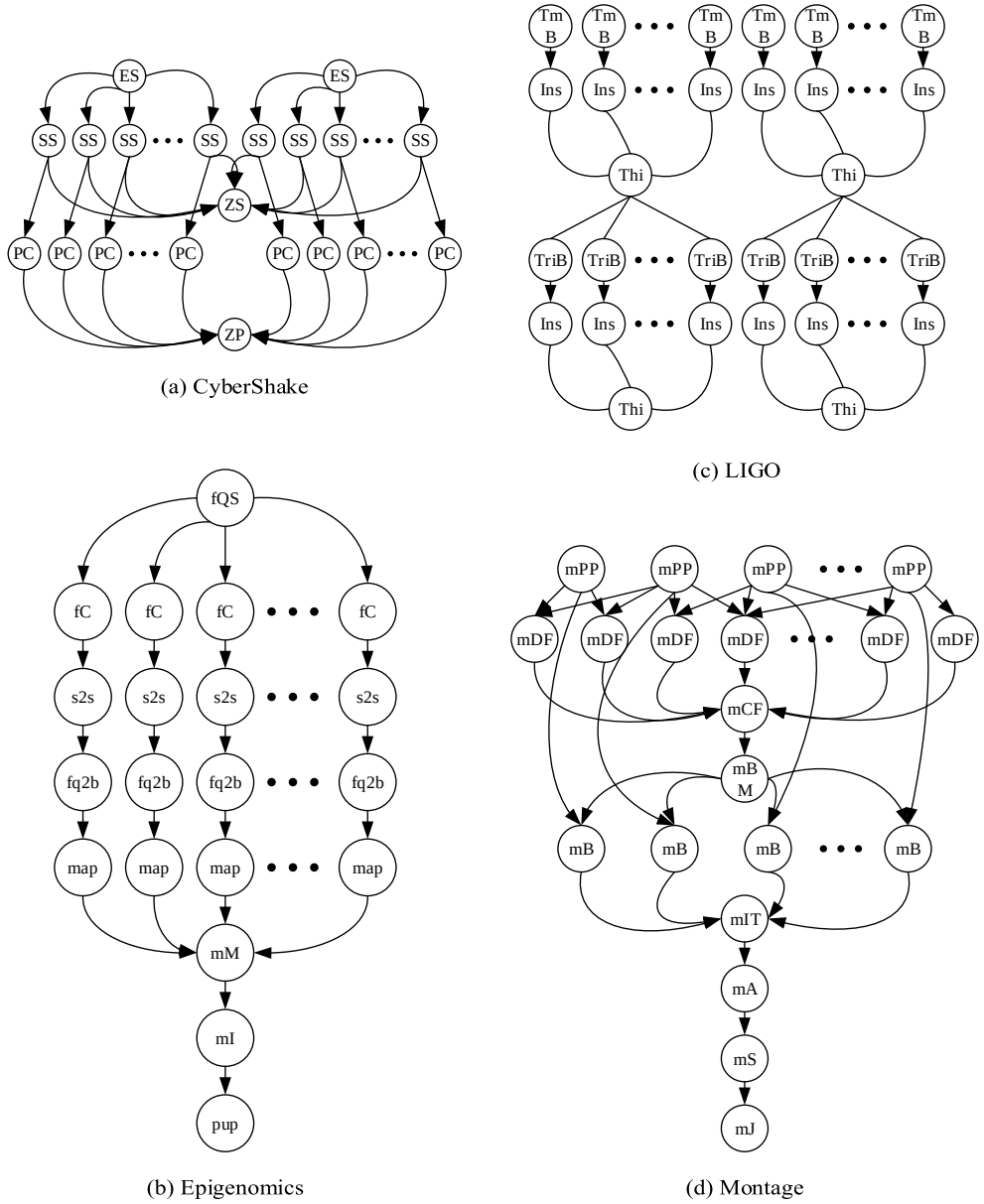


Figure 4. DAG models of four scientific workflows used

7 Solutions Using NSGA-III

As described in Sections 4 and 5, an NSGA-III was specifically implemented using MATLAB for finding optimal multi-objective solutions for all four workflows as stated in Section 6. In

order to assess and evaluate the multi-objective solutions, optimal single objective solutions were found first. Comparison and evaluation of the optimal multi-objective solutions can then be assessed by comparing with the single objective solutions.

As three objectives in this work are, *Cost*, *Makespan* and *VMutilization*, optimal single objective solution was generated by NSGA-III by means of simply using single fitness function for each objective. This resulted in the single best solution that NSGA-III could generate. In case of the multi-objective solutions, however, NSGA-III provides a set of Pareto-optimal solutions as the best optimal solution for all three objectives may not exist.

The list of NSGA-III parameters is summarized in Table 3. Parameters used in this study followed those proposed in DTLZ (3 to 15 multiple objectives were tested) [12] which reports that 91 reference points are suitable for three objectives and population size has to be at least as many as reference points and divisible by four. Hence, 92 were chosen as the number of population in this study.

Table 3. Parameters of NSGA-III

Parameter	Value
Scheduling objectives	3 (*Cost*, *Makespan*, *VMutilization*)
Length of chromosome (No. of tasks)	100 (CyberShake, Epigenomics, LIGO, Montage)
Reference points	91
Population size	92
Crossover operation	Single Point Crossover - Random selection - 50% of the parent population
Mutation operation	Gaussian Mutation - Random selection - 2 out of 100 positions - 50% of the parent population
Stopping criteria	At least 500 iterations and No progress occurs after 100 iterations onwards

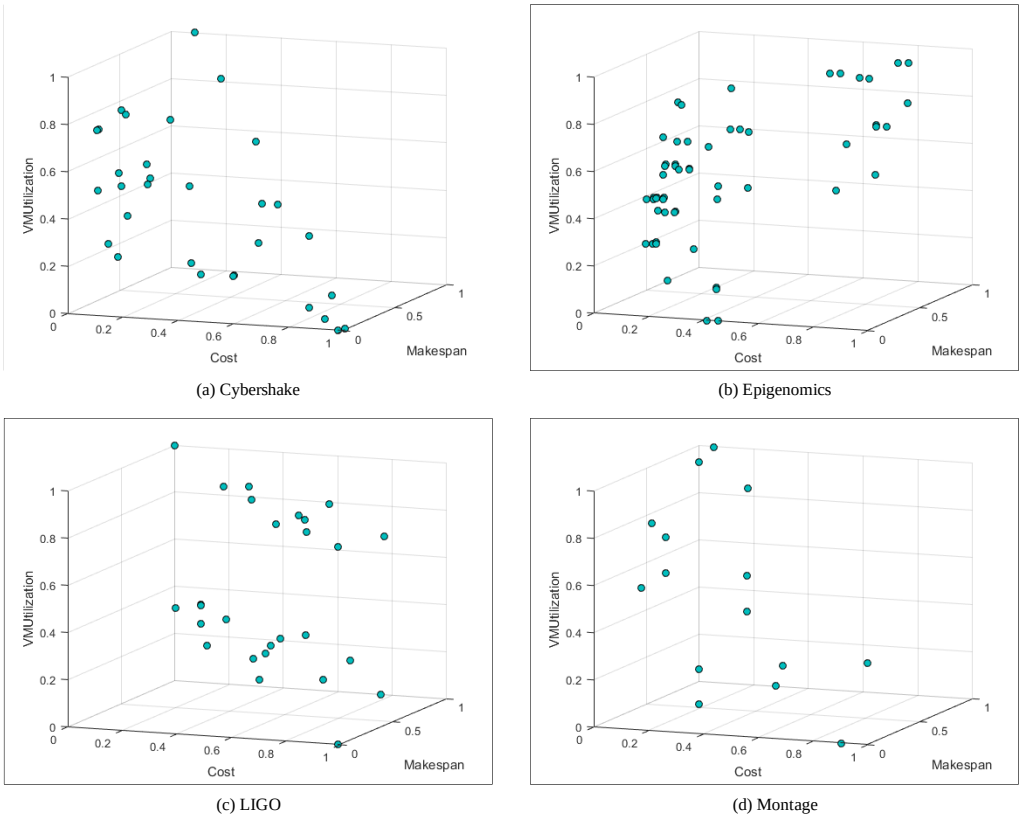
7.1 Results

As the three multi-objective solutions generated a set of Pareto-optimal solutions, there are numerous possible ways to present the results from NSGA-III for each scientific workflow. In this Section, the results are presented in two parts including the Pareto-optimal solutions and the comparison between single and multiple objectives of each workflow. For ease of presentation in a graph, all values of *Cost*, *Makespan* and *VMutilization* are normalized such that the lowest and highest values are 0 and 1 respectively. Note that the best value for *Cost* and *Makespan* are 0 while this is 1 for *VMutilization*.

7.1.1 Pareto-optimal Solutions

As stated in the Section 4 NSGA-III offers nondominated solutions having the best value at least one objective. To visualize the Pareto-optimal solutions of three objectives is capable and informative in order to explore the Pareto plane. Figure 5 depicts the optimal solutions of each workflow, each comprises 92 points. Note that different solutions yielded the same values for all objectives (as shown Montage workflow where many among possible 92 solutions yielded the same values). As in multi-objective optimization, the optimal solution where all criteria are satisfied may not exist. The same happened in this work for all workflows. Hence, the best solution has to be selected according to priorities of each requirement. Among the four

solutions, Montage is least distributed while Epigenomics is the most well distributed. In the next section, Pareto-optimal solutions at the average value are compared to the best solution from single objective.



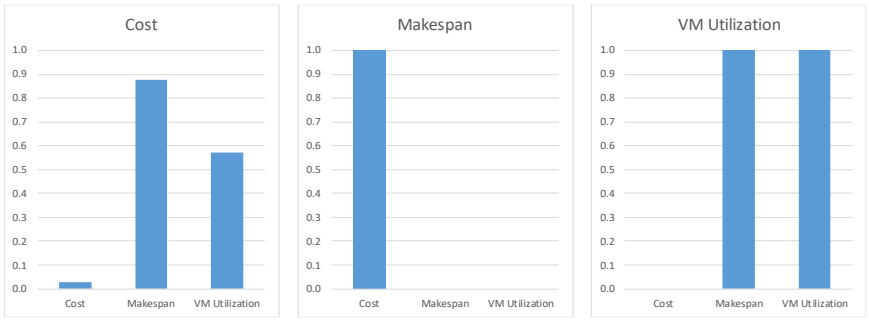
(a) Cybershake (b) Epigenomics

(c) LIGO (d) Montage

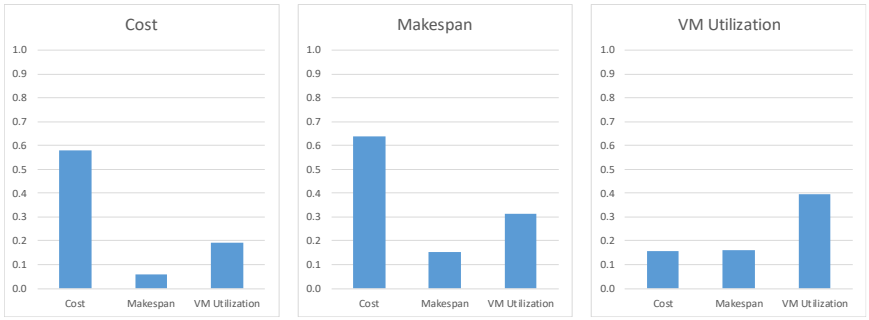
Figure 5. Pareto-optimal solutions of the four scientific workflows

7.1.2 Single VS Multiple Objective Comparisons

As NSGA-III yielded solutions of all objectives where the best, average and worst value occur at each objective for each workflow. Presenting them in all possible perspectives can overwhelm the merit and the discussion. Therefore, for each scientific workflow, the best solution from the single objective is depicted to compare with the Pareto-optimal solution at the average value where that particular objective occurs. Figures 6, 7, 8 and 9 depict these comparisons in terms of *Cost*, *Makespan* and *VMutilization* for CyberShake, Epigenomics, LIGO, and Montage workflows respectively. For example Figure 6(a) depicts the best solutions for each single optimal objective for CyberShake, where Figure 6(b) depicts solutions for the three multiple-objective values where each objective is at average. The rationale for comparing with the average solutions instead of the best in the three multiple-objective solutions is that selecting the best solution at each objective implies that very consideration/importance is given to the other two objectives.

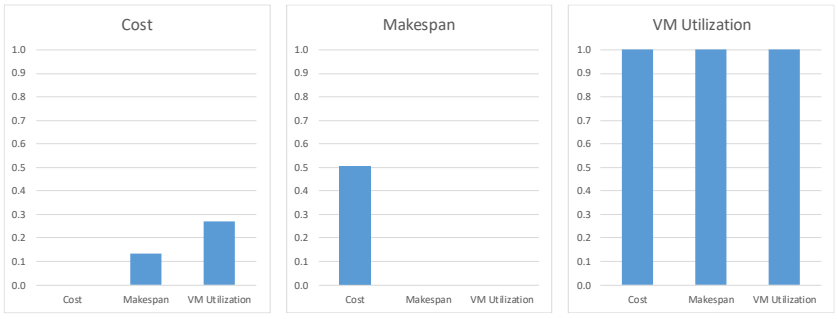


(a) Single objective solutions of CyberShake

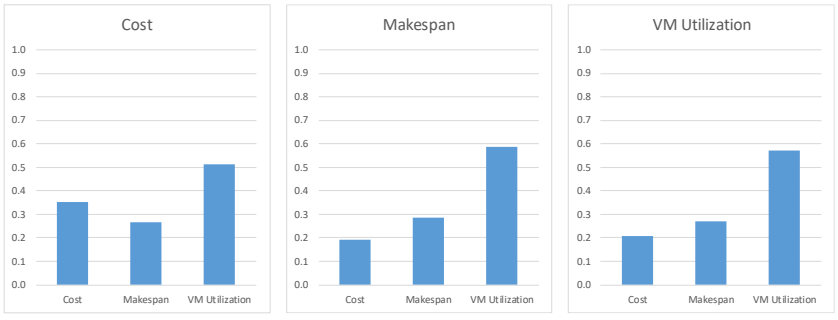


(b) Multi-objective solutions of CyberShake

Figure 6. Single & Multiple-Objective Solutions (CyberShake Workflow)

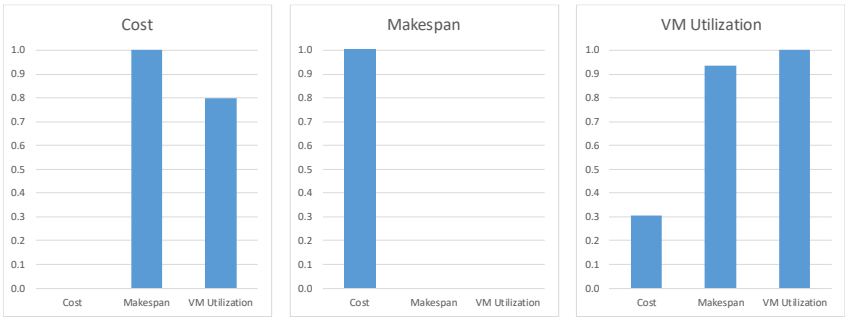


(a) Single objective solutions of Epigenomics

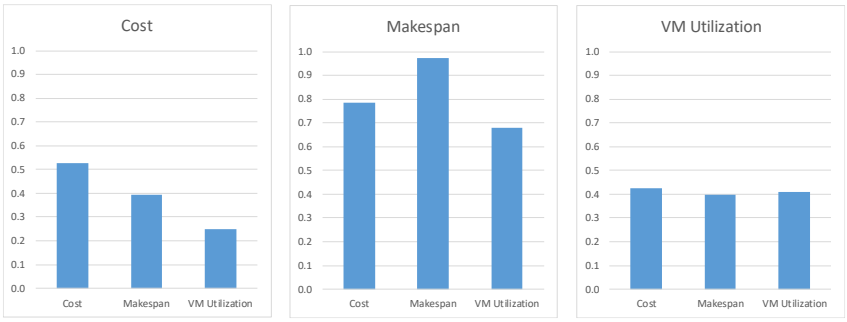


(b) Multi-objective solutions of Epigenomics

Figure 7. Single & Multiple-Objective Solutions (Epigenomics Workflow)

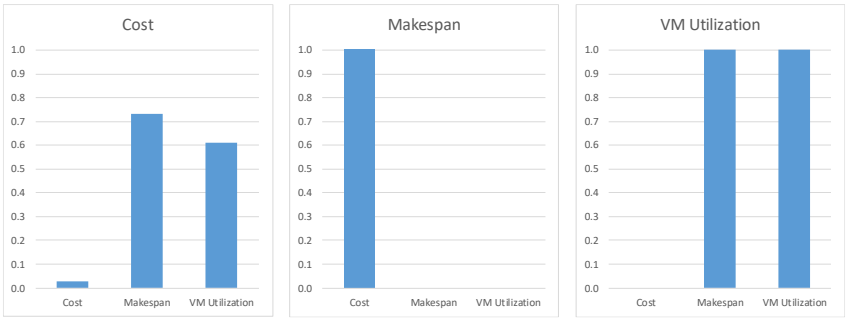


(a) Single objective of LIGO

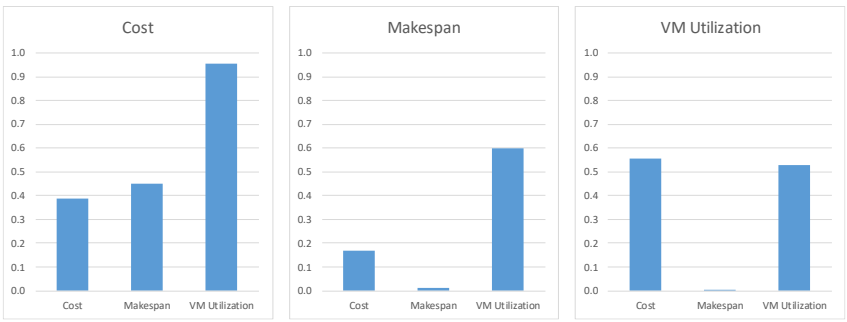


(b) Multi-objective of LIGO

Figure 8. Single & Multiple-Objective Solutions (LIGO Workflow)



(a) Single objective solutions of Montage



(b) Multi-objective solutions of Montage

Figure 9. Single & Multiple-Objective Solutions (Montage Workflow)

7.2 Discussions

Referring to Figures 6 to 9, the solutions reaffirm the common belief that *Cost* and *Makespan* are conflicting objectives as shown in cases of single objectives for all scientific workflows. It also reveals that *VMutilization* and *Makespan* are also conflicting objectives. Contrast to common suspicion that higher *VMutilization* implies lower *Cost*, this is not necessary true as shown in Epigenomics workflow (i.e. best *VMutilization* in Figure 7(a)).

Multi-objective solutions from all scientific workflows also reaffirm the advantage of considering multi-objective for such workflows. While no best solutions exist where all three objectives are optimized, none of the multi-objective solutions leads to worst or considerably high cost for any objective. Among the four scientific workflows, CyberShake configuration is the hardest for finding compromising solutions among all three objectives as *VMutilization* are considerably low for all three objectives. Previous work tends to concentrate on two multi-objective optimizations (i.e. *Cost* and *Makespan*), the result of this study suggests that inclusion of *VMutilization* in the multi-objective optimization is beneficial as optimizing *VMutilization* has a tendency to balance the values both *Cost* and *Makespan* too (i.e. the average solutions for *VMutilization* for all workflows do not show unsatisfactory values of *Cost* and *Makespan*, even though the average of *Cost* in Montage workflow is relatively high).

The results of this study reveal that in multi-objective optimization of scientific workflow, a trade-off between objectives must be considered and the configuration of the workflow probably has the most significant impact to this multi-objective optimization problem as no common factor can be concluded from the solutions in all four scientific workflows used in this study.

8 Conclusions and Future Work

Scheduling in cloud computing where resources are efficiently utilized has become multi-objective problem. Numerous studies of scientific workflow scheduling on cloud have been carried out from different perspectives. This study is the first to attempt multi-objective optimization of scheduling on cloud where *Cost*, *Makespan* and *VMutilization* are considered from user perspective. The state of art NSGA-III was used as the tool in the multi-objective optimization of four scientific workflows. The chromosome representation using the topological order of tasks and a workflow scheduling algorithm are proposed.

Apart from reaffirming the two conflicting objectives of *Cost* and *Makespan*, the study reveals that *VMutilization* ought not be overlooked as its inclusion does not lead to unsatisfactory solutions from both *Cost* and *Makespan* perspectives, but can offer a more balanced scheduling of scientific-workflows. Single objective scheduling ought to be avoided as it may compromise other important aspects. Nevertheless, configuration and dependency in a scientific workflow are the most crucial aspects in multi-objective optimization when determining an efficient scheduling.

Further studies can be extended from several perspectives, this study considers scheduling from user perspective. Similar attempts can be done from provider and broker perspectives. This suggests inclusion of energy consumption, energy sources and CO_2 Emissions in the multi-objectives. It is worth bearing in mind that the task can become very complex when many objectives are considered together. From NSGA-III perspective, it can also be applied to other areas of multi-objective optimization such as feature selection in data mining. Another direction for future studies is the investigation of other EA techniques in similar multi-objective optimization.

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MISTA 2017

Responsive Single Bus Corridor Scheduling Based on Machine Learning and Optimisation

Ruibin Bai · Ning Xue · Xia Li · Tianxiang Cui

1 Background and motivation

Buses are probably the second most important urban public transport behind subways. However, the innovations in technologies and services related to bus transport are lagging behind other transport systems. The bus timetables and schedules are often still very much created from traditional planning models and approaches based on some simple rough data estimation of travel demand and travel time. In many countries, bus timetables are given as a priori, which assumes repetitive customer demand for weekdays and weekends. These predefined timetables are too rigid for volatile passenger demands, often affected by factors like rain, temperature, events, etc. In some other places, the bus timetables are not fixed. In this regard, the operating companies often adjust the dispatching density (or dispatching headways) dynamically for different traffic and travel demand scenarios. This method is particularly useful when the travel demand is very high. Similar mechanisms are also used in some metro-line timetables, which are also defined by the start time, the finish time and the dispatching density (e.g. every 5 min). Practically, the determination of the dispatching density is primarily based on years of experience. In most cases, there are just two different dispatching densities for peak time and off-peak time respectively. However, because of some dynamic events (e.g. weather changes, road accidents), the classic dispatching density models (e.g. those by Sun and Zhang (2016) are problematic. Some of the issues can be illustrated by a real-life picture (Figure 1) taken from an electronic bus stop plate below, which shows the distribution of the buses along the bus route (each green light at the bottom of the picture stands for a bus).

One obvious issue is that the distribution of buses along the bus route is highly uneven, creating an unreliable bus service across different bus stops (e.g. bus users at some stops have to wait much longer than the expected waiting time). Additionally, the delays of buses would have negative impact on future bus dispatches at terminals. The root of the issues is the assumption of constant travel time between bus stops over time in this traditional bus dispatching model. Such an assumption often does

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Fig. 1 A real-life snapshot of an electronic bus stop plate with information of real-time bus distribution along the bus corridor.

not hold in urban areas in the event of accidents and fluctuations of road traffics. In this research, we aim to develop an improved bus dispatch model that can handle the fluctuations in travel time and demand. Therefore, we studied a time-dependent bus headway optimisation problem.

2 Literature review

There are numerous studies on optimising bus scheduling under various conditions. Here we primarily focus on the single bus route problem for relevancy. Some earlier studies adopted deterministic models while most studies have been focusing on dynamic strategies under random variables. Cortés et al (2011) proposed an integrated model that permits the options of short turning strategy and deadheading into a normal bus dispatching model. The benefits of all different options were evaluated. Sun and Zhang (2016) studied a bus headway optimisation which assumed a constant bus travel time and passenger arrival rate within the planning horizon. However, it is well accepted that bus scheduling problem has multiple random variables and therefore robustness of the bus services are also important. Chen et al (2009) analysed bus service reliability issues at different levels and defined three measurements for bus routes and stops. It was found out that the reliability is highly correlated to the length of bus routes. However, the study fails to analyse how the reliability might change over time. Daganzo (2009) investigated a dynamic bus holding scheme at pre-defined control points to in-vehicle passenger delay. However, this is a typical reactive, myopic approach and often leads to sub-optimal solutions over long runs. Eberlein et al (1998) studied a real-time deadheading scheme to skip some stops in the event of disruptive events so that the remaining schedules are not affected. The decisions to be optimised include the time when a deadheading should take place and by how many stops. Liu et al (2013) proposed a genetic algorithm to solve a similar problem under random travel time for near-optimal solutions. Although these dynamic scheduling strategics

(e.g. short turning and deadheading) are useful to address some interruptive incidents and improve bus service reliability, the solutions may be too myopic because the decisions are primarily made by considering the real-time data. Repetitive patterns are overlooked.

In this paper, we propose a new bus headway optimisation model that takes advantage of both the dynamic events data as well as the historical data so that our solution is more robust than the deterministic solution and less myopic than the aforementioned dynamic schemes. This is achieved by developing a mathematical model integrated with two machine learning modules to supply high quality forecasts for random variables based on both the real-time and historical data.

3 Problem description and model formulation

3.1 Problem description

The problem can be described as follows. Given a bus route with a dispatching terminal and two directions (outbound or direction 1 and incoming or direction 2), let S and K be the list of stops in direction 1 and direction 2 respectively. Denote V and W be the list of bus trips to be made over the planning horizon for the two bus route directions respectively. At any moment of decision making, CT, we want to determine the optimal dispatch headways (i.e. the gap between any two consecutive bus trips) at the terminal for all the future outbound bus trips. The objectives include the minimisation of the total passenger waiting time, minimisation of passenger total overload penalties and minimisation of the total bus operation costs. The model contains three types of parameters: user controlled parameters, real-time parameters and forecast parameters by machine learning modules. They are listed below:

3.2 Parameters and notations

3.2.1 User controlled parameters

- BusRoute: including types (cyclic, or symmetric dual control for symmetric single control), number of stops, total distance, travel time under normal conditions in both directions (upstream and downstream).
- Ts: Planning horizon start time. By default this is set to the current time.
- Tf: finish time of the planning horizon. Target planning period is 2-3 hours.
- n: the total number of buses available.
- B: the list of buses available with properties including capacity, per trip running cost, available time window (or unavailable time window), vehicle types (normal bus or emergency back-ups). Broken-down vehicles should NOT be included in this list.
- $g_{\max}$: maximum bus dispatching gap permitted at different period p.
- $g_{\min}$: minimum bus dispatching gap, if not given, default to 1 min.
- $MinRestTime$: Minimum rest time for a bus at a terminal.
- w_1, w_2, w_3: the weights for three penalty terms: total waiting time, overloading and running cost. $w_1 + w_2 + w_3 = 1$.

3.2.2 Parameters required from real-time monitor module

- The status of all buses defined in B, including their positions.
- The boarding and alighting data at each station in the most recent m trips. This can be empty if no bus trips are made during that day.
- The GPS trajectory data of the most recent m bus trips.

3.2.3 Parameters estimated by machine learning modules

- $r(p,k)$: Passenger arrival rate at time period p and stop k. unit: person/min.
- $a(p,k)$: proportion of bus load alighting at stop k during time period p. $0 \leq a(p,k) \leq 1$.
- $RT(d_0,i)$: the time for a given schedule bus trip i to reach final destination (i.e. final terminal). d_0 is the departure time of the bus trip i. If $d_0 < CT$, the trip is on-going. The time should be estimated based on a combination of the vehicle's current position and historical data.
- $T(k,k+1,p)$: trip time for a bus from stop k to $k+1$.
- T_1: the average total trip time in control direction 1. Time-independent.
- T_2: the average total trip time in control direction 2. Time-independent.

3.2.4 Decision variables and auxiliary variables

The solution is represented by two bus trip queues, V and W, for control direction 1 and direction 2 respectively. Both V and W are sorted by the planned departure times. The length of V and W should cover the entire planning horizon and can be estimated based on the average departure headways and average travel time for each direction.

- g_i: the dispatching headway for scheduled bus trip $i \in V$ in control direction 1.
- h_j: the dispatching headway for a scheduled bus trip $j \in W$ in control direction 2.
- d_i: departure time of scheduled bus trip $i \in V$ in control direction 1. Hence $g_i = d_i - d_{i-1}$.
- e_j: departure time of bus trip $j \in W$ in control direction 2, and $h_i = e_i - e_{i-1}$.
- d_i^k: departure time of bus trip $i \in V$ from station/stop k.
- e_j^s: departure time of trip $j \in W$ from stop $s \in S$.
- g_i^k: headway for trip $i \in V$ at bus stop $k \in K$. $g_i^k = d_i^k - d_{i-1}^k$.
- h_j^s: headway for trip $j \in W$ at bus stop $s \in S$ and $h_j^k = e_j^k - e_{j-1}^k$.
- L_i^k: passenger load for trip $i \in V$ at stop $k \in K$ for direction 1.
- L_j^s: passenger load for trip $j \in W$ at stop $s \in S$ for direction 2.

3.2.5 Other notations

- CT: current time.
- K:The set of successive stops in control direction 1, including start and finish terminals, indexed by k.
- S:The set of successive tops in control direction 2, including start and finish terminals, indexed by s.
- P: a set of continuous time periods of identical length τ, indexed by p.
- C_i, C_j: the capacity of vehicles used for trips i and j respectively.
- F_i, F_j: the fixed operation cost for running trips i and j respectively.

3.3 Objective function

The scheduling can be activated by a returning vehicle or a number of returning vehicles with a rolling planning horizon. Assume a current rescheduling point/time t_0 and a planning horizon consisting of P successive identical time periods, indexed by p.

The objective function consists of three penalty components, the total passenger waiting time O_1, the total passenger overloads O_2 and the bus service operation cost O_3. The total waiting time at all stations can be calculated as follows.

$$O_1 = \sum_{i \in V} \sum_{k \in K} r(p,k) \times (g_i^k)^2/2 + \sum_{j \in W} \sum_{s \in S} r(p,s) \times (g_j^s)^2/2 \tag{1}$$

The first term is for the control direction 1 and the second term for the direction 2. Period p can be estimated as $p = (d_{i-1}^k + d_i^k)/(2 \times \tau)$ for direction 1 and $p = (e_{j-1}^s + d_j^s)/(2 \times \tau)$ for direction 2.

The passenger overload penalty can be computed as follows.

$$\begin{aligned} O_2 = & \textstyle\sum_{i \in V} \sum_{k \in K} [T(k, k+1, p) \times \max\{0, r(p,k) g_i^k + (1 - a(p,k)) L_i^k - C_i\}] \\ & + \textstyle\sum_{j \in W} \sum_{s \in S} [T(s, s+1, p) \times \max\{0, r(p,s) g_j^s + (1 - a(p,s)) L_j^k - C_j\}] \end{aligned}$$

where p can be estimated similarly to the previous equation.

The total operation cost can be estimated by.

$$O_3 = \sum_{i \in V} F_i + \sum_{j \in W} F_j \tag{2}$$

Naturally one could model this as a multi-criteria or multi-objective optimisation problem. Unfortunately for real-life applications, bus planning practitioners generally cannot tolerate long computational time that may be required by a multi-objective optimisation approach. Moving O_1 and O_2 into the constraints is another good option but cautions should be made for a potential problem without feasible solution due to a lack of sufficient buses. The main focus of this paper is modeling the optimisation problem with (part of) the parameters coming from machine learning modules. We adopted a simple objective function which is a weighted sum of the above three components.

$$OBJ = \min_{g_i, h_j} (w_1 O_1 + w_2 O_2 + w_3 O_3) \tag{3}$$

Auxiliary variables can be calculated through the following recursive functions

$$g_i^k = d_{i-1}^k - d_i^k \quad \forall i \in V, k \in K \tag{4}$$
$$d_i^k = d_i^{k-1} + T(k-1, k, p) \quad \forall i \in V, k \in K \tag{5}$$
$$g_i^0 = g_i \quad \forall i \in V \tag{6}$$
$$d_i^0 = d_i \quad \forall i \in V \tag{7}$$
$$h_j^s = h_{j-1}^s - h_j^s \quad \forall j \in W, s \in S \tag{8}$$
$$h_j^k = h_j^{s-1} + T(s-1, s, p) \quad \forall j \in W, s \in S \tag{9}$$
$$h_j^0 = h_j \quad \forall j \in W \tag{10}$$
$$h_j^0 = h_j \quad \forall j \in W \tag{11}$$

For most applications, we can approximate $T(k-1,k,p)$ by $T(k-1,k)$. If this does not meet practical real-life requirements, $T(k-1,k,p)$ should also be estimated through machine learning modules where $p = (d^k_{i-1} + d^k_i)/(2 \times \tau)$ for direction 1 and $p = (e^s_{j-1} + d^s_j)/(2 \times \tau)$ for direction 2.

3.4 Constraints

The following constraints should be satisfied.

$$g_{\min} \leq g_i \leq g_{\max} \forall i \in V \tag{12}$$

$$g_{\min} \leq h_j \leq g_{\max} \forall j \in W \tag{13}$$

$$d_i + RT(d_0, i) - \overline{d_i} \geq MinRestTime \quad \forall i \in V \tag{14}$$

$$e_j + RT(d_0, j) - \overline{d_j} \geq MinRestTime \quad \forall j \in W \tag{15}$$

$$\sum_{i \in V} g_i \geq Tf - Ts \tag{16}$$

Constraints (12) and (13) ensure that the headways for each bus trips in both directions are between a pre-specified minimum and maximum. Constraints (14) and (15) make sure that drivers have a minimum rest time before their next trips start. Constraint (16) guarantees the full coverage of planning horizon in any feasible list of bus trips V. The constraint of the maximum number of vehicles used should be automatically satisfied while creating trip lists V and W.

4 Solution methods

The model developed in the previous section is highly non-linear and can be in large-scale if the granularity of periods P is significantly smaller than the planning horizon. Therefore heuristic approaches are proposed as the solution method for the problem. As illustrated in Section 3.2.4, the primary decision variables are the headways (g_i, h_j) for bus route directions 1 and 2. Other variables can be computed through these two set of decision variables. In this research, memetic algorithm was chosen as the solution method for this problem. The reasons are: first, although the problem is formulated as a single objective optimisation problem, naturally it has three conflicting objectives and one would need minimal efforts to extend the proposed method to a multi-objective version. Second, a natural solution encoding scheme will be a vector concatenating g_i and h_j. Because the problem is not tightly bound, the combination operators in GA could be very efficient in finding high quality regions and solutions. Finally, local search procedure in memetic algorithm will make sure the resulting solution is at least a local optimum.

One of the main contributions of this paper is the introduction of machine learning methods into a traditional optimisation problem. Through machine learning modules, much high quality parameters are estimated by utilising both the real-time traffic data and historical data at a much finer time granularity (defined by the size of the period p). The parameters estimated by the machine learning modules include passenger demand data ($r(p,k)$, $a(p,k)$ and travel time data ($RT(d_0,i)$, $DT(k,k+1,p)$, and $DT(p,k)$).

In this research, we use a popular SVR (supporting vector regression) method from the LIBSVM library (Chang, 2016) for the estimation of both the passenger demand data and travel time data.

5 Discussions and future research

Although the coding of the algorithm is completed already and some initial results with some toy instances are available, the full experimental results are not available yet. It is expected that complete computational results will be presented during the conference presentation.

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Benders Decomposition in SMT for Rescheduling of Hierarchical Workflows

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Abstract Real-life scheduling has to face many difficulties such as dynamic manufacturing environments with failing resources and urgent orders arriving during the schedule execution. Complete rescheduling, considering only the original objective function, in response to unexpected events occurring on the shop floor may yield schedules that are prohibitively different from the original schedule, which may lead to extra costs due to impacts on other planned activities. Our novel approach in the area of predictive-reactive scheduling is to allow for substitution of jobs which cannot be executed with a set of alternative jobs. Hence, this paper describes the model of hierarchical workflows suitable for dealing with unforeseen events using the possibility of alternative processes and proposes a new approach, based on the Satisfiability Modulo Theories (SMT) formalism, to recover an ongoing schedule from a resource failure. The experimental results show that the SMT approach using Benders decomposition is orders of magnitude faster than a Constraint Programming approach.

1 Introduction

Scheduling aims at allocating scarce resources to jobs in order to optimize certain objectives. There has been extensive research on this area in the past decades. Developing a detailed schedule in manufacturing environment helps maintain efficiency and control of operations.

In the real world, however, manufacturing systems face uncertainty owing to unforeseen events occurring on the shop floor. Machines break down, operations take longer than anticipated, personnel do not perform as expected, urgent orders arrive, others are canceled, etc. These disturbances may bring inconsistencies into the ongoing schedule. If the ongoing schedule becomes infeasible, the simple approach is to collect

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the data from the shop floor when the disruption occurs and to generate a new schedule from scratch. Because most scheduling problems are NP-hard, complete rescheduling is usually prohibitively time-consuming and generates a new schedule that is too deviated from the original schedule.

To avoid the problems of rescheduling from scratch, the continuous correction of precomputed predictive schedules (so-called reactive scheduling) is becoming more and more important. Reactive scheduling differs from predictive scheduling mainly by its on-line nature and associated real-time execution requirements. The schedule update must be accomplished before the running schedule becomes invalid, and this time window may be very short in some areas.

In this paper, we describe the model with the hierarchical structure of tasks that is suitable also for the field of predictive-reactive scheduling. Further, we model the rescheduling problem in the formalism called Satisfiability Modulo Theories (SMT) [14]. The goal of the model is to recover a schedule from a resource failure, exploiting the possibility of alternative processes. Also, we suggest how to leverage the Benders decomposition [6] by dividing the problem into a master problem and a subproblem where the subproblem passes the reason for inconsistency to the master problem. We experimentally compare these two models to each other, as well as against a Constraint Programming (CP) model.

We first contextualize this paper in terms of the related works. Section 3 then describes the scheduling model and the resource failure recovery problem tackled in this paper and the suggested approaches are described in Sect. 4. The experimental results are given in Sect. 5, and the final part points out possible future work.

2 Related Work

The approaches how to tackle dynamics of the scheduling environment can be divided basically into two branches according to whether or not the predictive schedule is computed before the execution starts [24]. If the predictive schedule is not computed beforehand and individual jobs are assigned to resources pursuant to some so-called dispatching rules during the execution, we talk about *completely reactive scheduling* or on-line scheduling. This strategy is suitable for very dynamic environments, where it is not known in advance which jobs it will be necessary to process. On the other hand, it is obvious that this approach seldom leads to an optimal or near-optimal schedule.

If the schedule is crafted beforehand and then updated during its execution, it is referred to as *predictive-reactive scheduling*. When correcting the ongoing schedule in response to changes within the environment, the aim is usually to minimize the schedule modification. The motivation for minimizing the modification of the schedule is that every deviation may lead to deterioration in the performance of the manufacturing system because of impacts on other planned activities based upon the original schedule.

There is an extensive literature on rescheduling [2,17,26], giving various generic approaches as well as ad-hoc procedures to particular cases. However, to the best of our knowledge, there is only scarce research carried out aiming at the possibility of re-planning in the field of predictive-reactive scheduling.

On the other hand, if a certain level of minor disruptions is anticipated, it can be taken into account when generating a baseline schedule, such as in [15], where the static schedules account for unforeseen prolongation of processing times of tasks. This area is usually referred to as *proactive scheduling*.

The scheduling model described further is based on the notion of *workflows*. Workflow, in general, may be understood as a scheme of performing some complex process, itemized into simpler processes and relationships among them. There exist many formal models to describe workflows [23] that include decision points and loops to describe repetition of operations, but in real-life applications, many workflows are obtained by decomposition of tasks, which is the motivation for the hierarchical structure of workflows. In this paper, we consider workflows that have a very similar structure to Temporal Planning Networks [19] but are enhanced by extra constraints and resources.

The logical constraints in a workflow indicate the connection with the first order logic and the problem of Boolean Satisfiability (SAT) [22]. Most SAT-solvers today are still based on variations of the DPLL procedure [9]. However, significant advances in SAT solving techniques were made in the last decades thanks to better implementation techniques such as the *two-watched literal* approach for unit propagation [13] and conceptual improvements such as *conflict-driven clause learning* and *restarts* [20, 5].

This progress made SAT-solvers applicable also to solving Constraint Satisfaction Problems (CSP) [18] in general. In the lazy clause generation approach [16], a Boolean formula corresponding to a finite domain CSP is lazily created during the computation. On the other hand, motivation of gaining the advantages of techniques used both in SAT-solvers and CSP-solvers lead to another hybrid approach, where some domain specific reasoning is implemented within a SAT-solver: SAT Modulo Theories (SMT) [14].

The temporal constraints in our scheduling model are linear inequalities so that it is suitable for SMT, using the linear integer arithmetic as the background theory. SMT solvers have already been used for solving combinatorial optimization problems [8], including some scheduling problems [1], and exhibited promising performance.

The problem we tackle has already been targeted by [4], where an ad-hoc heuristic-based algorithm is proposed which quickly finds near-optimal solutions, but supports temporal constraints only between primitive tasks, which limits its applicability. Also in [25], a very similar problem is addressed, with the main difference that the optimization objective is to minimize the work processed in vain, and a detailed experimental analysis is presented.

3 Scheduling Model

In this work, we use the model of workflows that match up the structure of Nested Temporal Networks with Alternatives [3], where the nodes of a network correspond to the tasks of a workflow. The tasks decompose into other tasks, referred to as their subtasks. There are two types of decomposition: *parallel* and *alternative*. The tasks that do not decompose further (i.e., leaves) are called *primitive tasks*. The primitive tasks correspond to executable operations and are associated with some additional parameters such as duration and resource.

The workflows as described define a number of feasible processes. A *process* is a subset of tasks selected to be processed. While a parallel task requires all its children to be processed, an alternative task requires exactly one of its children to be processed. If an arbitrary task is not in the process, none of its subtasks is in the process either. Hence, to ensure that an instance of a workflow is actually processed, its root task has to be in the selected process. An example of a workflow and a process (Fig. 1) [21] contains eight primitive tasks, three parallel tasks, and two alternative tasks.

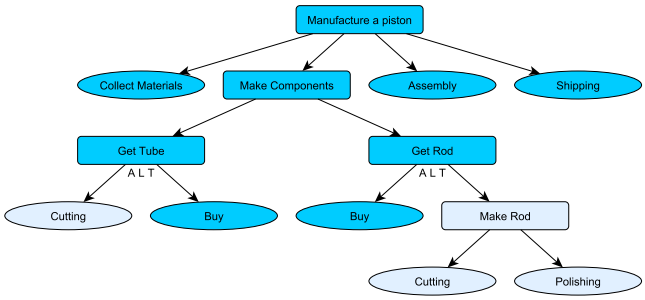

Fig. 1 An example of a workflow. The label 'ALT' beneath tasks stands for alternative decomposition; the other decompositions are parallel. Primitive tasks are ellipse-shaped. The highlighted tasks form an example of a selected process.

The nested structure may not be flexible enough to describe naturally some additional relations in real-life processes, for example when an alternative for one task affects the selection of alternatives in other tasks. In order to simplify the description of these additional relations between tasks, a pair of tasks can be bound by a *logical constraint*. Logical constraints include *implications* (if one task is in the process, the other task must be in the process too), *equivalences* (either both tasks must be in the process or neither of them can be in the process), and *mutual exclusions* (at most one of the tasks can be in the process). Further, there are simple temporal constraints [11] to determine the maximum (as well as minimum) time distance between two tasks, provided that both tasks are selected to the process.

Primitive tasks are processed on *resources*. We consider here that all resources are unary, which means that each resource may perform at most one primitive task at any time. Such resources are often referred to as disjunctive constraints. Each primitive task is specified by exactly one resource on which the primitive task is to be processed.

3.1 Scheduling Problem Definition

Formally, a scheduling problem S consists of three sets: $Tasks$, $Constraints$, and $Resources$.

3.1.1 Tasks

The set $Tasks$ of tasks is a union of three disjoint sets: $Parallel$, $Alternative$, and $Primitive$. For each task T except the *root* task, the function $Parent(T)$ denotes the parent task in the hierarchical structure. Similarly for each task T we define the set $Subtasks(T)$ of its child nodes as $Subtasks(T) = \{C \in Tasks \mid Parent(C) = T\}$.

The tasks from sets $Parallel$ and $Alternative$ are called compound tasks and they decompose to some subtasks, whereas the primitive tasks do not decompose:

$$\forall T \in Parallel \cup Alternative : Subtasks(T) \neq \emptyset$$
$$\forall T \in Primitive : Subtasks(T) = \emptyset$$

Let process $P \subseteq Tasks$ be the set of tasks selected to be processed. To make the process feasible, it is necessary to satisfy the following constraints:

$$\forall T \in P \cap Parallel : Subtasks(T) \subseteq P \tag{1}$$

$$\forall T \in P \cap Alternative : |Subtasks(T) \cap P| = 1 \tag{2}$$

$$\forall T \notin P : Subtasks(T) \cap P = \emptyset \tag{3}$$

$$root \in P \tag{4}$$

Note that without constraint (4), an empty set would be a feasible process.

Let S_i and E_i denote the start time and end time, respectively, of task T_i. We consider only non-interruptible tasks, and each primitive task T_i is specified by the duration D_i. The times for compound tasks are computed from the times of their subtasks. Thus we obtain the following constraints:

$$\forall T_i \in P \cap Primitive : E_i = S_i + D_i \tag{5}$$

$$\forall T_i \in P \setminus Primitive : S_i = \min\{S_j \mid T_j \in Subtasks(T_i) \cap P\} \tag{6}$$

$$\forall T_i \in P \setminus Primitive : E_i = \max\{E_j \mid T_j \in Subtasks(T_i) \cap P\} \tag{7}$$

3.1.2 Extra constraints

There are basically two types of constraints: logical and temporal. Logical constraints are of three types: implications, equivalences, and mutexes. The semantics of the constraints is as follows:

$$\forall (i \Rightarrow j) : T_i \in P \Rightarrow T_j \in P \tag{8}$$

$$\forall (i \Leftrightarrow j) : T_i \in P \Leftrightarrow T_j \in P \tag{9}$$

$$\forall (i \ mutex \ j) : T_i \notin P \vee T_j \notin P \tag{10}$$

The time distance between two distinct tasks may be restricted by a *simple temporal constraint*. This constraint can be written as a triplet (X_i, X_j, w_{ij}), where $w_{ij} \in \mathbb{Z}$, X_i is either S_i or E_i, and X_j is either S_j or E_j. The semantics is as follows:

$$\forall (X_i, X_j, w_{ij}) : T_i \in P \wedge T_j \in P \Rightarrow X_j - X_i \leq w_{ij} \tag{11}$$

Notice that temporal constraints may be between compound tasks, that is why the time variables S_i and E_i are needed for all tasks in the model.

3.1.3 Resources

Each primitive task is associated with exactly one resource where it can be processed. Let $T_i \in Primitive$, then the resource that can process the primitive task T_i is denoted R_i.

All resources in a schedule are unary, which means that they cannot execute several tasks simultaneously. Therefore, in a feasible schedule, the following holds:

$$\forall T_i, T_j \in Primitive \cap P, T_i \neq T_j : R_i = R_j \Rightarrow E_i \leq S_j \vee E_j \leq S_i \tag{12}$$

Note that the fact that each primitive task is associated with exactly one resource does not make the model less expressive than considering alternative resources for a primitive task as this may be modeled using alternative tasks. Similarly, more resources for a primitive task can be modeled via parallel decomposition to temporally synchronized primitive tasks.

3.2 Schedule

A schedule S (sometimes referred to as a resulting schedule or a solution) is acquired by determining the set P, and allocating the tasks from P in time, that is, assigning particular values to the variables S_i and E_i for each $T_i \in P$.

To make a schedule *feasible*, the process selection and the allocation must be conducted in such a way that all the constraints (1)–(12) in the problem are satisfied.

3.3 Rescheduling Problem

The problem we deal with is that we are given a particular instance of the scheduling problem along with a feasible schedule, and also with a disturbance arising from the shop floor. The aim is to find another schedule that is feasible with respect to the disturbance. The feasible schedule we are given is referred to as an *original schedule* or an *ongoing schedule*.

In what follows we restrict our attention to a disruption called a resource failure, which is often referred to as a machine breakdown. The resource failure may happen in a manufacturing system at any point in time, say t_f, and means that a particular resource cannot be used anymore, i.e., for all $t \geq t_f$ there is no task allocated to that resource.

Formally, let Sch_0 be the original schedule, with P_0 being the selected process in Sch_0, $S_i(Sch_0)$ and $E_i(Sch_0)$ the start times and end times in Sch_0; and R be the resource that failed at some time $t_f : t_f \in \mathbb{Z}, t_f \geq 0$. Next, let us define $Pinned$ as the set of primitive tasks that are from P_0 and, whose execution has finished if they were allocated to R or whose execution has at least started if they were allocated to an available resource, that is, $T_i \in Pinned$ if and only if:

$$T_i \in Primitive \cap P_0 \ \wedge \big((E_i(Sch_0) \leq t_f \wedge R_i = R) \vee (S_i(Sch_0) < t_f \wedge R_i \neq R)\big)$$

Finally, the aim is to find a new feasible (recovered) schedule Sch_1, with P_1 being the selected process in Sch_1, $S_i(Sch_1)$ and $E_i(Sch_1)$ the start times and end times in Sch_1, subject to the following constraints:

$$\forall T_i \in Pinned : T_i \in P_1 \wedge S_i(Sch_1) = S_i(Sch_0) \tag{13}$$

$$\forall T_i \in Primitive \setminus Pinned, R_i = R : T_i \notin P_1 \tag{14}$$

$$\forall T_i \in Primitive \setminus Pinned, R_i \neq R : T_i \in P_1 \Rightarrow S_i(Sch_1) \geq t_f \tag{15}$$

The constraint (13) ensures that the primitive tasks defined above as $Pinned$ are also in the new process and are not shifted in time. The constraint (14) ensures that each primitive task that is not in $Pinned$ and is associated with the failed resource is not in the new process P_1, and for the remaining primitive tasks, the constraint (15) ensures that no primitive task will be scheduled in the past.

The aim is to find the new schedule Sch_1 as similar to Sch_0 as possible. For this purpose, we minimize the objective function defined as the number of primitive tasks that were removed from the original schedule plus the number of primitive tasks that were added to the recovered schedule:

$$f = |\{T \in P_0 \cap Primitive \setminus P_1\}| + |\{T \in P_1 \cap Primitive \setminus P_0\}| \tag{16}$$

Note that the constraint (3) states that non-membership of a task implies non-membership of its subtasks. Hence, there cannot be any unneeded task in the process. This is coherent with the needs of real-life applications.

4 Resource Failure Recovery as Satisfiability Modulo Theories

In this section, we describe how we model the resource failure recovery problem in the SMT formalism. Recall that the task of an SMT solver is to find a satisfying assignment of a formula φ or report unsatisfiability. Such formula is usually represented in a Conjunctive Normal Form (CNF), i.e., as the conjunction of clauses, where clauses are disjunctions of literals corresponding to expression in a particular *theory*, in our case difference logic. To ease the description, we list the clauses in the form given by their semantics. Such clauses can be transformed to a CNF.

We use the application interface of the Z3 solver [10]. This solver provides two objects: *Solver*, which can determine the unsatisfiable core, and *Optimizer*, which allows the addition of weighted soft clauses to the formula, and then minimizes the sum of weights of soft clauses that are not satisfied by the assignment.

4.1 Global Model

For each task $T_i \in Tasks$, let us introduce propositional variables V_i that will be *true* if and only if $T_i \in P_1$, and numeric (non-negative integer) variables S_i and E_i determining the start time and end time of task T_i. Now we describe how the SMT formula φ to be satisfied is constructed.

For each compound task T_i, let us denote $Subtasks(T_i) = \{T_{i_1}, ..., T_{i_k}\}$. If $T_i \in Parallel$, then for each $j = i_1, ..., i_k$ the following unit clause is added to the formula φ:

$$(V_i = V_j) \tag{17}$$

If $T_i \in Alternative$, the following clause is added:

$$\left(ite(V_i; 1; 0) = \sum_{j=i_1}^{i_k} ite(V_j; 1; 0) \right) \tag{18}$$

The expression $ite(V_i; 1; 0)$ is evaluated as 1 if V_i is *true*, otherwise 0. It is straightforward to verify that satisfying the clauses (17) and (18) enforces the satisfaction of constraints (1)–(3). Note that we also tried modeling the constraint (2) in a purely logical way, i.e., using disjunction and at-most-one clauses instead of clause (18), but this did not lead to any measurable difference in run times. To satisfy also the constraint (4), we simply add the unit clause:

$$(V_{root}) \tag{19}$$

As to the time allocations for primitive tasks, that is, satisfying the constraint (5), it is enough to add the following clauses:

$$(E_i = S_i + D_i) \tag{20}$$

The time allocations for compound tasks is not that straightforward as the functions min and max are not in the formal specification of SMT. Hence for a parallel task we need clauses ensuring that the start time of the task is always less than or equal to its subtasks, and that it equals one of its subtasks. A similar construction is used for the end times. Formally, for each $T_i \in Parallel$:

$$(V_i \Rightarrow \bigvee_{j=i_1}^{i_k} S_i = S_j) \tag{21}$$

$$(V_i \Rightarrow \bigvee_{j=i_1}^{i_k} E_i = E_j) \tag{22}$$

And for each $j = i_1, ..., i_k$:

$$(V_i \Rightarrow S_i \leq S_j) \tag{23}$$

$$(V_i \Rightarrow E_i \geq E_j) \tag{24}$$

Since alternative tasks have at most one subtask in the process, it suffices to add, for each $T_i \in Alternative$, and for each $j = i_1, ..., i_k$:

$$(V_j \Rightarrow S_i = S_j) \tag{25}$$

$$(V_j \Rightarrow E_i = E_j) \tag{26}$$

It is easy to see that the satisfaction of clauses (21)–(26) ensures satisfying the constraints (6) and (7).

The logical constraints (8)–(10) in the format of $(i \rightarrow j)$, $(i \leftrightarrow j)$, and $(i \; mutex \; j)$ are handled by adding the following clauses, respectively:

$$(V_i \Rightarrow V_j) \tag{27}$$

$$(V_i \Leftrightarrow V_j) \tag{28}$$

$$(\neg V_i \vee \neg V_j) \tag{29}$$

Next, for each simple temporal constraint (11) (X_i, X_j, w_{ij}), where $w_{ij} \in \mathbb{Z}$, X_i is either S_i or E_i, and X_j is either S_j or E_j, we add:

$$((V_i \wedge V_j) \Rightarrow X_j - X_i \leq w_{ij}) \tag{30}$$

The disjunctive constraints (12) are handled by adding the following clauses, for each $T_i, T_j \in Primitive, T_i \neq T_j, R_i = R_j$:

$$((V_i \wedge V_j) \Rightarrow (E_i \leq S_j \vee E_j \leq S_i)) \tag{31}$$

To ensure that the primitive tasks from the set $Pinned$ are also in the new process and are not shifted in time, that is, to satisfy the constraint (13), add for each $T_i \in Pinned$:

$$(V_i) \tag{32}$$

$$(S_i = S_i(Sch_0)) \tag{33}$$

To satisfy the constraint (14), add for each $T_i \in Primitive \setminus Pinned, R_i = R$:

$$(\neg V_i) \tag{34}$$

And to satisfy the constraint (15), add for each $T_i \in Primitive \setminus Pinned, R_i \neq R$:

$$(S_i \geq t_f) \tag{35}$$

Note that the clauses (20) and (35) do not need the implication form $(V_i \Rightarrow expr)$, because in case $V_i = false$, the values of the time points S_i and E_i do not affect other variables and thus may be assigned arbitrary values (satisfying the clauses) by an SMT solver, which turned out to be slightly faster than the implication form.

Finally, the distance function f is optimized by adding soft clauses of weight 1. For each $T_i \in Primitive \cap P_0$, the following soft clause is added:

$$(V_i) \tag{36}$$

Oppositely, for each $T_i \in Primitive \setminus P_0$, the following soft clause is added:

$$(\neg V_i) \tag{37}$$

Notice that instead of all weights being equal to one, we could set the weights according to the additional cost if the corresponding primitive task is processed. Thus, instead of robust rescheduling, we could directly use the same model for cost-minimizing scheduling.

4.2 Benders Decomposition

The SMT model can be decomposed in two phases: first, compute the process P_1, that is, assign truth values to the variables V_i, and second, schedule the process, that is, assign time points to the variables S_i and E_i for each $T_i \in P_1$. If the second phase corresponds to an unsatisfiable problem, get the reason for inconsistency (unsatisfiable core) and go back to the first phase to find another process. Iterate until a feasible schedule is found or there is no other process to try.

4.2.1 Master Problem

Formally, we first build a formula φ_p consisting exactly of the clauses that do not involve time variables, i.e., the clauses (17)–(19), (27)–(29), (32), (34), (36), and (37). Then the SMT solver is run. If it reports 'infeasible', there is no solution to the problem. If it finds a satisfying assignment $\mathbf{V}$, we now build a formula φ_s exploiting the knowledge of $\mathbf{V}$.

4.2.2 Subproblem

The formula φ_s is build from the clauses involving the time variables. However, as we already know the values of all V_i, we add only the clauses that are not trivially valid.

The clauses (21)–(26), and (35) of the form $(V_i \Rightarrow expr)$, such that V_i is $true$, are added to the formula φ_s in the form $(expr)$. Similarly, the clauses (30) and (31) of the form $((V_i \wedge V_j) \Rightarrow expr)$, such that both V_i and V_j are $true$, are added to the formula φ_s in the form $(expr)$. Finally, the clauses (20), (33), and (35), such that V_i is $true$, are added without modification.

After the SMT solver is run on the formula φ_s, there are two possible results. If it finds a satisfying assignment, that is, a schedule, it is clearly the optimal solution

as the objective function concerns only the process selection. Otherwise, if it reports unsatisfiable, i.e., the process is not schedulable, we will obtain an unsatisfiable core, which is a set of clauses that cannot be satisfied together. Note that there may be many reasons for inconsistency (hence many unsatisfiable subsets of clauses) but from Z3 we will obtain just one unsatisfiable core. From the unsatisfiable core obtained, we will get a set of tasks that cannot be in a feasible process together, hence removing at least one such task from the process P_1 may lead to finding a schedulable process.

4.2.3 Unsatisfiable Cores

In order to get a set of tasks that cannot be in a feasible process together, we associate each clause in the formula φ_s with one or two tasks.

The clauses in the formula φ_s originating from the form $(V_i \Rightarrow expr)$ are associated with the task T_i. The reason for association with the task T_i is that if the clause in question is in the unsatisfiable core, then removing task T_i from the process P_1 may lead to finding another process that is schedulable. Note that in the case of clauses for alternative tasks (25) and (26) of the form $(V_j \Rightarrow expr)$, we indeed need to associate it with T_j, because in the case of alternative tasks it may be enough to replace the subtask in the process. The clauses (20) and (35) are also associated with task T_i.

Similarly, the clauses of the form $((V_i \wedge V_j) \Rightarrow expr)$ are associated with both tasks T_i and T_j. Finally, the clauses (33) for pinned tasks are not associated with any task, that is, these clauses are not tracked, because the primitive task T_i from the set $Pinned$ cannot be removed from the process. Note that for the set $Pinned$ the clauses (20) are not tracked either.

After obtaining an unsatisfiable core of the formula φ_s, we construct a new clause to be added to φ_p. Assuming the union of tasks that are associated with clauses from the unsatisfiable core is $T_k, ..., T_l$, the clause $(\neg T_k \vee \ldots \vee \neg T_l)$ is added to the formula φ_p. Then the SMT solver is run again to solve the updated formula φ_p, i.e., the master problem.

It is not obvious whether the decomposition approach with passing the unsatisfiable core from the one subproblem to the other should lead to improvement in terms of solving efficiency because learning nogood clauses is done internally by the DPLL procedure that an SMT solver uses. The clauses learned by the global model are potentially more powerful because they may use variables from both planning and scheduling aspects, but the Benders decomposition helps focus quickly on the most relevant explanations. We can observe in the experimental evaluation that indeed, the Benders decomposition finds solutions much faster, which is very important given the real-time aspect of the problem, but on the long run, the global model may sometimes be the best.

5 Experimental Results

In this section, we experimentally evaluate the global and the decomposed SMT model using the Z3 solver. The Z3 solver allows us to set several parameters including the engine. As to the `Optimizer`, the best engine for maximal satisfiability turned out to be the one set by parameter `wmax`, which is described in [7]. As to the `Solver` used for the formula φ_s, the best arithmetic solver turned out to be the one based on the Bellman-Ford algorithm.

time limit	SMT BD	SMT	CP
10 ms	60.1	10.4	59.1
100 ms	90.9	64.5	85.9
1 s	98	82.3	92.2
10 s	99.7	100	92.6
100 s	100	100	92.6

Table 1 Ratio of solved instances within a given time limit.

Also, we show results for a CP model using the IBM CP Optimizer [12], which provides a modeler with so-called interval variables that also have support for hierarchical decomposition, that is, interval variables may be optional, and there are also specialized constraints, such as *span*, *alternative*, and *no overlap*, which make it very easy to model. The only parameter of the optimizer we adjusted was `DefaultInferenceLevel`, which we set to `Extended`.[1]

The problems are randomly generated as follows. First, we generate the desired number of primitive tasks and build the hierarchical structure of tasks. Then we easily find the process P_0, which may be done in linear time when there are no extra logical constraints, and then allocate the primitive tasks from the process P_0 on resources in such a way that the resources are used without empty gaps, which is again easy when there are no extra temporal constraints. Afterward, we add the desired number of extra logical constraints as well as extra temporal constraints in such a way that the already crafted schedule is still feasible. In order to maximize the number of feasible processes in a workflow, and thus to make the problems harder, a compound task is an alternative task if the task has at least one primitive task as its subtask, otherwise it is a parallel task.

We performed experiments with instances consisting of 100 primitive tasks in a workflow. Duration of each primitive task is a number greater than 0 and less than 15 generated uniformly at random, each task has 2–5 subtasks, and the number of resources is 2–8. The number of temporal constraints and the number of logical constraints are 0–99. Logical constraints are implications or mutexes with equal probabilities $\frac{1}{2}$. (We omit equivalences as they are merely pairs of implications.) Temporal constraints are added in pairs $(X_i, X_j, dist)$ and $(X_j, X_i, -dist)$, where each X is S or E with equal probabilities $\frac{1}{2}$, and $dist = X_j - X_i$, if $T_i, T_j \in P_0$, otherwise it is a randomly generated number from the interval $[-makespan, makespan]$, where $makespan$ is the end time of the root task in the original schedule. Such temporal constraints with zero slack are motivated by real-life scheduling applications where some jobs are temporally synchronized. The time t_f of a resource failure is set to 0, otherwise enlarging the set of pinned tasks only reduces the search space.

Experiments were performed on a Dell PC with an Intel(R) Core(TM) i7-4610M processor running at 3.00 GHz with 16 GB of RAM. Table 1 shows the ratio of solved instances (in percentage) within a given time limit.

Given the time limit of 100 seconds, the average run times for SMT with Benders decomposition, SMT, and CP were respectively 151.276 ms, 479.54 ms, and 7443.424 ms. The number of solvable instances was 356 out of 1000 instances.

However, as depicted in Fig. 2, the global SMT model is slower for most instances than the CP model, which in turn is not able to solve some instances in a reasonable

[1] All the source code can be downloaded from
https://drive.google.com/open?id=0BxOh5Kp8klV3OFBCZWJYQVVmSTA

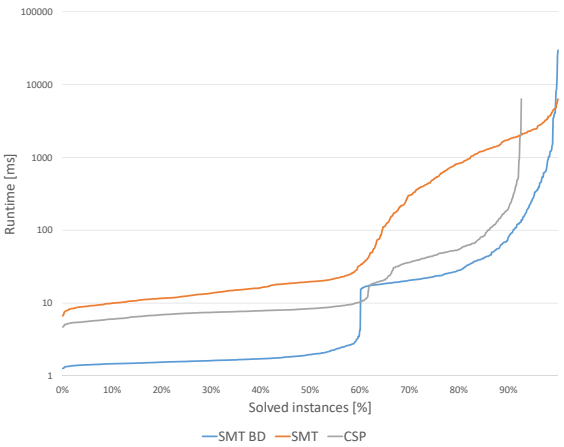

Fig. 2 Ratio of solved instances for SMT with Benders decomposition, SMT, and CP.

time limit	SMT BD	SMT	CP	Ad-hoc
10 ms	60.4	11.6	56.8	84.8
100 ms	97.2	61.9	99.7	90.4
1 s	99.9	82.3	99.9	93.3
10 s	100	100	99.9	96.9
100 s	100	100	99.9	98.8

Table 2 Ratio of solved instances within a given time limit.

time. CP either solves a problem within one second, or it does not solve it at all. This happens also for very small instances. The reason is a temporal inconsistency between a task and some of its ancestors, which the CP engine is not able to efficiently detect because the inconsistency is hidden in disjunctions.

We also tried the decomposition in the CP model, as well as a hybrid approach, using SMT for finding a process and CP for scheduling, but this did not lead to any better results, mainly because finding the conflicting set of constraints using the method `RefineConflict` provided by IBM CP Optimizer turns out to be more time-consuming than using the `Solve` method itself.

In order to compare these methods also against the ad-hoc heuristic [4], we generated the problems with the same parameters except the temporal constraints $(S_i, S_j, dist)$ and $(S_j, S_i, -dist)$ are added only between primitive tasks. The results (Table 2 and Fig. 3) confirm that the ad-hoc method quickly finds a near-optimal solution when there is some, but when there is no solution, it is not able to prove it as it only tries a given number of instantiations in the scheduling phase, so-called cut-off limit. Hence the run times directly depend on the cut-off limit, which we set to 100000.

The meaning of the ad-hoc column is the percentage of instances where the heuristic terminated in the given time limit either giving a feasible (near-optimal) solution or claiming infeasibility (which may be even incorrect). The results indicate that the heuristic is not very effective. However, the runtimes of the heuristic strongly depend on the given cut-off limit, which is the maximum number of attempted allocations during search. The smaller the cut-off limit, the faster the runtime, but for too small cut-off limits there is a chance of incorrectly claiming infeasibility. Now, the average

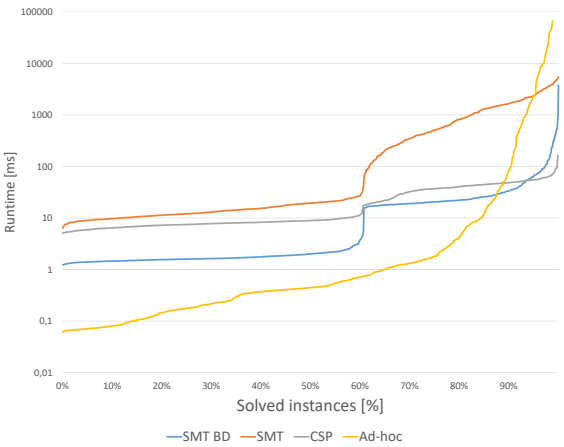

Fig. 3 Ratio of solved instances for SMT with Benders decomposition, SMT, CP, and Ad-hoc algorithm.

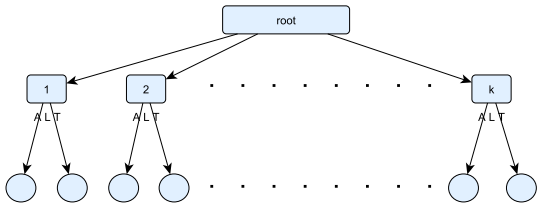

Fig. 4 A pathological case with the exponential number of processes for $l = 2$; primitive tasks associated with the failed resource are omitted.

run times for SMT with Benders decomposition, SMT, CP, and ad-hoc heuristic were respectively 22.904 ms, 479.51 ms, 121.02 ms, and 1925.823 ms. The number of solvable instances was 392 out of 1000 instances.

It is also noteworthy that allowing temporal constraints only between primitive tasks makes the problem easier for all aforementioned approaches, which is most apparent for CP. Nevertheless, we find this comparison not very relevant because it is only for a restricted set of problems, where temporal constraints are allowed only between primitive tasks, which is required by the ad-hoc heuristic.

It may seem that the time limit of 100 seconds is too short. Recall that the goal is to reschedule in case of breakage during the execution. Because of this real-time aspect, time-consuming algorithms might lead to a failure of the scheduling mechanism, so the short timeout is on purpose. However, to examine how the models perform in extremely hard instances, which we deem those having an exponential number of processes, while none of them being schedulable, we contrive the following example (Fig. 4).

Let us have k alternative tasks, all of them being subtasks of a root parallel task (hence necessarily selected in the process) and each of them having $2l$ primitive subtasks of duration 10. Next, let us have two resources, such that l primitive tasks from each alternative task are to be processed on the resource that fails (at time 0), and the remaining l primitive tasks on the resource that stays available. Clearly, since we need to schedule k primitive tasks and for each of them there are l options as to which

k	SMT BD	SMT	CPwB	CP
2	34	3	22	
3	96	84	102	
4	118	41	129	
5	310	304	803	
6	1,345	5,860	4,552	
7	16,801	922,857	53,860	
8	491,676		639,049	

Table 3 Runtimes in milliseconds for the pathological case.

primitive task to select, there are l^k processes. To make sure that there is not enough resources for scheduling k primitive tasks (one from each alternative task), but enough for scheduling $k-1$, we add the simple temporal constraint limiting the makespan $(S_{root}, E_{root}, 10k-5)$. In this case, the decomposed model in each iteration discovers an unsatisfiable core whose clauses are associated with all the k primitive tasks in the process, and hence indeed tries l^k processes.

For $l = 2$, Table 3 shows the run times in milliseconds, based on the number of alternative tasks k, for which the computation terminated in one hour.

The results show that even in the case where the decomposed model has to examine an exponential number of processes, it is still the fastest. However, the exponential growth clearly indicates that none of the methods can solve this example in a reasonable time for larger values of k.

Notice that the CP Optimizer is already not able to solve the instances for $k \geq 2$ within an hour. The column CPwB shows run times when we bound the ends of interval variables by $10k$ and the starts of interval variables by $10(k-1)$. However, for the hierarchical scheduling problems described, it is not easy to obtain a bound on the makespan that would bring measurable benefit but would not prune out some solutions. On the contrary, all our attempts to expedite the SMT solver by deducing some extra clauses turned out to be (sometimes significantly) counterproductive.

6 Conclusions

This paper proposes using the hierarchical model with alternatives in the field of predictive-reactive scheduling. The model we described makes it possible to replace tasks in the process by other tasks that are not in the process, i.e., to re-plan some subset of the schedule. We focused on the resource failure recovery problem and proposed two ways how the problem can be modeled in the SMT formalism. The experimental results showed that the SMT-based approaches are more suitable for this problem than a CP-based approach. We find our main contribution in the decomposed SMT approach yielding unexpected speed-up, especially when compared to IBM CP Optimizer, which is designed particularly for scheduling problems.

Since our approach seems well suited to other types of disruptions, such as an urgent order arrival, it could be addressed as future work. It might also be of interest to evaluate the suggested methods on a set of instances based on real world data. Further, it could be beneficial to enhance the decomposition approach for problems where the objective function involves changes of the time allocations.

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Abstracts

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Optimal solutions for minimum-cost sequential system testing

Roel Leus · Salim Rostami · Stefan Creemers

1 Introduction

We consider a multi-component complex machine (subsequently referred to as a "system") that undergoes periodic functionality checkups. We examine in which order the components of the system should be tested so as to diagnose the functionality of the system and its potential malfunctioning components with minimum cost and/or time. With these challenges in mind for preventive and corrective maintenance, the importance of the *sequential testing problem* was highlighted in the U.S. Air Force more than sixty years ago [1]. The goal of sequential system testing is to discover the state of a system, which is either up (working) or down (not working), by testing its components one by one. The testing continues until the system state is identified. The objective is to find a sequence of tests that minimizes the total expected cost of the sequential diagnosis. As equipment becomes more complex and expensive, the number of required tests and the inspection costs increase, and consequently, the sequential diagnosis becomes more expensive.

Depending on the type of the system under investigation, test failures may have different consequences. A k:n system (or *k-out-of-n*) is up if at least k out of the n components are functioning, and is down if $(n-k+1)$ components or more are down. Minimum-cost test sequencing for k:n systems with general precedence constraints between the tests is studied by Wei et al. [2]. An *n-out-of-n* (or serial) system is up if all the n components are functioning. Hence, its testing terminates if a single failure is encountered. In a 1:n (or parallel) system, on the other hand, a single working component suffices to guarantee the functionality of the whole system. Consequently, the diagnosis ends at the first success. Sequencing problems for serial and parallel systems are equivalent: an optimal solution to a 1:n system with success probabilities p is also optimal for an n:n system with success probabilites $1-p$. In this work we focus on test sequencing for n:n systems under general precedence constraints.

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The n:n case with general precedence constraints is known to be strongly NP-hard [3]. To the best of our knowledge, no research has been specifically devoted to developing exact algorithms for this problem. Simon and Kandane [4] describe conditions for a diagnostic strategy to be optimal for series or parallel systems with general precedence constraints, but they do not develop algorithms to find such a strategy. The scientific work to date on sequential testing has mainly focused on identifying special cases that are polynomially solvable; see for instance Monma and Sidney [5]. For an extensive review of the literature we refer to Ünlüyurt [6]. We will benchmark our computational results against the performance of the procedures in [2], where a more general problem is solved.

2 Problem statement

We consider a set $N = \{1, \ldots, n\}$ that contains n tasks, each of which corresponds to an inspection or a test (the two terms can be used interchangeably) with a non-negative testing cost c_i and a success probability $p_i \in [0; 1]$; the test outcomes are independent. The tests are subject to technological precedence constraints represented by a set $E \subset N \times N$, which is a partial order relation, such that $(i, j) \in E$ implies that test i should be executed before test j. A solution is then a sequence $\mathbf{s} = (s_1, s_2, \ldots, s_n)$ of the tests that minimizes the total expected cost of the diagnosis $\sum_{j=1}^{n} (\prod_{i=1}^{j-1} p_{s_i}) c_{s_j}$, where the cost c_{s_j} of the activity s_j in the j-th position is incurred only if all the preceding activities succeed, which is the case with probability $\prod_{i=1}^{j-1} p_{s_i}$. Obviously, a sequence is feasible if and only if it extends E.

A compact formulation for this sequencing problem uses binary "linear ordering" variables x_{ij} (in line with Potts [7]), where $x_{ij} = 1$ if job i precedes job j in the corresponding sequence, and $x_{ij} = 0$ otherwise. The problem can then be stated as follows:

$$\text{(CF)} \qquad \min \quad \sum_{j=1}^{n} c_j \prod_{i=1}^{n} [(p_i - 1)x_{ij} + 1] \tag{1}$$

subject to

$$x_{ij} + x_{ji} = 1 \qquad \forall \{i, j\} \subset N \tag{2}$$

$$x_{ij} + x_{jk} - x_{ik} \leq 1 \qquad \forall \{i, j, k\} \subset N \tag{3}$$

$$x_{ij} = 1 \qquad \forall (i, j) \in E \tag{4}$$

$$x_{ij} \in \{0, 1\} \qquad \forall \{i, j\} \subset N \tag{5}$$

Constraints (2) enforce the asymmetry property for every pair of activities, and Constraints (3) demand that the ordering be transitive (which also eliminates cycles). The precedence constraints in E are imposed via Constraint set (4).

3 Solution methods

Although correct, the formulation (CF) is not very practical due to its non-linear character. We examine two different ways to work around this issue. The first method ensues from applying Dantzig-Wolfe decomposition to formulation (CF), where we

construct the set of all integer solutions $x_j^m = (x_{1j}^m, x_{2j}^m, \ldots, x_{nj}^m)$ to the constraints (4)-(5), $\forall j \in N$ and $m \in M_j$. More specifically, we have $x_j = \sum_{m \in M_j} z_j^m x_j^m$, with $\sum_{m \in M_j} z_j^m = 1$ and $z_i^m \in \{0, 1\}$. This substitution leads to a linear formulation with a large number of columns, which can be solved by means of column generation. The corresponding pricing problem has a non-linear objective, but can be solved using dynamic programming (DP). The search for integral x can then be embedded within a branch-and-price (B&P) framework.

The second solution approach that is tested, is to apply a DP-recursion directly to the original sequencing problem. In detail, each state $Y \subseteq N$ of the recursion represents a smaller-size sequencing instance, where the first $n - |Y|$ positions of the sequence are already filled and the remaining positions are still undecided. The state space contains all the feasible states that can be visited during the diagnosis procedure, where a state Y is feasible if it respects the precedence constraints, that is, $\forall (i, j) \in E : i \in Y \Rightarrow j \in Y$. Similar recursions have been tested earlier for other scheduling problems [8,9], where it was shown that a careful memory management is crucial in the implementation.

4 Main findings

We have performed experiments on benchmark datasets, the results of which will be presented during the conference. As it stands, the DP-recursion with judiciously implemented memory management is the clear winner of the two tested algorithms. The B&P-algorithm does not deliver competitive performance, mainly due to the rather weak LP-bounds. Despite this unfavorable comparison for the linear formulation, the application of similar decompositions for other scheduling problems with a non-linear objective in a compact intuitive formulation, e.g. for project scheduling for maximum net present value [10], has not been attempted yet and might lead to viable procedures (partly also because there are less alternative solution methods available).

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Symmetry breaking in a special case of the RCPSP/max

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1 Introduction

This contribution was motivated by a practical problem arising in scheduling liquid handling systems that automate laboratory procedures. The problem can be modelled quite naturally as a special case of the well-studied Resource Constrained Project Scheduling Problem with Generalized Precedence Relations (RCPSP/max)[1]. We prove the existence of a symmetry in the problem and show how the default search of IBM's CP Optimizer 12.6 [7, 8] benefits from the removal of this symmetry. The approach is tested on 177 real world test instances obtained from our industry partner.

2 Motivation

The liquid handling system is designed to process multiple laboratory tests in parallel with the objective of processing all the tests in the least possible time, i.e. minimising makespan. In order to process a test, a protocol must be followed. This protocol details the set of activities, that must be completed in order to successfully process the test as well as the timings between these activities. Due to the sensitivities of the chemical reactions involved in the tests, these timings specify both a minimum and a maximum amount of time, commonly referred to as minimum and maximum time-lags or generalized precedence constraints.

Scheduling the resources of automated systems, e.g. robots, vacuums, heating/cooling instruments, to process multiple tests in parallel in the least amount of time can be modelled as a special case of the RCPSP/max. In practice schedules must be generated quickly so that systems do not stay idle while a solution is being determined.

3 Problem Description

We are given a set of jobs (tests) $\mathcal{J}$ which must be completed according to a set of protocols $\mathcal{P}$. A job $j \in \mathcal{J}$ must be processed according to its protocol, $p(j) \in \mathcal{P}$. This

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protocol defines both the set of activities, V_j, that must be processed, and the set of precedence relations between these activities, A_j. We denote the set of all activities and precedence relations by $\mathcal{V} = \bigcup_{j \in J} V_j$ and $\mathcal{A} = \bigcup_{j \in J} A_j$ respectively.

A *schedule* is an assignment of start times to each activity, $S = \{S_{i,j} | (i,j) \in \mathcal{V}\}$, where $S_{i,j}$ denotes the start time of activity i from job j. The time at which activity $(i,j) \in \mathcal{V}$ is completed is called the *completion time* and is denoted by $C_{i,j}$. Each activity, $(i,j) \in \mathcal{V}$, has a fixed duration, $d_{i,j}$, and must be scheduled without pre-emption,

$$C_{i,j} = S_{i,j} + d_{i,j}, \quad (i,j) \in \mathcal{V} \tag{1}$$

A generalized precedence relation, $(i,j,i',j') \in \mathcal{A}$, states that between the start time of activity $i \in V_j$ and the start time of activity $i' \in V_{j'}$ there exist minimum and maximum timelags, denoted by $\delta^{min}_{i,j,i',j'}$ and $\delta^{max}_{i,j,i',j'}$ respectively,

$$\delta^{min}_{i,j,i',j'} \leq S_{i',j'} - S_{i,j} \leq \delta^{max}_{i,j,i',j'}, \quad (i,j,i',j') \in \mathcal{A} \tag{2}$$

A schedule is *time feasible* if all of the duration and timelag constraints, i.e. (1) and (2), are feasible.

To process the activities we are given a set of renewable resources $\mathcal{R}$. Each resource $k \in \mathcal{R}$ has a fixed capacity of R_k units. In order to be processed, activity $(i,j) \in \mathcal{V}$ consumes $r_{i,j,k}$ units at the beginning of the activity, $S_{i,j}$, and then releases $r_{i,j,k}$ units at it's completion, $C_{i,j}$. A schedule S is said to be *resource feasible* if at each time t over some time horizon T the demand for resource $k \in \mathcal{R}$, denoted by $r_k(S,t)$, does not exceed the capacity R_k, i.e.,

$$r_k(S,t) = \sum_{(i,j) \in \mathcal{V}: S_{i,j} \leq t < C_{i,j}} r_{i,j,k} \leq R_k, \quad t \in T, k \in \mathcal{R} \tag{3}$$

A schedule S is called *feasible* if it is both time and resource feasible. The objective is to find a feasible schedule with the minimum makespan.

Furthermore protocols are designed in such a way that if the set of jobs contains a single job, $\mathcal{J} = \{j\}$, then the activities in the protocol form a complete linear ordering, i.e. there exists a schedule S such that $S_{1,j} \leq S_{2,j} ... \leq S_{n_j,j}$. For a given schedule, we will refer to jobs in which all activities are ordered by index, as having a *standard ordering*. Additionally the earliest schedule associated with this complete linear ordering is optimal, and can be found in polynomial time by, for example, a modified label correcting algorithm [2].

4 Properties

For the RCPSP/max even finding a feasible solution in general is NP-Complete [1]. However in this special case it is possible to find a feasible solution in polynomial time, albeit a very poor one, by completing each job in standard ordering, one at a time. This is a similar property to that found in the job shop problem with time-lags problem (JSTL) and is referred to as a canonical solution [3]. The problem of finding an optimal solution however remains NP-Hard, as single-machine problems with minimal and maximal timelags between activities are in general NP-Hard [9].

Removing symmetries is a technique used extensively when solving combinatorial optimization problems [4]. A symmetry is a function that maps each solution to

an equivalent solution. Symmetry breaking is the process of eliminating a number of equivalent solutions, ideally all but one, in order to reduce the search space while ensuring that an optimal solution still exists. Historically, the most used method of symmetry breaking involves adding constraints to the basic model.

For project scheduling problems specifically, Kovács and Váncza [5, 6] define the notion of a *progressive pair*, which represents two well-defined sets of activities that characterise repetitive patterns on a graph theoretical basis. They show that symmetries can be removed by inserting additional Start-Start precedence constraints between equivalent activities of the two sets in the progressive pair. Our contribution can be seen as an extension of their work when generalized precedence relations are also considered.

More explicitly, with respect to the problem being considered in this work, we prove the following,

Theorem 1 *Given a feasible schedule S and an arbitrary ordering of jobs O, there exists a feasible permutation of the schedule S^{π}, where for any two jobs, $j, j' \in \mathcal{J}$, such that $j \prec j' \in O$ and both jobs have the same protocol, $p(j) = p(j')$, all activities in j' must start at the same time or after their corresponding activity in j, that is $S^{\pi}_{i,j} \leq S^{\pi}_{i,j'}$ for all $i \in V_j$*

During the conference, a sketch of the proof to Theorem 1 will be provided. It is possible to remove this symmetry by introducing an additional set of Start-Start precedence constraints between corresponding activities. Activity-on-Node (AoN) networks are arc-labelled and node-labelled directed graphs that are commonly used to visualise project scheduling problems. Activities are represented by nodes and precedence relations by arcs. In Figures 1 and 2, we see the AoN for two jobs with the same protocol with and without the additional symmetry breaking constraints.

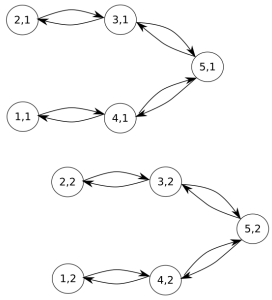

Fig. 1: The AoN for two jobs with the same protocol

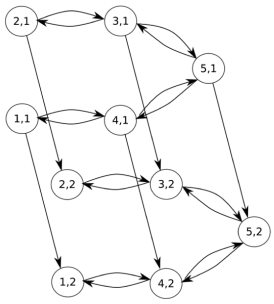

Fig. 2: The AoN for two jobs with the same protocol with the additional set of precedence constraints to break symmetries

5 Computational Study

IBM's CP Optimizer 12.6 was tested on real-world test instances with and without the symmetry breaking constraints. It was found that the approach with the symmetry breaking constraints outperforms the approach without them on 110 of the 177 instances and greatly minimised the standard deviation for the time taken to find an initial feasible solution, 7.3s and 17.6s respectively.

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Mathematical model for a high quality examination timetabling problem: case study of a university in Malaysia

Nurul Farahin Jamaluddin • Nur Aidya Hanum Aizam • Nurul Liyana Abdul Aziz

Abstract This paper presents mathematical programming models for a real-world university examination timetabling problem (UETP) related to a university in Malaysia. The aim of this study is to produce an examination timetable that most closely conforms to the demands of the communities. Hence, a new binary model is developed. The model will consider most of the preferences from the timetabling communities, which complicates the current scheduling system in the related university. We tested the model in two cases of different preferences on a real-world examination timetabling dataset at the university, for verification purposes. Case 1 is tested with single value of preference for all exam assignments to room and timeslot. Case 2 is tested using preferences in the range between 1 (least preferred) and 5 (most preferred) values of exam assignments to room and timeslot. Computational results are reported and analysed directly using the AIMMS mathematical software with CPLEX 12.6.3 as the solver. The results are then compared in term of computational time and the optimal value of objective function achieved to the existing timetable. The study clearly shows that the model generated performed well and the solutions are successfully optimized in which the exams are scheduled in compliance to the student's demand. This model therefore increases the quality level of the examination timetabling.

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Variation of Demands for a New Improvised University Course Timetabling Problem Mathematical Model

Nurul Liyana Abdul Aziz • Nur Aidya Hanum Aizam • Nurul Farahin Jamaluddin

Abstract University course timetabling is a well-known management problem amongst researchers, thus the rich body of literature. However, published articles are mainly on improved solution approaches and ignored human preference. Most researchers present different constraints and ignored human preference. This however, limits the model application to other universities. In our study, we aim to bridge the gap by acknowledging these varieties of demands. In the process of generating our mathematical model, we have gone through meticulously researches that have been carried out in the past years to determine the demands and level of preferences of individuals involved directly with the timetable. The varieties of demands were clarified from a survey conducted. Hence an improvised UCTP model was developed, which involves a super set of constraints. To verify the compatibility of our mathematical model, we illustrate with real data from a Tunisian university. The paper divides the timetabling problem into two parts: case study 1 and case study 2. Case study 1 considers all the requirements of Faculty of Economics and Management Sciences of Sfax (FEMSS) and the resulting timetables were compared qualitatively with those generated by the Timetabling Heuristic Approach. In case study 2, several additional constraints listed are considered. The experimental results confirm the applicability of our improvised model towards real problem.

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MISTA 2017

Multi-commodity flow and station logistics resolution for train unit scheduling

Raymond S K Kwan • Zhiyuan Lin • Pedro J Copado-Mendez • Li Lei

1 Introduction

In UK and many other countries, self-propelled train vehicle units with a fixed number of carriages, e.g. 2-car and 4-car units, are commonly used. This is in contrast to locomotive pulled formations with a variable number of carriages. Scheduling a wide-area rail network with route and time-of-the-day dependent seat demands, optimizing the coupling and de-coupling of train units is often a key feature. Train unit movements are quite restrictive because the tracks they run on are shared. Therefore rather than to create a new empty running journey to redistribute a unit to elsewhere, the unit may be scheduled to be attached to some trains already in the timetable. Also, no additional drivers would be needed. However, a side consequence is that some train trips will be overprovided with seats by such train units in transit to serve the real high demand trips. Since seat demand data is often not easy to determine, inference from schedules in the past is heavily relied upon and overprovision in the current round of scheduling therefore may have long lasting effects on future schedules.

To achieve an optimized flow of train unit resources over the rail network during a working day is complex and difficult. The problem is made more complex in ensuring all the train movements are conflict free at individual train stations. Typical station layouts include tracks that are blind-ended or through running. Some platforms may be short limiting how many train units are allowed to be coupled. To achieve an operable blockage-free schedule may involve reassigning some linkages, re-ordering how multiple units are coupled, shunting units between platforms and sidings, etc. The detailed logistics at each train station obviously could have some rippling effects across all other stations in the network.

A multi-stage approach is proposed for the train unit scheduling problem. In the first stage, a multi-commodity flow problem is solved temporarily regarding each station as a single point with no infrastructure details and without fixing how multi-units are ordered when they are coupled to serve a train trip. The second stage resolves potential station conflicts in the first stage solutions. The resolution process is performed on each individual train station. The results of the second stage would include some alternative resolution plans so that the third

stage process can finalize compatible station plans across the whole network. It is anticipated that because of the real world nature of the scheduling problem, a fourth stage would be needed allowing the human planners to assess and fine tune the schedules interactively.

This paper presents an overall framework and the on-going research on its component stages. The research has been carried out in collaboration with some UK train operating companies and tested on ScotRail, Transpennine Express, Great Western Railway and some past rail franchise bid datasets. Research progress status and relevant results will be presented at the conference.

2 Multi-commodity flow

Given a (tentatively) fixed timetable of train trips for one working day, a directed acyclic graph (DAG) is constructed where the train trips are represented by nodes. A source node and a sink node represent the beginning and end of the working day. Arcs represent potential linkages between a pair of nodes. Different train unit types are multi-commodities. The problem is to select paths from source to sink in the graph such that all the train trips are covered meeting the seat demands. Where the paths overlap, the corresponding train units are coupled subject to compatibility constraints and some coupling bounds.

The multi-commodity flow model is formulated as an integer linear program (ILP) [1]. Apart from minimizing the number of train units used, other quality measures such as total mileage are also incorporated in the objective function. The ILP is computationally very challenging to solve. Therefore, a specialized solver has been derived based on branch-and-price [2]. The main features of the solver include local convex hull techniques [3] for more efficient computation regarding constraints on seat demands and unit coupling type compatibility and bounds. Some specialized branching techniques have also been derived.

In practice, train operators often cannot specify seat demands precisely for each train trip. Deviations from the norm may be caused by many different circumstances and factors. Passenger count surveys are only snapshots that may not always yield accurate inferences. Seat capacities provided in historic schedules may also be unreliable because the scheduling process might have deviated from the seat demands originally specified. Hence, bi-level seat demands are accommodated [4]. For each train trip, the lower minimum seat demand is a hard constraint. A higher seat demand may also be specified such that they would be satisfied as much as possible without using additional train units.

The above solver has demonstrated the ability to solve small to medium sized real-life problem instances within practical time. For larger and harder instances, a hybridized algorithm called SLIM [5] has been developed. SLIM is driven by an iterative improvement heuristics, which aims at converging from a low quality initial reduction of the DAG to a minimally sized DAG that is sufficient to yield a (near) optimal solution. In every iteration, the size-reduced DAG is passed to the core ILP solver above to derive a solution. Because of the aggressive reduction of the DAG, the ILP solver needs little computational time in each iteration. SLIM also benefits from being well suited for parallelization.

On-going research on the ILP solver above includes coping with integer fixed charge variables more efficiently and catering for a richer variety of real-life problem variations and constraints, For SLIM, the focus is on maintaining a good balance between DAG size and search intensification/diversification.

3 Station level logistics

The multi-commodity flow schedule yielded in section 2 has left two operational aspects open to be determined before the solution can be fully operable. First is the unit coupling order in a trip served by multi-units. Second is the precise activity plan required to implement a linkage between an arrival and a departure. For example, suppose a unit arrives on route A and is scheduled to departure on route B next. And suppose route B uses a different platform, the

re-platforming implies some movements of the unit within the station that must not be blocked in any way.

The multi-commodity flow network level solution can be easily transformed into a station-by-station view. At each station, the partial solution consists of a list of arrivals, a list of departures, and a set of linkages connecting the two lists. A linkage also includes information about the unit(s) and platforms assigned. The movements of a unit to implement a linkage is called a "linkage shunting plan" and the collection of linkage shunting plans at a station is called a "station shunting plan". For example, the assignment of a unit on arrival to serve a departure 20 minutes later is a linkage; and if the arrival and departure concerned take place at different platforms, a linkage shunting plan would be needed to re-platform the unit – feasibility of such an activity depends on the time gap available and whether the path of movement is free. Within a station shunting plan, all its linkage shunting plans must be conflict free, i.e. not blocking each other. Since each linkage can have many possible linkage shunting plans, their possible combinations in forming a station shunting plan would be prohibitively numerous to be fully enumerated. Hence, an estimation approach is proposed [6]. The linkages are classified according to characteristics of unit coupling, platform, track type (blind-end or through track) for both the arrival and the departure linked. Each classified linkage type has some associated shunting rules and parameters for determining an estimated minimum shunting time required. Those linkages having time gaps below their corresponding minimum shunting times are deemed infeasible, but they would have been prevented when the DAG was formed. On the other hand, time gaps well above minimum are deemed to pose no problem in deriving a suitable linkage shunting plan. Precise linkage shunting plans are then sought for the remaining linkages, during which relevant unit coupling orders will also be resolved. Finally, any other undetermined unit coupling orders will be determined.

Station logistics requires comprehensive studies of real operations to abstract. Hence, investigations and station site visits with collaborating operators have been carried out and their analyses are on-going.

4 Optimized and operable network-wide train unit schedules

Many UK train operating companies are already using the interactive TRACS-RS [7] system without an optimizer for train unit scheduling. Trials with some collaborating operators have demonstrated that the optimized multi-commodity flow solutions this research produced can be uploaded onto TRACS-RS and the station logistics can be resolved through its interactive facilities. The research described in section 3 will lead to minimal need for interactive station logistic resolution. On-going research is investigating a mathematical approach for finally integrating the prospective individual station logistics. In this approach, the original DAG is transformed into a multi-graph in which some nodes will be extended into multiple nodes representing alternative coupling orders. The objective is to find an optimal selection of alternative coupling order nodes and their associated arcs to be used across the network.

Data statement Part of the data used for this research may be commercially sensitive. Where possible, the data that can be made publicly available is deposited in http://archive.researchdata.leeds.ac.uk/.

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On the exact solution of a large class of parallel machine scheduling problems

Teobaldo Bulhões · Ruslan Sadykov · Eduardo Uchoa · Anand Subramanian

1 Introduction

We consider the problem of scheduling a set of jobs J ($n = |J|$) on a set of machines of different types $k \in M$ ($m = |M|$) without preemption. A job j is not allowed to be processed before its release date r_j, and its processing time on a machine of type $k \in M$ is denoted as p_j^k. Also, s_{ij}^k denotes the setup time required to process job j immediately after job i on a machine of type k. Each job j is associated to a cost function $f_j(C_j)$ defined over its completion time C_j. The objective function is to minimize $\sum_{j \in J} f_j(C_j)$. This function is very general and can model many criteria. For example, suppose each job has an earliness $E_j = \max\{d_j - C_j, 0\}$ and a tardiness $T_j = \max\{C_j - d_j, 0\}$ that is computed based on its due date d_j. A classical objective is to minimize the total weighted earliness and tardiness given by $\sum_{j \in J}(w'_j E_j + w_j T_j)$, where w'_j and w_j are penalty coefficients associated with job j. Remark that cost functions that include earliness penalties are not regular and may have optimal solutions that include idle times between jobs. In this work we present a novel exact algorithm that is capable of solving problem $R|r_j, s_{ij}^k| \sum f_j(C_j)$ and the large class of problems that can be derived as particular cases from it. The proposed algorithm consists of a branch-cut-and-price approach that combines several features such as non-robust cuts, strong branching, reduced cost fixing and dual stabilization. To our knowledge, this is the first exact algorithm for unrelated machines with earliness and/or tardiness that can solve consistently instances with more than 20 jobs. We report improved bounds for instances of problems $R|r_j, s_{ij}^k| \sum w'_j E_j + w_j T_j$ and $R|| \sum w'_j E_j + w_j T_j$ with up to 80 and 120 jobs, respectively.

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2 Mathematical formulation

We start by defining some notation required to introduce the mathematical formulation. Let T be an upper bound on the maximum completion time of a job in some optimal solution. Let $N_k = (V_k = R_k \cup O_k, A_k = A_k^1 \cup A_k^2 \cup A_k^3 \cup A_k^4)$ be the acyclic graph associated with each machine type $k \in M$, where set $R_k = \{(j,t,k) : j \in J, t = r_j + p_j, \ldots, T\}$ contains the vertices associated with the jobs. We adopt the convention that idle times only occur after the job has been processed. We assume that job j has been processed when arriving at vertex (j,t,k), but note that j did not necessarily finished at time t because of the existence of idle times. Set $O_k = \{(0,t,k) : t = 0,\ldots,T\}$ contains the vertices associated with a dummy job 0. For brevity, we denote arc $((i,t,k),(j,t+s_{ij}^k+p_j^k,k))$ as (i,j,t,k). Set $A_k^1 = \{(i,j,t,k) = ((i,t,k),(j,t+s_{ij}^k+p_j^k,k)) : (i,t,k) \in R_k, (j,t+s_{ij}^k+p_j^k,k) \in R_k, j \in J \setminus \{i\}\}$ contains the arcs connecting the vertices of R_k. Set $A_k^2 = \{(0,j,t,k) = ((0,t,k),(j,t+s_{0j}^k+p_j^k,k)) : (j,t+s_{0j}^k+p_j^k,k) \in R_k, j \in J\}$ contains all arcs connecting the vertices from O_k to R_k. Set $A_k^3 = \{(j,0,t,k) = ((j,t,k),(0,T,k)) : (j,t,k) \in R_k\}$ contains all arcs connecting the vertices from R_k to O_k. Set $A_k^4 = \{(j,j,t,k) = ((j,t,k),(j,t+1,k)) : (j,t,k) \in R_k \cup O_k, (j,t+1,k) \in R_k \cup O_k\}$ contains the arcs associated with idle times. Let f_a be the cost of an arc $a = (i,j,t,k) \in A_k^1 \cup A_k^2$, which is the cost incurred if job j finishes to be processed at time $t + s_{ij}^k + p_j^k$. Note that $f_a = 0, \forall a \in A_k^3 \cup A_k^4$. Moreover, let set $R_k^j = \{(i,t,k) \in R_k : i = j\}$ denote the vertices associated with job j. Finally, for each subset $S \subseteq V_k$, let $\delta^-(S)$ and $\delta^+(S)$ be the sets representing the arcs entering and leaving S, respectively. The proposed arc-time-indexed formulation is as follows.

$$\text{(F1)} \quad \min \sum_{k \in M} \sum_{a \in A_k} f_a x_a \tag{1}$$

$$\text{s.t.} \quad \sum_{k \in M} \sum_{a \in \delta^-(R_k^j)} x_a = 1, \qquad \forall j \in J \tag{2}$$

$$\sum_{a \in A_k^2} x_a \leq m_k, \qquad \forall k \in M \tag{3}$$

$$\sum_{a \in \delta^-(\{v\})} x_a - \sum_{a \in \delta^+(\{v\})} x_a = 0, \qquad \forall k \in M, \forall v \in V_k \setminus \{(0,0,k),(0,T,k)\} \tag{4}$$

$$x \geq 0, x \text{ integer} \tag{5}$$

Objective function (1) minimizes the completion time dependent costs. Constraints (2) state that each job $j \in J$ must be processed exactly once. Constraints (3) impose that at most m_k machines of type $k \in M$ can be used. Constraints (4) are related to the flow conservation. Define P_k as the set of paths in graph N_k that start at vertex $(0,0,k)$ and end at vertex $(0,T,k)$. Let b_p^a be the number of times path p traverses $a \in A_k$ and let λ_p be a binary variable that assumes value 1 if p is in the solution. Formulation F1 can be rewritten in terms of variables λ_p by means of a Dantzig-Wolfe decomposition as follows.

$$\text{(DW-F1)} \quad \min \sum_{k \in M} \sum_{p \in P_k} \Big(\sum_{a \in A_k} b_a^p f_a \Big) \lambda_p \tag{6}$$

$$\text{s.t.} \quad \sum_{k\in M}\sum_{p\in P_k}\Big(\sum_{a\in\delta^-(R_k^j)} b_a^p\Big)\lambda_p = 1, \qquad \forall j \in J \tag{7}$$

$$\sum_{p\in P_k}\Big(\sum_{a\in A_k^2} b_a^p\Big)\lambda_p \le m_k, \qquad \forall k \in M \tag{8}$$

$$\lambda \text{ binary} \tag{9}$$

3 Branch-cut-and-price algorithm

This section briefly describes the proposed branch-cut-and-price (BCP) algorithm over formulation DW-F1. The solution of every node has the following three steps. **(i)** A lower bound for the problem is obtained by solving the linear relaxation of DW-F1 (possibly with cuts and branching constraints) through column generation. The pricing subproblem for a machine type $k \in M$ corresponds to finding a least-cost path in graph N_k where arc costs are associated with reduced costs. In this step, the pricing subproblem is solved by a labeling algorithm and a dual stabilization technique is applied. **(ii)** A reduced cost fixing procedure is executed for every machine type $k \in M$ to remove from graph N_k the arcs that can not be in a solution that improve the current upper bound. **(iii)** Cuts are separated and added to the master problem. Steps **(i)**, **(ii)** and **(iii)** are repeated as long as a tailing off condition is not reached and the pricing time is smaller than a predefined threshold, after which a strong branching technique is applied. The cuts adopted are obtained by a Chvátal-Gomory rounding of the n constraints (7). Such cuts are often referred to as non-robust, since each single cut requires additional states in the dynamic programming algorithm used for solving the pricing subproblem. Therefore, those cuts should be added in a limited fashion to avoid combinatorial explosion. To mitigate the impact of the Rank-1 cuts on the pricing subproblem, we have adopted the limited arc memory technique described in [4].

4 Computational results

Table 1 reports preliminary results obtained by the proposed BCP over the $R|r_j, s_{ij}^k|\sum w'_jE_j + w_jT_j$ instances of [3] and the $R||\sum w'_jE_j + w_jT_j$ instances of [1]. The former instances were derived from the instances of [1] by adding two types of sequence-dependent setup times: small and large. All experiments were conducted on a Intel Xeon E5-2680 v3, running at 2.5 GHz with a single thread and a time limit of 12 hours. In this table, **Improv (%)** denotes the average percentage improvement over the best known solution (BKS) which were obtained by running the methods devised in [1,2], and **#New** corresponds to the number of new improved solutions. Regarding the $R|r_j, s_{ij}^k|\sum w'_jE_j + w_jT_j$ instances, it can be observed that most of them were solved to optimality and several new improved solutions were found. Also, it appears that the instances with large setup times are harder to be solved. More precisely, we can verify that the quality of the lower bound obtained by BCP as well as the upper bound found by running the heuristic proposed in [2] are worse for such instances. On the other hand, only 8 $R||\sum w'_jE_j + w_jT_j$ instances considered were not solved to optimality. In contrast, 497 such instances were not solved by the method presented

in [1]. Moreover, the use of Rank-1 cuts improved significantly the performance of our method, resulting in much smaller BCP trees and CPU times.

Table 1 Average results for $R|r_j, s^k_{ij}|\sum w'_j E_j + w_j T_j$ and $R||\sum w'_j E_j + w_j T_j$

Instances			BCP				BKS		[1]		
n	m	Setups	#Solved	Gap (%)	Time (s)	#Nodes	Improv. (%)	#New	#Solved	Gap (%)	Time (s)
40	2	small	60/60	0.00	224	1.10	0.120	22	-	-	-
60	2	small	60/60	0.00	1695	3.53	0.330	42	-	-	-
60	3	small	60/60	0.00	2109	10.6	0.408	47	-	-	-
80	2	small	60/60	0.00	5832	5.70	0.136	41	-	-	-
80	4	small	48/60	0.52	16368	91.93	0.264	50	-	-	-
40	2	large	60/60	0.00	931	2.80	0.760	46	-	-	-
60	2	large	58/60	0.06	10589	23.16	1.340	58	-	-	-
60	3	large	45/60	1.21	20691	85.76	1.560	55	-	-	-
80	2	large	28/60	1.32	35368	48.76	0.798	54	-	-	-
80	4	large	10/60	3.91	39479	120.43	0.385	27	-	-	-
40	2	no	60/60	0.00	312	3.37	0.000	0	26/60	0.16	52
60	2	no	60/60	0.00	708	3.27	0.001	1	7/60	0.89	109
60	3	no	60/60	0.00	428	2.93	0.010	5	7/60	0.82	120
80	2	no	59/60	0.00	2425	5.36	0.003	3	2/60	0.90	134
80	4	no	60/60	0.00	964	3.86	0.063	15	0/60	4.54	228
90	3	no	60/60	0.00	2018	4.70	0.033	20	1/60	2.52	153
100	5	no	59/60	0.02	3397	26.73	0.103	27	0/60	8.83	297
120	3	no	56/60	0.04	10775	16.72	0.072	22	0/60	4.12	165
120	4	no	58/60	0.007	7944	17.66	0.171	31	0/60	6.98	217

As for future work, we intend to test our algorithm on other particular cases such as the single and parallel machine problems studied in [6] and [5], respectively.

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Robotic Cell Scheduling Considering Energy Consumption of Robot Moves

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1 Introduction

We consider a robotic manufacturing cell consisting of serial machines that produce identical parts. A material handling robot is responsible for loading/unloading the machines and transferring the parts between the machines. Most of the studies in this literature consider the problem of determining the sequence of robot moves that maximizes the throughput rate. These studies assume that the robot moves are performed with its maximum speed, which leads to higher energy consumption cost. However, reducing the speed of some moves may lead the same production rate with a lower cost. In this study, we consider the problem of determining both the optimal robot speeds and the optimal robot move sequence. The objectives to be considered are the minimization of the total energy consumption and maximization of the throughput rate. The energy consumption function is convex and nonlinear with respect to the robot speed. We study the tradeoff between the throughput rate and the energy consumption of the robot. We utilize the epsilon constraint approach for this bi-criteria problem by writing the throughput objective as a constraint and formulate the problem as a Mixed Integer Nonlinear Program for the general *m*-machine cells. We consider two-machine cells in detail and develop analytical results by utilizing the KKT optimality conditions. We develop a heuristic approximation algorithm to determine the optimal robot speeds for each possible robot move cycle corresponding to a given throughput rate.

Acknowledgements

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Scheduling container transportation through conflict-free trajectories in a warehouse layout: A local search approach

Emmanouil Thanos · Tony Wauters · Greet Vanden Berghe

1 Introduction

Container storage and transportation procedures constitute a crucial element of warehouse operations. Efficient transport scheduling significantly reduces serving time in settings which involve large numbers of dynamic orders. Due to limited warehouse space, container stacks frequently form highly-confined traffic networks for transferring vehicles. Pickup, relocation and dispatching operations are further constrained by precedence and piling restrictions arising from containers' physical properties and their positions within the stacks. These factors impose great difficulties in terms of finding globally optimal solutions for real-time applications.

A significant number of heuristic approaches have been developed for addressing scheduling and routing of Autonomous Agents and Automated Guided Vehicles [7]. The two components are generally handled at distinct decision levels: vehicles are first assigned to transportation requests and their routing is subsequently determined to ensure good quality conflict-free paths for each request while also respecting deadlines or priorities [2, 5, 8]. Less frequently conflict inspection precedes scheduling to completely eliminate overlapping routes [6], thus possibly excluding good-quality solutions which could be resolved in a warehouse context by adding waiting times. Other notable works focusing on conflict resolution utilize fixed itineraries, assuming vehicles' assignments are predetermined [1, 3, 4]. Concerning container relocations, extensive research has revealed that there has been insufficient research towards integrated approaches capable of simultaneously addressing relocation and routing objectives. An extended literature review and research justification will be presented within the full paper.

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The present work proposes an integrated Local Search (LS) approach for scheduling container boxes' transport requests in a warehouse layout. The model formulates the detailed setting of a warehouse traffic network including stacks of container boxes, alongside operational vehicles and their pickup, placement and transfer functions, as detailed in Section 2. The formulation presented in Section 3 incorporates multiple real-world movement restrictions for vehicles, different direction and conflict rules for the network's edges, in addition to precedence constraints and piling restrictions concerning container boxes' physical properties and their positions within the stacks.

The proposed approach, detailed throughout Section 4, models LS neighborhoods which permit various moves for the existing schedule. Possible decisions are execution order, vehicles assignments, dispatching locations or path selections for the scheduled transports. At each LS node the developed algorithm constructs the routing schedule for the utilized vehicles and imposes waiting times to resolve conflicts and maintain safety distances, while satisfying all precedence constraints and piling restrictions. Constructed schedules are evaluated in terms of their total makespan.

2 General setting

The problem considers a warehouse where container boxes of various sizes are stored, as illustrated in **Fig. 1**. There exist one or more entrances where boxes arrive and from where they are stored, as well as exit points where boxes must be transfered to before being sent out to customers. Stored boxes are piled into numerous column stacks at fixed locations within the warehouse. Stack locations, entrances and exit points are connected through multiple narrow aisles while sequences of neighboring stacks form additional aisles. Within stack aisles, fixed distances are assumed between neighboring stacks.

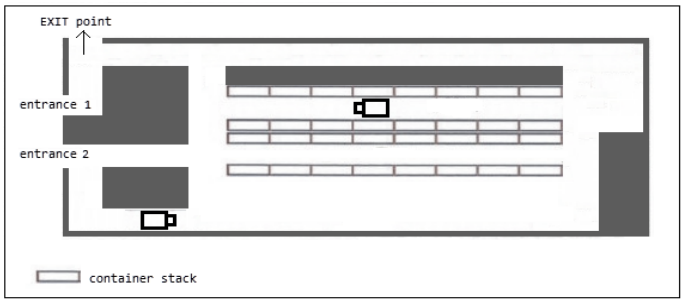

Fig. 1 Warehouse setting

All transports are executed by a limited number N_V of special, small single-type vehicles which may be autonomously or manually guided. They move forward and backwards through the aisles, with a reduced backward speed and sequentially perform box pickups and deliveries. Within the problem's network graph, edges therefore correspond to the aforementioned aisles, as shown in **Fig. 2**. Graph nodes represent the entrances, exit points, intersections and stack locations, with each edge connecting exactly two graph nodes. Each stack is associated with a sequence of container boxes,

while boxes are similarly associated with their current stack, their position within the stack and certain physical properties such as height and weight. Placing a box atop a stack may not be permitted, depending on the physical properties of the box to be placed and those currently located in the stack.

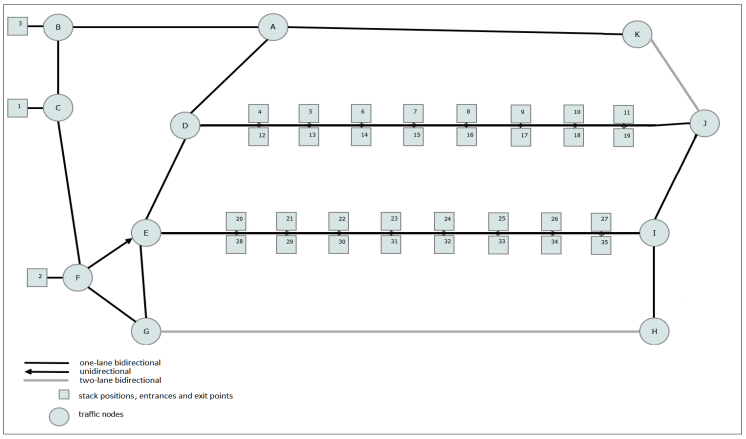

Fig. 2 Network graph

Depending on aisles' position and width, their corresponding edges may be unidirectional, one-lane bidirectional or two-lane bidirectional. At certain intersections they may additionally form narrow angles which prevent vehicles from taking direct turns. In such cases vehicles are forced to first turn onto a different edge at the intersection before reversing to continue onto the target edge.

Conflict possibilities exist when two vehicles are scheduled to traverse the same edge simultaneously. When vehicles are moving towards the same direction or they are performing a pickup/placement operation, conflicts may occur for any edge if the minimum safety distance is not preserved. When they travel in opposite directions, conflicts may occur only on one-lane bidirectional edges at any point the two vehicles employ the edge simultaneously.

3 Model

All the aforementioned features are incorporated into the construction of paths for the network graph. Path generation occurs as a preprocessing step of the proposed algorithm and involves the construction of the set S_P of all possible paths between each pair of graph nodes and for both starting orientations. Generated path instances contain all the required information concerning how vehicles must traverse graph edges from source to destination node, namely: moving directions and orientations, potential waiting times for pickup/placement operations and travel durations on path edges.

Transport requests express the requirement of transferring a container box from its current location (entrance/stack) to a set of possible destinations (stacks/exit points). A request r is defined by a pickup location PU_{loc}, a set of possible destination locations

S_{PL} and a container box B. Each request must be assigned to exactly one vehicle which must travel from its current location to PU_{loc}, pick up B from the top of the associated stack, travel from PU_{loc} to a location $PL_{loc} \in S_{PL}$ and finally unload B atop the destination stack. Pickup and placement completion times Tr_{pu} and Tr_{pl} are considered equal to the ending times of the corresponding operations.

A precedence constraint is a triple $< op, r1, r2 >$, where op is an operation (PU/PL) and $r1$ and $r2$ are transport requests. It expresses the requirement that the defined operation associated with $r1$ must be executed before $r2$'s. For the purposes of this work, it is assumed that the set S_R of transport requests to be processed and scheduled at each call of the LS algorithm includes all the auxiliary requests potentially arisen due to relocation requirements, with the size of typical instances reaching up to approximately 150. The set S_C of precedence constraints possibly restricting their execution order is generated upon S_R and the requirement that all container boxes are at the top of the right stack at their associated request's pickup completion time.

A route is a path with waiting times possibly enforced in-between its pairs of consecutive edges. A routing schedule consists of an assignment of each transport request $r \in S_R$ to a vehicle V, in addition to the pair of routes which must be followed to execute r's pickup and placement operations and a triple $< TO, Tr_{pu}, Tr_{pl} >$ of timestamps defining take-off time, pickup and placement completion time, respectively. A routing schedule is valid if (i) all precedence constraints are satisfied by the resulting execution order, (ii) all placement operations are feasible at the time they occur and (iii) all pairs of overlapping routes are conflict-free.

Given a warehouse network graph modeled with the aforementioned properties, the set of paths S_P, the set of transport requests S_R, the set of precedence constraints S_C and the number of available vehicles N_V, the problem consists of finding a valid routing schedule which minimizes the total makespan. The total makespan M is equivalent to the latest placement completion time of the schedule and calculated using Equation 1 as follows:

$$M = \max_{r \in S_R} Tr_{pl} \tag{1}$$

4 Local search framework

The general idea of the proposed LS algorithm is that an LS node indirectly defines a routing schedule by gradually routing all available vehicles. An LS node is defined by:

- a vector R_{ins} of all requests to be inserted into the schedule, defining the order in which they will be processed
- for each $r \in R_{ins}$, a quadruple $< V, PL_{loc}, PATH_{pu}, PATH_{pl} >$,
  where V is the vehicle assigned to r, $PL_{loc} \in S_{PL}$ the selected destination for placing box B, $PATH_{pu}$ the path V will follow to move from its current location to PU_{loc} and $PATH_{pl}$ the path V will follow to move from PU_{loc} to PL_{loc}

For the evaluation of a candidate node, all requests in R_{ins} are sequentially inserted into the initially empty schedule in the following manner: beginning from the ending time of the latest route associated with V, every edge in $PATH_{pu}$ is iteratively checked for conflicts with regard to the routes of every other vehicle. Whenever a conflict is encountered, the problematic edge is shifted later into V's route imposing waiting times for V on $PATH_{pu}$'s graph nodes. If the conflict is not resolvable then a deadlock is

detected and the routing schedule is rendered invalid. The ending time of the last edge of $PATH_{pu}$, which contains the pickup operation, is stored as pickup completion time Tr_{pu} of r. V is similarly routed through $PATH_{pl}$ to provide placement completion time Tr_{pl}. After each operation the data concerning associated stacks and containers is updated.The cost of a node is the total makespan of its resulting routing schedule, calculated using Equation 1.

An LS node n is feasible if the resulting routing schedule is valid. The neighborhood $N(n)$ of n is the set of all feasible nodes obtainable from n by applying one of the following operations:

- MOVE_BACK(r) : move r one position back in R_{ins}
- MOVE_FORWARD(r) : move r one position forward in R_{ins}
- CHANGE_VEHICLE(r, V') : assign a different vehicle V' to r. $PATH_{pu}$ is assigned default value.
- CHANGE_PLloc(r, L') : select a different placement location $L' \in S_{PL}$ for r. $PATH_{pl}$ is assigned default value.
- CHANGE_PUpath(r, P') : select a different path P' connecting V's starting location with PU_{loc}
- CHANGE_PLpath(r, P') : select a different path P' connecting PU_{loc} with PL_{loc}.

Beginning from an initial LS node with default values, the neighborhood of the current node is explored at each iteration. During the search different variable selection strategies are applied depending on the schedule's current state. Simple value selection strategies are utilized for each variable based on path lengths, location proximity, vehicle availability and stack loads, properties which are also employed to provide default values when required. The LS is incorporated within a meta-heuristic, the details and performance of which will be presented at the conference. Nodes are selected for expansion according to the evaluation function and the utilized meta-heuristic.

Concerning schedule construction, multiple techniques are employed to reduce recalculations and accelerate the conflict inspection procedure. For problems involving small numbers of vehicles, the possibility of calculating time-difference windows for all pairs of paths at the preprocessing step is also examined. All optimization techniques will also be detailed during the presentation.

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An Integrated Optimization of Dynamic Product Family Configuration and Supply Chain Configuration

Takuya Tsuboi · Tatsushi Nishi · Yoshitaka Tanimizu · Toshiya Kaihara

Abstract In this paper, we consider an integrated optimization of dynamic product family configuration and supply chain configuration. The problem is formulated as a mixed integer linear programming model. Two types of methods: a sequential method and a simultaneous method are applied to solve the problem. Computational experiments are provided to show the effectiveness of the simultaneous method from practical data.

1 Introduction

Mass customization is a strategy to meet customer's needs while mass-producing products. One of the methods to achieve mass customization is a modular production. In the method, a variety of final products are manufactured by assembling some modules. Therefore, the optimal planning of Bill of Materials (BOM) is required. The number of product types is limited due to the budget and common modules are available even if different products are manufactured. The product family configuration determines the selection of final products and their modules under the budget constraint. The supply chain configuration is concerned with the selection of production facilities and the production quantity of modules and products to be manufactured. Conventionally, the product family configuration and the supply chain configuration decisions have been regarded as separated problems. In many cases, a sequential method is adopted

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to determine the supply chain decision after the product family configuration is determined. However, it is expected that the profit becomes larger by simultaneously optimizing the dynamic product family configuration and supply chain configuration taking into account for production cost, transportation cost and inventory cost. Hence, the integrated optimization of dynamic product family configuration and supply chain configuration is required. Fujita et al. (2013) formulated a simultaneous decision problem of product family configuration and supply chain configuration as a mixed integer nonlinear programing problem [1]. Bertrand et al. (2016) proposed a mixed integer linear programing problem for the integrated optimization problem [2]. A Stackelberg game model [3], a two-phase approach [4] were used to solve the integrated problems. However, these studies considered the problem within a single period. In this paper, we propose a multi-period model for simultaneous optimization of product family configuration and supply chain configuration.

2 Mathematical model

Product family configuration is the selection of products to be manufactured and the construction of the BOM for each product. The objective of product family configuration problem is to maximize the profit by selecting the products to sell to customers and reducing the costs related to the development and manufacturing of modules and products. The module is a part of product and it has several functions. Each module possesses a substitutability. Therefore, each product can be manufactured by assembling some modules selected from candidate modules. Unit production cost of module and unit assembly cost of product are incurred in the supply chain configuration. On the other hand, the development and manufacturing costs are incurred for each type of selected modules and products in the product family configuration. We need to select the best modules for final products to be manufactured from the combination of modules. The supply chain configuration concerns with the operation of production facilities, the production quantity of modules and products, product inventory, transportation quantity of the modules and product sales. In the model, there are multiple production facilities. The modules are produced at the supplier and the products are manufactured at the production sites and they are directly solved to customers. Therefore, transportation quantity of modules from a supplier to a production facility and sales of products are also included in supply chain configuration. The total profit to be maximized is the sales profit minus the total cost. The total cost is composed of production cost, transportation cost, assembly cost, inventory cost, distribution cost, facility maintenance cost, development and manufacturing cost. In the proposed model, the product family configuration and supply chain configuration can be changed dynamically according to the cost and the demand for each product. The proposed model has two major characteristics. First, we can only change the product family configuration in some specific periods because the product family configuration cannot be changed frequently in practice. Therefore, the same product family is used until the next product family determination period once the product family configuration is determined. Second, since we assume a budget constraint for all time periods, interactions in different decision timing are considered. The constraints in the proposed model are listed as follows:

1. All selected modules must be produced when manufacturing a product.
2. The modules and products can be produced within a finite capacity.

3. The production quantity of products at a facility is equal to the quantity of corresponding modules.
4. Inventory balancing constraint in each period.
5. To manufacture selected products to satisfy customers' demands in each period

The problem is formulated as a mixed integer linear programming problem and it is solved by a general-purpose solver.

3 Computational results

We investigate the effect of the simultaneous optimization. The proposed model was solved by both a sequential method (SDM) and a simultaneous method (SOM). In SDM, we determine the product family configuration first. Then, the optimal supply chain configuration is determined to maximize the total profit when the product family configuration is fixed. In SOM, we determine the product family configuration and the supply chain configuration simultaneously to maximize the total profit. Three cases are assumed. The parameters for each case are shown in Table 1.

Table 1 Planning horizon and product family determination periods for three cases

Case	1	2	3
Planning horizon	4	6	4
Product family determination periods	1, 3	1, 3, 5	1, 2, 3, 4

Table 2 Computational result of Case 1,2

Solution method	SDM	SOM	SDM	SOM
Case	1	1	2	2
Sales profit	545460.8	605349.6	805145.6	874458.3
Total cost	221825.0	242600.5	324912.7	348065.2
Total profit	323635.8	362749.1	480232.9	526393.1
Computational time(s)	0.59	32.32	1.38	2470.38

Table 2 shows the computational results of SDM and SOM for each case. SOM obtains higher total profit in the two cases. This is because final products are manufactured with less costs and delivered to customers with higher price in SOM. It means that it is possible to manufacture larger number of products by using the budget efficiently than SDM. On the other hand, the computational time of SDM is shorter than that of SOM. In SDM, the 0-1 knapsack problem is solved when the product family configuration is determined. There are smaller number of binary variables when the product family configuration is determined. The computational time of SOM significantly increases as the number of the product family determination period increases.

We investigate the validity of the multi-period model. The performance of the single period model and the multi-period model are compared in Table 3. Both models utilize the SOM. In the single period model, the single period problem is solved in each period. The result of single period model is represented by the sum of the results

for each period. It is confirmed from Table 3 that the profit is increased by the multi-period problem. The total profit of multi-period model is larger than that of the single period model because inventories are efficiently used to deliver the products when the sale price is higher by stocking them as inventories. The effectiveness of SOM with the multi-period model is confirmed.

Table 3 Comparison of results between single period model and multi-period model

Model	Single period	Multi-period
Case	3	3
Sales profit	541938.2	638604.3
Development cost	26297.9	31660.7
Production cost	65587.6	79990.7
Assembly cost	24414.0	30117.7
Inventory cost	0.0	988.8
Transportation cost	43653.5	54510.4
Distribution cost	15783.6	18761.3
Facility maintenance cost	32157.0	40339.6
Total cost	207893.6	256368.8
Total profit	334044.5	382235.4
Computational time(s)	2.11	724.09

4 Conclusion

In this paper, an integrated optimization of dynamic product family configuration and supply chain configuration model has been developed. We have shown that a simultaneous determination of the product family configuration and supply chain configuration leads to larger total profit. The performance of the simultaneous optimization method is superior to a conventional sequential method from computational results. On the other hand, it will be necessary to develop an efficient solution approach since the total computational time for the simultaneous method is so large.

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A Batch-oblivious Approach For Scheduling Complex Job-Shops with Batching Machines: From Non-delay to Active Scheduling

Karim Tamssaouet · Stéphane Dauzère-Pérès · Claude Yugma · Jacques Pinaton

1 Introduction and Problem Description

The production of microelectronic devices is a highly complicated and cost intensive process, particularly in the front-end where the processing of wafers takes place. Depending on the product types and on the technology, a wafer goes through more than 600 process steps over a period of a few weeks. In this context, scheduling decisions have a substantial impact on key performance indicators such as throughput and cycle time, see Monch et al. (2011). Among the complex workshops in a wafer manufacturing facility, the diffusion area is of critical importance. The processes in this area are performed on two types of equipment: cleaning and furnace machines.These machines can process several lots simultaneously. Moreover, the processing durations can be very long compared to other operations in the fab. Due to the dynamic arrival of jobs, an important decision is whether it makes sense to wait for the arrival of future jobs in the case of incomplete batches, or to start non-full batches.

This work deals with a real-life complex scheduling problem arising in the diffusion area, but can be applied in many other industrial contexts. We consider scheduling a set of jobs that have different sizes, different priorities and arrive dynamically to the work area. Each job is associated with a route that describes the sequence of operations that the job should go through. Each operation can only be performed on a set of qualified machines, and its processing duration depends on the selected machine. A significant constraint to consider is p-batching, the machines can process multiple operations in parallel. On a qualified batching machine, an operation can be batched only with a subset of operations with the same batch family. Some of the machines are subject to sequence-dependent setups, such that the amount of time required to configure a machine to process a subsequent operation depends upon the machine's

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configuration that was used by the precedent operation. We aim to optimize different regular mono-objective functions from the literature.

Our proposition relies on the novel approach proposed by Knopp et al. (2017) for complex job-shop problems with batching machines. In this approach, an adaptation of the classical conjunctive graph, introduced by Roy and Sussmann (1964), is made to represent batches through arc labels. Using this new representation, called batch-oblivious graph, the schedules are constructed and improved during the graph traversal. The obtained schedules are non-delay in the sense that no machine is kept idle while an operation is waiting for processing. So instead of non-delay schedules, this work proposes generating active schedules by inserting an idle time if it is relevant to increase the batch size. This new way of constructing schedules aims at reaching quickly good solutions within a heuristic.

2 Batch Oblivious Disjunctive Graph

Most existing solution approaches for Complex Job-Shop scheduling problems with batching machines rely on the disjunctive graph representation of Ovacik and Uzsoy (2012). This representation introduces dedicated nodes to represent explicitly batching decisions.A novel batch-oblivious modeling is presented by Knopp et al. (2017). Like a classical conjunctive graph, the batch-oblivious conjunctive graph uses nodes to uniquely represent operations and arcs to represent precedence constraints on routes and resources. Instead of inserting additional nodes and arcs to model batches, they are coded in the arc weights. This new representation has many advantages. It reduces the structural complexity of the graph. It allows reusing ideas and techniques for a less complex problem like the move proposed by Dauzère-Pérès and Paulli (1997) for the flexible job shop scheduling problem. Last but not least, it is possible to propose an integrated algorithm that computes start dates and improves the solution during the graph traversal by filling underutilized batches through a combined resequencing and reassignment strategy. In this work, we adopt the same representation in the form of a batch-oblivious modeling. We propose a new integrated algorithm that modifies the solution differently during the graph traversal.

3 Towards Active Schedules

First, it is important to clarify the notion of active and non-delay schedules within this work. The schedules that are provided by the construction algorithm of Knopp et al. (2017) are qualified as non-delay schedules. When a node is visited during the graph traversal, the construction algorithm gives this node the earliest start date. The last condition only defines what is called semi-active schedules in the scheduling literature (Pinedo (2016)). So, in addition to the earliest start date, when the algorithm searches for potential operations that can complete a non-full batch, it advances only those that can be integrated to the batch without delaying its start time.

Theoretically, it is always possible to construct an optimal solution using this construction algorithm on the batch-oblivious conjunctive graph. However, it may take a lot of moves before obtaining such a solution. A new construction algorithm that outputs active schedules can constitute a useful shortcut that accelerates the search by quickly constructing good solutions. The schedules are called active as we intentionally

insert idle times. Instead of restricting the search to operations that can complete a batch without delaying its start date, we extend the search for any operation that can be added to the batch without delaying its start date by more than a given maximal delay.

Whatever the situation, we assume that there is no benefit in delaying the processing starting date of a batch by more than its processing duration. If this maximal delay is fixed to zero, we get back to the non-delay construction algorithm. We then want to quickly construct a near-optimal solution by providing as parameter the maximal delay for each batch family that is between these limits. Similar to the proposed idea by Cigolini et al. (2002) for online scheduling on batching machines, instead of being constant, this parameter is used to calculate actual maximal delays that depend on the batch filling ratio.

We utilize a heuristic based on the idea of Greedy Randomized Adaptive Search Procedure by Feo and Resende (1995). A starting construction heuristic, as the one proposed by Yugma et al. (2012), creates the initial solutions: It ranks the jobs in the order of their ratio between the due date and weight, and the best insertion position is selected for each operation . To improve an initial solution, we use simulated annealing. As neighborhood operator, we use the integrated move defined by Dauzère-Pérès and Paulli (1997) and that does not differentiate between reassignment and resequencing of operations.

4 Preliminary Results and Perspectives

We partially implemented our proposition and conducted preliminary numerical experiments using the instances given by Knopp et al. (2017). In this set, there are 15 industrial instances and 15 randomly generated ones. As a regular objective function, we chose the total weighted completion time. The results are preliminary because of the partial implementation of the idea. We restricted the possibility of delaying the processing start date to batches that only have one operation. We still then use the non-delay strategy in case of all non-full batches of more than one operation on machines that can process more than two batches.

The obtained results are summarized in Table 1. It shows the comparison between the obtained solutions by the active construction algorithm versus the ones obtained by the non-delay construction according to several indicators. For the active algorithm, each instance is solved in four runs by changing the maximal delay: a batch can be delayed by at most 25% , 50%, 75% or less strictly that its processing duration.

	$\leq 25\%$	$\leq 50\%$	$\leq 75\%$	$< 100\ \%$
Average Relative Deviation	-0,55%	-0,15%	-0,20%	-0,43%
# Improved Solutions	9	3	7	9
Minimum Relative Deviation	-16,31%	-16,36%	-16,24%	-11,99%
# Deteriorated solutions	8	9	6	6
Maximum Relative Deviation	7,21%	7,21%	9,57%	7,72%

Table 1 Comparison between non-delay construction algorithm and active construction one using different maximal delays

The relative deviation is computed as the $(f(S) - f(S_{ref}))/f(S_{ref})$, where S is the active schedule, S_{ref} is the non-delay solution and $f()$ is the regular objective function. In all cases, the average relative deviation shows the dominance of the active construction algorithm. Depending on the accepted maximal delay, we can notice that the results are quite different. If we take the case where the maximal delay is fixed as the processing duration ($< 100\%$), the active construction algorithm is better than the non-delay one. 9 instances over 30 are improved while solutions of less quality are obtained for only 6 instances with the worst relative deviation of 7.8%. The same conclusion can be made about the case where the maximal delay is limited to the quarter of the processing duration.

Although promising, these results are still preliminary. Definitive conclusions can be made when the whole proposition is implemented. We intend to perform extensive experiments to better analyze the benefit of going from non-delay to active schedules. These experiments will also aim at studying how the allowed maximal delay impacts the solution quality. Finally, as the non-delay construction algorithm is a particular case of the active one, we have as objective to design an adaptive construction algorithm that uses a suitable maximal delay depending on the solution space exploration. While obtaining the advantage of accelerating the search through the construction of active schedules, the ability to visit any solution is maintained through the non-delay construction algorithm.

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Three upper bounds for the speed meeting problem

Benoit Cantais · Antoine Jouglet · David Savourey

1 Introduction

In a speed meeting problem, people are gathered in a place where tables are disposed to meet each other. The set of persons that each person wishes to meet is known. At regular intervals, the persons are asked to get up and are redistributed among the tables. A distribution of persons among the tables is called a *round*. Speed meetings are getting popular in business networking. It allows, in a short period of time, a large group of investors and businessmen to meet each other and find business opportunities.

We consider the problem with M tables of C seats, T meeting rounds and a set a persons $X = \{1, \ldots, N\}$. $G = (X, U \subseteq X^2)$ is an oriented graph such as $(i, j) \in U$ means that person i wishes to meet person j (see example on Figure 1). A meeting $(i, j) \in U$ is considered as realized if persons i and j are seated at the same table during at least one meeting round. At round $t \in \{1, \ldots, T\}$, each person i can be seated at only one table and at most C persons can be seated at table $m \in \{1, \ldots, M\}$. Given the number of rounds T, the goal is to distribute the persons around the tables at every round to maximise the total number of wished meetings realized. We use a reduction from *CLIQUE* [3] to demonstrate that the problem is *NP-Complete*.

As far as we know, this problem has not been treated yet. Some well known problems can be seen as particular cases of the speed meeting problem. In the social golfer problem, the persons do not want to meet more than once in a given number of rounds. In the fully social golfer problem [4], every person has to meet every other person exactly once. The Oberwolfach problem [2], the 36 officers problem [7], the Kirksman's schoolgirl problem [1] and the block design theory [6] are connected to the speed meeting problem. In the speed dating problem [5], the special case where the capacity of the tables is 2 is considered.

In this talk, we present three methods to find an upper bound for the speed meeting problem.

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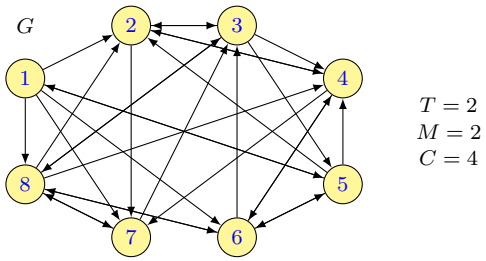

Fig. 1 An example of an instance of a speed meeting problem.

2 Upper Bounds

Let P be a partial solution to an instance of the problem, *i.e.* the tables are partially assigned for each round. We propose three methods to find an upper bound to the best solution that can be reached by completing P. In this abstract, the upper bounds algorithms will be illustrated through the example given on Figure 1 and the partial solution presented on Figure 2.

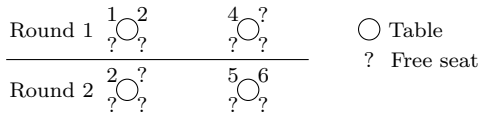

Fig. 2 A partial solution for the example problem.

2.1 An upper bound relying on the relaxation of the identities of the persons

From the original problem, we build a relaxation by keeping only the number of wished meetings of each person. Then, we solve optimally the relaxed problem where we suppose that the persons want to realize the given number of wished meetings, with no constraint on the identities of the persons to meet.

Let $w_{i,t}, i \in \{0,\dots,N\}, t \in \{0,\dots,T\}$ be a variable representing the number of wished but not realized meetings by i after round t in a solution of the relaxation. $w_{i,0}$ corresponds to the remaining number of wished meetings for i in P. The goal is to minimise $\sum_{i=1}^{N} w_{i,T}$. If i is added to a seat at round t, then $w_{i,t+1} = min(w_{i,t} - (C - 1), 0)$.

We use the algorithm presenter in and prove that it gives the optimal solution to the relaxed problem. For each round t, the algorithm sorts the persons by number of meetings left $w_{i,t}$ in descending order. The $min(M * C, N)$ first persons of this list are selected and added to the seats. Then, the algorithm computes $w_{i,t+1}$ for each person $i \in \{0,\dots,N\}$ and iterates again for the next round $t + 1$.

2.2 An upper bound relying on the resolution of a maximum cost flow problem

When completing P, for each round t, two kinds of meetings appear. Let m_t^{taken} be the number of meetings realized between persons that are already seated in P and persons

that are newly seated. Let m_t^{free} be the number of meetings realized only between persons that are newly seated. The second method is based, for each round t, on a separated computation of an upper bound for m_t^{free} and m_t^{taken}.

To compute an upper bound to m_t^{taken}, we solve a maximum cost flow problem in a transport network. The maximum cost flow is searched from the node *Source* to the node *Sink*. A node $P_{t,i}$ is added for each person i available at round t and a node $T_{t,m}$ for each table m. The arcs and their capacities are constructed as described in Table 1.

Arc	Capacity	Cost
$(Source, T_{t,m})$	Free seats for table m	0
$(T_{t,m}, P_{t,i})$	1	Meetings realized if i added to table m
$(P_{t,i}, Sink)$	1	0

Table 1 Construction of the maximum cost flow problem.

Given a round t, let $G_t^{free} = (F_t, V_t \subseteq F_t \times F_t)$ be an oriented graph such as $(i,j) \in V_t$ if (i,j) is a wished meeting not realized in P. The number of remaining free seats varies among the tables in P. This is a different version of the speed meeting problem with only one round and the tables having different capacities. The value of the optimal solution corresponds to an upper bound to m_t^{free}. We use the algorithm from section 2.1 to compute an upper bound. Figure 3 presents the problems associated with each round for the example problem.

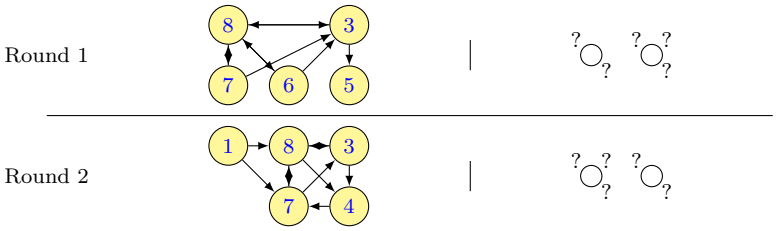

Fig. 3 The subproblems to compute an upper bound for m_t^{free} for round 1 and 2 of the example problem.

2.3 An upper bound relying on the resolution of a maximum flow problem

The third upper bound is based on the construction of a flow network and the resolution of a maximum flow problem going from a *Source* node to a *Sink* node. We create a node $T_{m,t}$ for each table m and round t that has free seats, a node $P_{m,t,i}$ for each person i that can be added to table m during round t and a node $M_{i,j}$ for each meeting (i,j) wished and not realized in P. The arcs and their capacities are constructed as described in Figure 2. A flow of value 1 going through a path $[Source, T_{m,t}, P_{m,t,i}, M_{i,j}, Sink]$ corresponds to a wished meeting (i,j) realized by adding i to table m, round t. In this relaxed problem, during a round t, a person that is not seating can be added to more than one table.

Arc	Exists if	Capacity
$(Source, T_{m,t})$	$\emptyset$	Upper bound for number of meetings table m, round t
$(T_{m,t}, P_{m,t,i})$	$\emptyset$	Upper bound for number of meetings if i added table m, round t
$(P_{m,t,i}, M_{i,j})$	j seating table m, round t	1
$(P_{m,t,i}, M_{j,i})$	j seating table m, round t	1
$(P_{m,t,i}, M_{i,k})$	k free at round t	1
$(M_{i,j}, Sink)$	$\emptyset$	1

Table 2 Arcs and their capacities for the construction of the flow network.

In this talk, we present preliminary results based on a set of crafted instances to compare the efficiency of the differents upper bounds methods applied in a branch and bound algorithm.

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Modeling a Practical University Course Timetabling Problem with Reward and Penalty Objective Function: Case Study FTSM-UKM

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1 Introduction

Generating a generally satisfied course timetable is a difficult task faces by most of educational institutions in every semester. This is due to a large number of students, lecturers, and courses which are inter-connected, and has objectives and constraints that need to be satisfied in order to generate a feasible timetable[1]. The University Course Timetabling Problem (UCTP) is defined as to schedule lectures (courses) that are conducted by lecturers within a given number of timeslots and allocate these courses into available rooms satisfying certain constraints[2]. There are normally 5 basic components in the UCTP model[3], which are:

- ***C*** - a set of courses to be scheduled;
- ***T*** – a set of timeslots to allocate the courses in;
- ***L*** - a set of lecturers conducting the courses;
- ***S*** - a set of student groups taking the courses;
- ***R*** - a set of class rooms to hold the courses.

There are two types of constraints needed to be considered when solving the UCTP problem: the hard constraints and soft constraints. The hard constraints need to be satisfied to ensure the feasibility of the generated timetable. Therefore, no hard constraint is allowed to be violated in the solution. However, the soft constraints are allowed to be violated, as the soft constraints are used to measure the general satisfaction of the generated timetable. Any violation of the soft constraints will result in a reduction of the timetable's satisfaction level. Therefore, a good quality solution of the UCTP should violate no hard constraints and minimize the violations of the soft constraints.

The UCTP is a Non-Polynomial-complete problem which cannot be solved in a polynomial time[4]. In the other word, the UCTP is very difficult to solve for optimality. As an educational timetabling problem, the UCTP has gathered great research interests in both operations research and artificial intelligence fields since the 1950s[5]. In the early days, researchers are mainly

concentrating on developing models and solution approaches for a single institution[6][7][8]. A survey regarding the early works in UCTP can be found in [9].

Since the approaches developed in the early days were tested on different specifically designed UCTP models, researchers at that time were impossible to compare their solution approaches with others. This issue caused the emergence of the first benchmark UCTP model, which is published for the First International Timetabling Competition (ITC-2002)[10]. To the current point of view, this benchmark model is a great reduction of real-world problems which only involves the basic components collected from real-world problems[11]. This benchmark model has greatly contributed the research field and provides a platform to compare different solution approaches. The simplified model, i.e. ITC-2002 creates a gap between research and practice. To bridge this gap, on the 2nd International Timetabling Competition, two new benchmark models were published, which are post enrollment-based CTP (ITC-2007-track 2)[12] and curriculum-based CTP (ITC-2007-track 3)[13]. The major difference between these two models is on how the conflictions between courses are defined. For the post-enrollment-based problem, as an extension of previous benchmark problem, the conflictions is defined based on the students' enrollment information. However, the confliction for the curriculum based CTP is based on the curriculum provided by the institution. Furthermore, lecturers are considered in the curriculum-based CTP, whilst not in the post enrollment-based CTP. These two datasets have significantly increased the problem complexity and make the problems closer to real-world situations[14].

Although many research has been conducted on these benchmark datasets, these works were rarely implemented on a real-world problem[11]. This might be due to the difference of the constraints between benchmark UCTP model and practical problem model is different from each other. Hence, an effectively modeling of a practical problem is important. [15] had stated some factors that may result in the uniqueness of different UCTP models. Such as a high number of elective courses. These elective courses are normally formed into groups which students need to attend *m* out of *n* courses in each group. Another factor may influence the problem model is the *course sections*, which represent for splitting a course into multiple sections. Furthermore, *course configuration* is another factor may affect the problem model. Some courses may require conducting different types of teaching events such as lectures, labs, and tutorials. These factors may lead to a more complex model when dealing with real-world problems.

It is always important to consider human's preference when designing a UCTP model[16]. When modeling a UCTP problem, the *objective function* should be carefully designed to reflect the general satisfaction of a generated timetable. Traditionally, the objective functions are designed based on measuring the violation of soft constraints. In both benchmark and practical UTCP models, *the objective function* is designed only based on how many times a undesired pattern appeared in solution. However, the user desired patterns in a timetable seem to be ignored by the research society.

Our previous work in [17] has conducted an investigation of factors which may influence the satisfaction on Fakulti Teknologi dan Sains Maklumat (FTSM), Universiti Kebangsaan Malaysia (UKM). In [17], two questionnaires have been designed and distributed to the lecturers and students in FTSM, UKM to collect their satisfaction information on every possible schedule patterns. The result has identified the user's undesired and desired patterns in a course timetable, and how strong they like or dislike.

This work extends our previous work[17] and introduces a new practical course timetabling problem at FTSM, UKM (FTSM-UCTP). The new practical problem contains more constraints compared with benchmark problems. Additionally, we also introduced a new objective function to measure the quality of the timetable by giving both penalty when undesired pattern appeared and reward when the desired pattern appeared.

This paper is organized as follow: In Section 2, we present the description of the course timetabling problem in FTSM, UKM. The formulation of the problem is presented in Section 3. Section 4 proposed a new objection function with penalty and reward mechanism.

2 FTSM Course Timetabling Problem (FTSM-UCTP)

The dataset FTSM-UCTP SemII_16/17 presented in TABLE I is real data used in FTSM for Semester II, year 2016/2017. In this dataset, the total number of courses is 74 exams with 21 students groups, required to be scheduled in 5 days (Mon-Fri) from 8:00 to 21:30. The features of the datasets can be found in TABLE I.

TABLE I. FEATURES OF FTSM-UCTP SEMII_16/17

Feature	Value
number of undergraduate courses	42
number of postgraduate courses	25
number of CITRA courses	7
number of class events	75
number of tutorial events	34
number of lab events	30
number of lecturers	71
number of teachers	5

Feature	Value
number of student groups for undergraduate courses	21
number of available timeslots for post-graduate events	15
number of available timeslots for undergraduate events	14
number of available timeslots for CITRA events	5
number of available room for post-graduate class events	3
number of available room for undergraduate class/tutorial events	7
number of available room for undergraduate tutorial events only	1
number of available lab room for undergraduate lab events	11

In FTSM-UCTP, the faculty has three types of courses in each semester: undergraduate courses, postgraduate courses, and CITRA courses. The CITRA courses can be enrolled by only non-FTSM students. Therefore conflictions raised by students taking CITRA courses are out of concern when scheduling the timetable. Furthermore, the post-graduate courses are only registered by post-graduate students in FTSM and have already pre-scheduled in the timetable. Neither the time nor classrooms can change. Hence, when generating the timetable, the confliction between any courses and CITRA or postgraduate courses only comes from lecturers availability. Furthermore, some undergraduate courses are elective courses. These courses are formed by groups which a student can only take one from each group in this semester. One thing needs to be noticed that each elective course requires only one class event to be scheduled.

Each course may require any number of events(sections) to be conducted weekly. There are three types of events for the courses in FTSM-UCTP which are class events, tutorial events, and lab events. The attendance required of an event depends on the type of the event. For class events, all students taking this course should attend all the class event of this course. Whilst, for lab and tutorial, if the course requires multiple events of lab or tutorial type, each student only need to select one event from this type to attend. The duration of these events depends on the type of the course they belonging to. The undergraduate and CITRA courses require all their events to be scheduled in a two-hour timeslot. Whilst, for the post-graduate courses' events, the durations are three hours. For the elective courses, which students are only allowed to select one from each group, all the class-type events in the same group should be scheduled at the same time to avoid student takes more than one courses from the group.

All the events are required to be scheduled in 5 weekdays (Monday to Friday) every week, from 08:00 to 21:30. As mentioned earlier, the duration of each type of courses is different, 2 hours for both undergraduate and CITRA courses, and 3 hours duration for postgraduate courses. All the undergraduate courses can be scheduled into 3 timeslots: 08:00- 10:00, 10:00 - 12:00, and 12:00 – 14:00, and for postgraduate courses, their activities should be scheduled into 3 timeslots: 09:00 to 12:00, 14:00-17:00 and 18:30 - 21:30. The CITRA courses' activities can be scheduled in 16:00-18:00 (Monday to Friday). Whilst, on Friday, there are only two timeslots available for undergraduate courses' activities which are: 08:00- 10:00 and 10:00 - 12:00. Due to both students and lecturers need to pray every Friday, the timeslots from Monday to Thursday is different with Friday. It should be noticed that if we failed to find a feasible arrangement in the three timeslots for an undergraduate event, it is acceptable to schedule them in the timeslots reserved for CITRA courses. In this case, a penalty should be applied to indicate a quality decrease of the timetable.

There are two types of instructors in FTSM: lecturers, and teachers. Each course is conducted by at least one lecturer. When scheduling a class event, the lecturers that were pre-

assigned to the course should be scheduled together. However, the lab or tutorial event require occupying only one instructor (either lecturer or teacher). The teachers that were pre-assigned to the course can only be assigned to handle lab and tutorial events, but not the class-type activities. In the FTSM-CTP, the instructors (lecturers and teachers) have already been assigned to each course, but not the event. The instructor(s) from a course should be assigned to this event based on the event type and availability.

To schedule the undergraduate courses, we have 7 classrooms (for lectures), 1 tutorial room, and 11 labs (this include 2 common labs for general courses and 9 specific labs for certain courses such as the multimedia lab for multimedia courses, network lab for network courses). These rooms and labs are with different facilities and capacities. The number of students attending an activity should not exceed the capacity of the room holding it. Another issue needs to be considered is each event may require different facilities. Therefore, the events should only be held in one of the rooms that fulfilling the facility requirements.

3 Problem Formulation

The FTSM-UCTP can be stated as follows:

$\boldsymbol{C = \{c_1, c_2, \ldots, c_n\}}$ a set of courses that need to be scheduled;
$\boldsymbol{E} = \{e_1, e_2, \ldots, e_n\}$ a set of events for courses requires to be scheduled, including classes, labs, and tutorials;
$\boldsymbol{E'}$ a set of events that have been scheduled ;
$\mathbf{E^* = E \cup E'}$ a set of all events in the problem
$\boldsymbol{E_c \subset E^*}$ a set of all events belongs to course $\boldsymbol{c}$;
$\boldsymbol{T} = \{t_1, t_2, \ldots, t_n\}$ a set of timeslots to schedule the teaching activities in;
$\boldsymbol{D} = \{d_1, d_2, \ldots, d_5\}$ a set of day from Monday to Friday;
$\boldsymbol{R} = \{r_1, r_2, \ldots, r_n\}$ a set of classrooms to locate the teaching activities;
$\boldsymbol{S} = \{s_1, s_2, \ldots, s_n\}$ a set of students groups participating the teaching activities;
$\mathbf{I} = \{i_1, i_2, \ldots, i_n\}$ a set of all instructors, including lecturers and teachers;
$\boldsymbol{L \subset I}$ a set of lecturers conducting teaching activities;
$\boldsymbol{Teacher \subset I}$ a set of teachers that can only conduct tutorial and lab activities;
$\boldsymbol{C_{Elective} \subset C}$ a set of elective courses in the same group, namely elective courses group;
$\boldsymbol{G}$ a set of all the elective courses groups; $\boldsymbol{E_{class} \subset E^*}$ a set of all class events;
$\boldsymbol{t_e \in T}$ the timeslots event $\boldsymbol{e}$ scheduled in;
$\boldsymbol{r_e \in R}$ the classroom event $\boldsymbol{e}$ allocation in;
$\boldsymbol{s_e \subset S}$ the set of students attending event $\boldsymbol{e}$;
$\boldsymbol{I_e \subset I}$ the set of lecturers conducting event $\boldsymbol{e}$;
$\boldsymbol{e_E^i}$ the $\boldsymbol{i_{th}}$ event in event set $\boldsymbol{E}$;
$\boldsymbol{d_t \subset D}$ the day of timeslot $\boldsymbol{t}$ from;
$\boldsymbol{E_d^s}$ the set of events taken by student $\boldsymbol{s}$ in day $\boldsymbol{d}$;
$\boldsymbol{T_{candidate}(e)}$ the candidates timeslots event $\boldsymbol{e}$ can schedule in;
$\boldsymbol{R_{candidate}(e)}$ the candidates room event $\boldsymbol{e}$ can allocate;
$\boldsymbol{E(C)}$ a set of all events requested by every courses in set $\boldsymbol{C}$;
$\boldsymbol{C_e}$ the course event $\boldsymbol{e}$ blongs to;
$\boldsymbol{C_U \in C}$ a set of undergraduate courses;
$\boldsymbol{C_P \subset C}$ a set of post-graduate courses taken by post-graduate students.(Already scheduled);
$\boldsymbol{C_{CITRA} \subset C}$ a set of CITRA – courses taken by non-faculty students (partially scheduled);

Based on the information we gathered from the faculty management office, we are introducing the hard constraints of the FTSM-UCTP model. These hard constrains are used to define a generated timetable's feasibility. The hard constraints of the FTSM-CTP are:

H1: All events required to be scheduled should be scheduled and scheduled only one time.

$$\prod_{e=1}^{|E|-1} \lambda_e = 1 \ \ and \ \sum_{e=1}^{|E|} \lambda_e = |E| \tag{1}$$

Where

$$\lambda_e = \begin{cases} 1, & if\ event\ \boldsymbol{e}\ is\ scheduled \\ 0, & otherwise \end{cases} \tag{2}$$

H2: No student can takes more than one event at the same time.

$$\sum_{i=1}^{|E|-1} \sum_{j=i+1}^{|E|} shareStudent\left(\boldsymbol{e}_E^{\boldsymbol{i}}, \quad \boldsymbol{e}_E^{\boldsymbol{j}}\right) \times sameTime\left(t_{e_E^i}, t_{e_E^j}\right) = 0 \tag{3}$$

Where

$$shareStudent(e_i, e_j) = \begin{cases} 1, & S_{e_i} \cap S_{e_j} \neq \emptyset \\ 0, & S_{e_i} \cap S_{e_j} = \emptyset \end{cases} \tag{4}$$

$$sameTime(e_i, e_j) = \begin{cases} 1, & if\ t_{e_i}\ and\ \ t_{e_j}\ are\ intersected \\ 0, & otherwise \end{cases} \tag{5}$$

H3: Each room can only allocate one event at the same time.

$$\sum_{i=1}^{|E|-1} \sum_{j=i+1}^{|E|} sameRoom\left(\boldsymbol{e}_E^{\boldsymbol{i}}, \quad \boldsymbol{e}_E^{\boldsymbol{j}}\right) \times sameTime\ \left(t_{e_E^i}, t_{e_E^j}\right) = 0 \tag{6}$$

Where

$$sameRoom(e_i, e_j) = \begin{cases} 1, & r_{e_i} = r_{e_j} \\ 0, & r_{e_i} \neq r_{e_j} \end{cases} \tag{7}$$

H4: No instructor can conduct more than one event at the same time.

$$\sum_{i=0}^{|E^*|-1} \sum_{j=i+1}^{|E^*|} I\left(\boldsymbol{e}_{E^*}^{\boldsymbol{i}}, \quad \boldsymbol{e}_{E^*}^{\boldsymbol{j}}\right) \times T\ \left(t_{e_{E^*}^i}, t_{e_{E^*}^j}\right) = 0 \tag{8}$$

Where

$$I(e_i, e_j) = \begin{cases} 0, & I_{e_i} \cap I_{e_j} = \emptyset \\ 1, & I_{e_i} \cap I_{e_j} \neq \emptyset \end{cases} \quad (9)$$

H5: For the events belongs to the same course, the class events should not be scheduled in the same day with its' tutorial or lab events

$$\sum_{i=1}^{|E|-1} \sum_{j=i+1}^{|E|} \Big(sameCourse(\boldsymbol{e_E^i}, \boldsymbol{e_E^j}) \times sameDay(d_{t_{e_E^i}}, d_{t_{e_E^j}}) \times \Big(isClass(\boldsymbol{e_E^i}) + \big(1 - isClass(\boldsymbol{e_E^j})\big) \Big) \Big) = 0 \quad (10)$$

Where

$$sameCourse(e_1, e_2) = \begin{cases} 1, & C_{e_1} = C_{e_2} \\ 0, & C_{e_1} \neq C_{e_2} \end{cases} \quad (11)$$

$$sameDay(d_1, d_2) = \begin{cases} 1, & d_1 = d_2 \\ 0, & d_1 \neq d_2 \end{cases} \quad (12)$$

$$isClass(e) = \begin{cases} 1, e \in E_{class} \\ 0, otherwise \end{cases} \quad (13)$$

H6: The room holding certain activities must be able to provide the facilities.

$$\forall e \in E \ (R_e \in R_{candidate}(e))$$

H7: Each event must be scheduled to its designed timeslots based on course type

$$\forall e \in E \ (t_e \in T_{candidate}(e))$$

H8: The class type event of all the courses in each elective course groups should be scheduled with time intersection between each other.

$$\forall C \in G \ \Big(\sum_{i=1}^{E(C)-1} \sum_{j=i+1}^{E(C)} isClass(e_{E(C)}^i) * isClass\left(e_{E(C)}^j\right) * \Big(1 - sameTime\left(t_{e_{E(C)}^i}, t_{e_{E(C)}^j}\right)\Big) = 0 \Big)$$

In this section, we have proposed the formulation of the FTSM-CTM model and listed 8 hard constraints to identify a generated timetable's feasibility. It should be noticed that, due to the complexity of this problem, the constraints **H7** could be relaxed if an undergraduate event cannot find a feasible arrangement. Indeed, it is allowed to schedule it into the timeslots reserved for CITRA events. Of cause, the penalty would be raised once it happened. Whilst, the CITRA events are necessary to be allocated in the timeslots reserved for CITRA courses.

4. Objective Function with Penalty and Reward Mechanism

The objective function is an equation to be optimized when solving optimization problems. For the UCTP, the objective function should reflect a course timetable's satisfaction level, in another

word, the quality. Commonly, for both benchmark and real-world UCTP models, the objective function is designed based on the violation of the soft constraint in the model which is calculating a weighted summation of the product of the number of appearance of an undesired scheduling pattern multiplied by the weight representing the level of undesirable.

However, this type of objective function design strategy creates a gap between research and real-world situations. In the real-world situation, the general satisfaction of a solution in UCTP may not only influence by the appearance of an undesired schedule pattern in the timetable, but also depends on the appearance of user preferred patterns. Therefore, we have conducted an investigation of the factors that may affect the general satisfaction level from both lecturers and students aspects in [17]. The work in [17] had designed two different questionnaires (i.e. one for lecturers and one for students in FTSM), that lists all the possible schedule patterns to collect how these patterns' appearance influenced their satisfaction level. The respondents are required to marked the influences between -5 (Most-Preferred) to +5 (Rejected). The result had successfully identified both the undesired patterns and the preferred patterns, together with the degree of their feelings.

Therefore, in this section, we propose a new objective function for the FTSM-UCTP based on the work in[17]. Our new objective function contains two part: the penalty mechanism and the reward mechanism. As a minimization optimization problem, the new objective function should be designed as the penalty of the solution's quality arose from the appearance of undesired patterns in a timetable minus the reward caused by the preferred patterns' appearance, as in Equation 14.

$$\textbf{MINIMISE}\ \ objective\ function\ (S) = totalPenalty(S) - totalReward(S) \tag{14}$$

Table II and III list the set of schedule patterns ($P = P_{students} \cup P_{instructors}$) affects the solution quality in FTSM-CTP, together with how much (w_p) a patten *p* will affect the general satisfaction of a solution in students and lecturers' aspect. A posititive(+) value of *w* represants a penalty value that should be given to the solution quality once the pattern appeared in the solution. These patterns are named as the *panealty-patterns* ($P_{penalty}$). On the other hand, the *reward-patterns* (P_{reward}) are the patterns with negtive(−) *w* value, that will improve the quality of the solution when it appears in the timetable. All the patterns listed in TABLE II and TABLE III can also be categorized in two categories. The first category is the *events_day* patterns, which describes what and how are the events of an instructor or student are aranged in a day. These patterns are : P_S1 to P_S5 and P_L1 to P_L4. The other category is the *event_timeslots* patterns, which describes an arrangment of cetain type event is scheduled on a specific timeslot. These patterns are: P_S6, P_S7 and P_L5 to P_L8. It is important to notice that the schedule patterns listed here are only the patterns that are concerned by instructors or students. The patterns they do not cared have been filltered out by the investigation we conductd in our previous work [17].

Therefore, The objective function of the FTSM-UCTP can be transformed to the following format as equation 15, where the *w* can be either positive (represent for the penalty as in the literature) or negative which represent a reward mechanism improve the solution quality. Hence, the new objective function as equation 15 is more accurate to illustrate the general satisfaction level of the UCTP in the real world.

$$\boldsymbol{MINIMISE}\ \ objective\ function\ (S) = \tag{15}$$

$$\sum_{s=1}^{|SG|}\sum_{d=1}^{|D|}\sum_{p=0}^{|P_{students}|}\left(followed\ \left(s_{s,}d_d,p(P_{students},p)\right)\times w_{p(P_{students},p)}\right)+\sum_{i=1}^{|I|}\sum_{d=1}^{|D|}\sum_{p=0}^{|P_{instructors}|}\left(followed\ \left(i_{i,}d_d,p(P_{instructors},p)\right)\times w_{p(P_{instructors},p)}\right)$$

Where

$$follow(u,d,p)=\begin{cases}1, & events\ of\ user\ \boldsymbol{u}\ are\ scheduled\ as\ pattern\ \boldsymbol{p}\ in\ day\ \boldsymbol{d}\\ 0, & otherwise\end{cases} \quad (16)$$

TABLE II. SCHEDULE PATTERNS AND INFLUENCE ON THE SOLUTION PENALTY FOR STUDENTS[17]

	Students Schedule Patterns ($P_{students}$)	**Type**	**w**
P_S1	Attending only one class event (not tutorial or lab) in a day (no other events in this day)	Reward	-2
P_S2	Attending only one tutorial/lab event in a day (no other events scheduled in this day)	Reward	-1
P_S3	Attending two consecutive class events only (not lab/tutorial) in a day	Reward	-3
P_S4	Having a two-hour free time between two class events in a day	Penalty	1
P_S5	Having a two-hour free time between two lab/tutorial events in a day	Penalty	1
P_S6	Schedule a class event on afternoon CITRA session (16:00-18:00)	Penalty	2
P_S7	Schedule a lab/tutorial event in afternoon CITRA session (16:00-18:00)	Penalty	2

TABLE III. SCHEDULE PATTERNS AND INFLUENCE ON THE SOLUTION PENALTY FOR INSTRUCTORS[17]

	Instructors Schedule Patterns ($P_{instructors}$)	**Type**	**w**
P_L1	Conducting two class-type events in a day	Penalty	3
P_L2	Conducting one class-type event and one lab/tutorial event in a day	Penalty	1
P_L3	Conducting two lab/tutorial events in a day	Penalty	1
P_L4	Conducting three or more events (including class, tutorial or lab) in a day	Penalty	4
P_L5	Conducting faculty-level class events on Monday (7 or more student groups enrolled)	Penalty	1
P_L6	Undergraduate class event is scheduled on CITRA session (16:00-18:00)	Penalty	3
P_L7	Undergraduate lab/tutorial event is scheduled CITRA session (16:00-18:00)	Penalty	1
P_L8	Class event is scheduled on early morning (08:00 - 10:00)	Reward	-1

5. Conclusions

In this work, we have introduced a new real-world university course timetabling problem model in Fakulti Teknologi dan Sains Maklumat, Universiti Kebangsaan Malaysia (FTSM-CTP). This practical model contains 3 types of courses which are the postgraduate courses, undergraduate courses, and CITRA courses. Each course may require any number of teaching events to be scheduled on weekly basis. There are three types of teaching events required by the courses in this problem: class events, tutorials, and labs. Each type of events has a different attending requirement and instructor occupation policy. Additionally, elective courses are organized as course groups, such that student should only select one course to take from each group. The available rooms and timeslots to assign an event in are predefined in this model based on the type of the event and type of the course it belongs to. The hard constraints of the FTSM-CTP have also reported.

We have identified the limitation of the standard objective function that was designed based on the appearance of user undesired schedule patterns. Therefore, in this work, we introduced a new objective function that involved both penalty and reward mechanism to evaluate the quality of a course timetable for the FTSM-UCTP. The new objective function bridged the gap between research and real-world situations by evaluating the quality of the course timetable based on users' preferred and unpreferred schedule patterns in the evaluation process. The ongoing research will focus on solving the FTSM-CTP by the graph-based constructive approach.

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Hybrid Heuristics for Multiple Orders per Job Scheduling Problems with Parallel Machines and a Common Due Date

Jens Rocholl • Lars Mönch

1 Introduction, Problem Setting and Analysis

We consider an earliness-tardiness (E/T) scheduling problem for parallel identical machines that is motivated by process conditions found in semiconductor wafer fabrication facilities (wafer fabs). Front opening unified pods (FOUPs) transfer wafers, thin discs made from silicon in modern 300-mm wafer fabs. Each wafer might contain up to 1000 integrated circuits (chips). There exists only a limited number of FOUPs in each wafer fab to avoid a congested automated material handling system (AMHS) [6]. Recently, a smaller number of wafers is required to fill the chips of a customer due to a decreased line width and an increased area per wafer. Consequently, there is often a need to group orders of different customers into one FOUP. Such scheduling problems that include an integrated grouping problem are called multiple orders per job (MOJ) problems (see [5] for a more detailed discussion). The researched scheduling problem can be described as follows. There are $j = 1,\ldots,F$ jobs available that can be processed on m identical parallel machines. Each job is associated with a FOUP and has a capacity K given in number of wafers. The set of all orders is denoted by O. Each order $o \in O$ has a size s_o given in wafers. It belongs to an order family $f(o) \in \{1,\ldots,L\}$. Only orders of the same family can be processed together in a single job j at the same time. All orders of the same family l have identical processing times P_l. Since all orders in a job are processed at the same time, the processing time of job j depends only on the family of the orders that belong to this job, i.e., we have $\tilde{p}_j = P_l$ when the orders of the job belong to family l. This situation is called lot processing environment. A nonrestrictive common due date d for all the orders is assumed, i.e., we have $\sum_{o \in O} p_o \le d$ where p_o is the processing time of order $o \in O$. We have $N = \sum n_l$ orders in total where n_l is the number of orders that belong to family l. All orders are available at $t = 0$, i.e., we have $r_o \equiv 0$ for all $o \in O$. The completion time of order o is C_o. Using the three-field notation from scheduling theory, the scheduling problem considered in this paper can be represented as

$$P_m \mid moj(lot), incompatible, d_o \equiv d \mid E/T, \qquad (1)$$

where P_m indicates m identical parallel machines, $moj(lot)$ refers to the lot processing environment, $d \equiv d_o$ to the nonrestrictive due date, *incompatible* to the different order families, and E/T to the E/T measure that is given by $E/T = \sum_{o \in O} \left\{ (d - C_o)^+ + (C_o - d)^+ \right\}$ where we set $x^+ = \max(x,0)$ for abbreviation. We know from [10] that even the single-

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machine version of problem (1) is NP-hard. Therefore, we have to look for efficient heuristics. Recently, MOJ scheduling problems have attracted a lot of interest from researchers (cf. [5] for a survey and [8], [10], [11] for specific MOJ scheduling problems). However, it seems that MOJ scheduling problems with a common due date are only rarely discussed so far in the literature. There are a few papers that deal with scheduling problems with a common due date on single and parallel batch processing machines (cf. [4], [7], [9]). Here, a batch is a group of jobs that are processed together at the same time on the same machine. Although these problems are somehow similar to problem (1) according to the grouping decisions, the major difference is that in problem (1) only a maximum number of jobs is allowed, while such a constraint does not exist for the number of batches in batch scheduling problems. The most pertinent reference for this research is [10] where the scheduling problem $1|moj(lot),incompatible,d_o \equiv d|E/T$ is considered and two genetic algorithms (GAs) are proposed. However, problem (1) deals with parallel machines which complicate the problem to a large extent.

It is possible to derive structural properties for certain classes of optimal schedules of problem (1) by using arguments similar to those in [3] for the problem $1|d_j \equiv d|\sum w_j E/T$ where w_j is the weight of job j. Note that the number of orders in a job plays the role of the weights considered in [3]. We denote the set of jobs on machine k that are not tardy by A_k whereas the corresponding set of tardy jobs on this machine is abbreviated by B_k. The number of orders included in job j is denoted by $|O_j|$. The most important of these properties are the following ones:

Property 1: There exists an optimal schedule for each instance of problem (1) where a job completes its processing at time d on each nonempty machine.

Property 2: The jobs in the set A_k are sequenced in non-decreasing order of the ratio $|O_j|/\tilde{p}_j$, while the jobs of the set B_k are sequenced in non-increasing order of this ratio in each optimal schedule for an instance of problem (1).

Note that the jobs in A_k are sorted according to the weighted longest processing time (WLPT) rule, whereas the jobs in B_k are sorted according to the weighted shortest processing time (WSPT) rule. This leads to a V-shaped schedule. If the jobs are assigned and sequenced, the job formation problem is a generalized assignment problem (GAP) that can be solved as a binary program using a commercial solver. We abbreviate this procedure by MOJAP.

2 Decomposition Heuristics

In the course of solving problem (1) we have to make job formation, assignment, and sequencing decisions. These steps can be carried out by a simple list scheduling approach and by hybrids that combine biased random-key GAs (BRKGAs) [2] with the simple heuristics. For the simple heuristic, we start from a list of orders that are sorted in non-decreasing order of the order size s_o. Starting from the top of this list, a job will be filled with orders until this is possible for capacity reasons in a first fit (FF) manner. If there is capacity left, the last inserted order is removed and starting from the bottom of this list, the first order that fits will be inserted in a best fit (BF) manner. After this, this procedure is repeated using a new job. We obtain a set of formed jobs by using this procedure that is motivated by bin packing-type procedures [8]. Similar to [1], the jobs are sorted first according to a given rule. The next job then is assigned to the sets A_k and B_k in an alternating manner on the first or a last position, respectively. The machine with the smallest value $\Delta_j := |C_j - d|$ is chosen to place job j. Here, the completion time of job j is denoted by C_j. Based on property 1, we know the completion time of the first job to be placed on an empty machine. We use the abbreviation

FFBF-SS for the simple heuristic. If the jobs are sorted according to the WSPT rule, it is called FFBF-SS(WSPT).

The first BRKGA variant considers a representation that consists of an random-key array of length F. It allows for determining the sequence and the family of the jobs. Based on the array entry v for job j we decode the family of job j by $\lceil Lv \rceil$ while the fractional quantities $Lv - \lfloor Lv \rfloor$ determine the sequence of the jobs. The FFBF-SS decodes this job sequence into a feasible schedule. If the E/T value is smaller than a given dynamically adjusted threshold the MOJAP procedure is used to improve the schedule. Finally, MOJAP is applied to the best schedule. This family rank-based procedure is abbreviated by BRKGA-FR. The second BRKGA variant first applies FFBF-SS(WSPT) to form jobs. These jobs are then sequenced using the BRKGA approach. The FFBF-SS is then used for the resulting sequences to decode the sequence into a schedule. The set of tardy batches is resequenced using WSPT, while the set of early or on-time jobs is resequenced using WLPT to ensure the structure according to property 2. Depending on the quality of the schedule MOJAP might be applied to further improve the schedule. Finally, MOJAP is applied to the best schedule. This job rank-based procedure is abbreviated by BRKGA-JR.

3 Computational Results

All the heuristics are implemented in the C++ programming language. The brkgaAPI framework [12] is used for the BRKGA-type algorithms while CPLEX is applied as commercial solver. We conduct computational experiments for $N = 120$ orders that belong to $L \in \{3,10\}$ families. $m \in \{2,6\}$ parallel machines are used in the experiments. The order sizes s_o are generated according to the discrete uniform distribution $DU[(v-1)/2,(3v+1)/2]$ for $v \in \{3,5\}$. The FOUP capacity is set as $K := 12\beta + 1$ for $\beta \in \{1,2\}$, whereas the number of FOUPs is chosen as $\max(\lceil Nv/12\beta \rceil + 1, L)$. Overall, 160 problem instances are considered. The results are shown in Table 1. We show the E/T value for each of the three algorithms relative to the smallest E/T value obtained by any of the algorithms.

Table 1: Computational Results for Problem (1)

m	L	β	v	FFBF-SS(WSPT)	BRKGA-FR	BRKGA-JR
2	3	1	3	1.035	1.008	**1.001**
		1	5	**1.000**	1.044	1.018
		2	3	1.062	**1.002**	**1.002**
		2	5	1.031	**1.001**	1.009
	10	1	3	1.036	1.008	**1.001**
		1	5	1.009	1.069	**1.000**
		2	3	1.051	**1.000**	1.001
		2	5	1.029	**1.005**	**1.005**
6	3	1	3	1.120	1.005	**1.001**
		1	5	1.015	1.038	**1.002**
		2	3	1.131	**1.000**	**1.000**
		2	5	1.124	1.003	**1.002**
	10	1	3	1.119	**1.005**	1.007
		1	5	1.035	1.050	**1.000**
		2	3	1.098	**1.001**	1.006
		2	5	1.130	**1.003**	1.015

Best results are marked in bold. The allowed computing time is 50 second per instance if less than 30 FOUPs are available and 180 seconds per instance for the remaining instances. Five independent replications of the BRKGA-type heuristics are performed for each instance to obtain statistically reasonable results. We see from Table 1 that the two BRKGAs clearly

outperform FFBF-SS(WSPT). In the case of six parallel machines and three families, the BRKGA-JR slightly outperforms the BRKGA-FR. We conclude that hybrids of metaheuristics and simple heuristics are able to determine high-quality solutions in a fairly short amount of computing time.

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The "Orchestrator" approach to multimodal continental trip planning.

Lukas Bach • Dag Kjenstad • Carlo Mannino

Abstract The orchestrator is a new, decentralized approach to trip planning in large multimodal networks. The solution technique, which resembles the overlay graphs approach [1], is based on an implicit and possibly recursive decomposition of the overall multi-modal network into a collection of multi- and mono-modal networks equipped with a specialized trip planner. Each subnetwork and associated solution algorithm can be created and maintained by different and independent organizations/developers and can be hosted by different platforms/servers. The orchestrator receives trip queries and, exploiting some pre-computed information, decomposes each trip query into a suitable sequence of queries to specific subnetworks in the available collection. The new approach, developed within the EU Horizon 2020 project BONVOYAGE is able to handle large collections of subnetworks significantly reducing response time.

1 Introduction

The orchestrator is a decomposition approach to solve the routing problem on a multimodal network *N*. It orchestrates a set of solution algorithms (solvers) acting on different multi- or single-mode subnetworks of the original network *N*. Depending on the specific query the orchestrator will pick up a suitable subset of solvers, run them, collect the partial solutions and compose them into a solution to the original query. For instance, if the request regards a trip from city A to city B, the orchestrator may invoke a solver in charge of city A, a solver in charge of city B, and a solver capable to find extra-urban path from the boundary of city A to the boundary of city B. Another example of such orchestrating approach regards the matching of two different trips in freight transportation scenarios, or the identification multimodal trips through successive and coordinated invocations to single-mode solvers.

The major features of the orchestrator approach are:

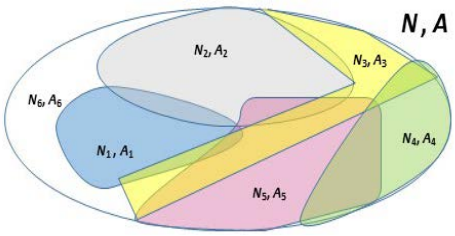
1. The decomposition allows for significant speed-ups in answering queries;
2. The different solvers operating on the subnetworks and made available may be designed and implemented by different organizations;

3. The decomposition is metric-independent so it can be used in conjunction with different objective functions. The only required property is that such functions are separable in terms which may be associated with the components of the decomposition.

Our technique is inspired by some recent developments in decomposition approaches for route planning. In particular, in [2] the authors advocate the use of separator-based methods to compute optimal paths in large networks. Such methods are based on a pre-processing phase in which the topology of the network is exploited in order to define a suitable decomposition into subnetworks; and a metric-dependent customization in which the best solution to the current query is computed and returned. According to [2], this approach outperforms all other techniques on complex, realistic scenarios as for example the contraction hierarchies approaches, see [4]. Alternative techniques may perform better when tackling problems with fixed metric (e.g., shortest time) or other simplified scenarios. For example, contraction hierarchies which may be efficiently efficient for certain problems show a poor behavior against user preferences (see [3]). In contrast, separator-based methods can tackle multiple metrics, consider U-turns, avoid left-turns, consider multiple restrictions, include real-time information, road closures, etc. Another interesting property of separator-based methods is that they can take into account time-dependent edge costs – that is the cost $l(u, v)$ associated with the directed edge (u, v) depends on the time when the edge is actually reached during the route. For such reasons they are very suitable to tackle planning problems which include public transport (see [7]). In [6] the authors describe a novel approach to integrate several independent single mode planners in a search. The technique is based on a construction of an abstract *metagraph* which in turn represents the different modes and planners.

1. Basic definitions and approach

The orchestrator may be viewed as a pair (N, A), where N is a multi-modal network, and A is an algorithm to find an optimal route (for a class of objective functions) between any origin/destination pair within N.



A multi-modal network is a graph $N = (V, E)$ where V is a set of nodes and E is a multiset of directed edges (i.e. ordered pairs of nodes). Each edge is associated with a single modality. So, for instance, if the nodes represent crosses in the road map of a city, then an edge between two crosses may be a car edge, a bike edge, a bus edge, etc. Metric data can be associated with each edge of the multi-modal network.

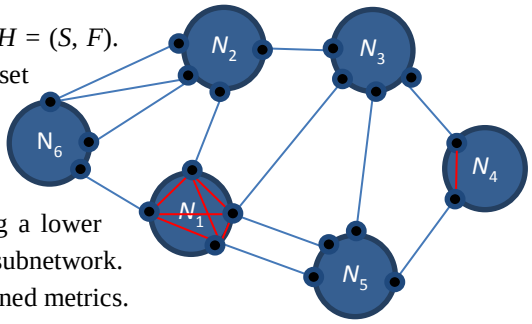
To perform its task, the orchestrator relies on a set of solvers $A_1, \ldots, A_q$ to compute an optimal route in a corresponding multi- or single-mode subnetwork $N_1, \ldots, N_q$ with the property that $N_1 \cup \ldots \cup N_q = N$. In other words, the subnetworks provide a coverage of the original network and they are not necessarily disjoint. Solvers may correspond to different algorithms but also different implementations of the same algorithm.

The pair (N_i, A_i) is a soloist. Observe that different soloists$(N_i, A_i), (N_q, A_q)$ may share the same solver or may share the same network, or even the same network and the same solver but with different implementations.

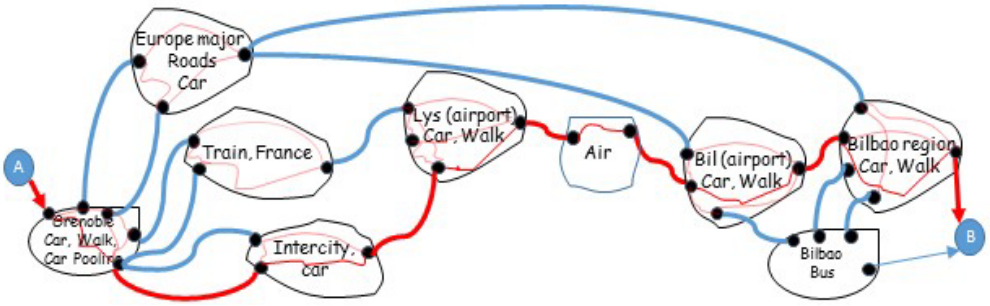
Once a query is submitted the orchestrator N, A identifies a suitable sequence of soloists to answer the query, construct the proper query for each soloist, collect answers from soloists and finally compose a unique solution.

As mentioned, the orchestrator generalizes the ideas in the overlay graph decomposition approach to routing. The crucial differences are that

1. The reference graph $N = (V, E)$ is a multi-modal network and may be partly unknown.
2. The elements in the decomposition are not simple cells identified by a set of nodes, but they are (potentially multi-modal) subnetworks of N along with a solution algorithm, namely soloists $(N_1, A_1), \ldots, (N_q, A_q)$.
3. The concept of boundary nodes is replaced by that of connection nodes. Connection nodes represent entry and exit points of the subnetworks associated with soloist. They may and may not coincide with (a subset of) the boundary nodes of the subnetwork (defined as for overlay graphs). The may or may not be geographically localized. Observe that connection nodes may actually correspond to extended areas within the associated subnetworks. In general, connections nodes must be chosen carefully at the registration of a new soloist into the BONVOYAGE platform. Again, we denote by S_i the set of connection nodes in subnetwork N_i.
4. The overlay graph is replaced by the orchestrator graph $H = (S, F)$. The nodes of the graph are the connection nodes. Each set S_i induces a clique in H, i.e. for every ordered pair of distinct nodes $u, v \in S_i$ there is an edge $(u, v) \in F$, called connection edge. With every connection edge (u, v) we also associate a distance $l_H(u, v)$ representing a lower bound on the minimum distance from u to v. in the subnetwork. Distances are evaluated according to one or more pre-defined metrics.



5. If nodes $u, v \in S$ belong to different subnetworks, there is an edge (u,v) in H only if it is possible to "transfer" from u to v. The length $l_H(u, v)$ is the distance between the connection nodes in terms of transfer time, and may also be 0 if the two nodes correspond to the same geographical location.



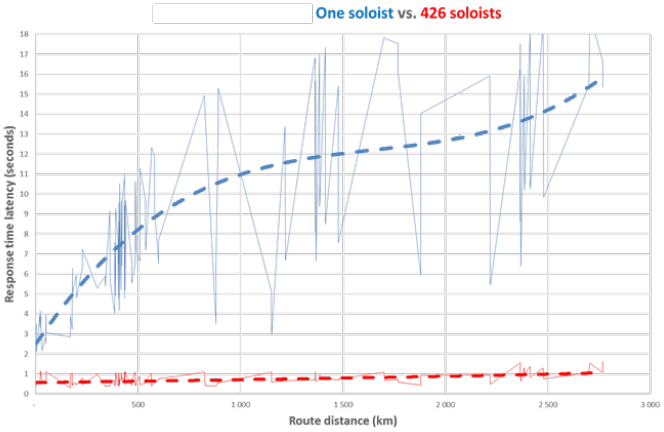
6. When a request for a route from A to B is received, the orchestrator looks for one or more shortest routes in the orchestrator graph. Once the route(s) is (are) identified, the actual value is computed with the current lengths. Observe that since pre-computed distances are not necessarily exact – in general they are lower bounds – and in addition they are computed before they are actually used, the resulting solutions are not necessarily optimal. In the figure, we present a possible response of an orchestrator with several multimodal soloists, when asked to return a multimodal trip (car, train, airplane) from a location in Grenoble to a location in Bilbao.

2 Preliminary computational results

The BONVOYAGE project is still ongoing, and the full implementation (at continental level) of our approach is scheduled by May 2018.

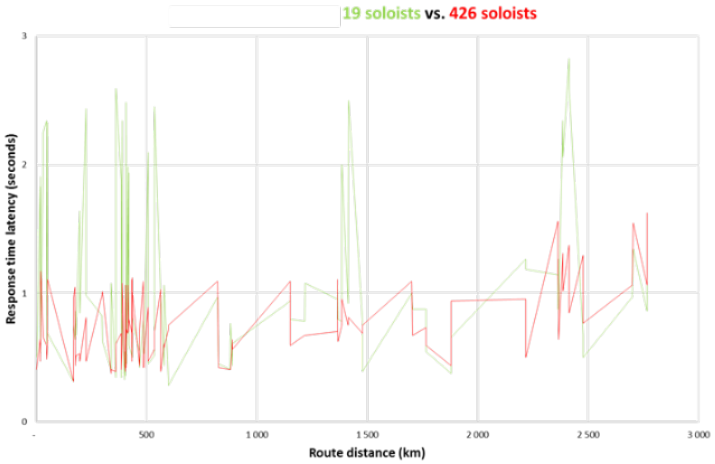
The main purpose of these initial experiments is to test the ability of the orchestrator decomposition algorithm to tackle large real-life instances of trip planning. In this test campaign, we did not intend to verify, for instance, the ability of the specific soloists to solve their own problem. We just assume they can do it in reasonable time. We decided to test the Orchestrator exploiting Norwegian data; indeed, the case covers the entire country with very detailed road data. For the tests, we have several different Orchestrator and Soloist setups using the detailed map information. In the different experiments, the number of soloists orchestrated by the tool grows from 1 to 426. In our experiments, we only consider a single road modality, and each soloist correspond to an area of Norway. The first Orchestrator setup manages only one soloist corresponding to the entire country. In the second set up, Norway is split into administrative regions: accordingly, we have 19 soloists $(A, G_1), \dots, (A, G_{19})$ each associated with the road network of the corresponding region. In the third set up, Norway is split into 426 counties, and the corresponding 426 soloists are defined as previously. The underlying road data for Norway does contain a total of 1.659.821 nodes and 1.789.444 arcs.

These soloists all return a single solution, which is the optimal solution. Hence, we have three different experimental cases for the orchestrator that we refer to as, Country, Regions and Counties. For all three cases, we use the same set of requests for testing. Every request is simply an ordered pair of points (A, B), each point identified by its coordinates. The orchestrator returns the best trip from A to B and from B to A. The requests are constructed by generating a set of 11 points and then taking all ordered pairs in the set, leading to 110 different requests. The points used for testing are spread around the country such that we get a sample of short (distance wise) requests and longer requests. The distance of these 110 requests range from 3 km to 2768 km.



The edge weights on the orchestrator graph represent approximation on the travel time between two connection points within a given soloist. To establish such approximations, the Orchestrator is equipped with a learning mechanism, therefore we also expect a speed-up if we run it a several times with the same (or similar) requests. Next, we present the computational results on our three test cases. In the figure on the left we see a comparison of our instances solved by an Orchestrator using a single soloist compared to an Orchestrator using 426 soloists. The Orchestrator receives the request and solves it on the Orchestrator graph. Next, from this solution it identifies the soloists that can (potentially) be used to find the optimal path. In the One soloist case, it always finds that

the requests fall fully within the soloist and forwards the request to the single soloist. In the County case, it finds multiple, but significantly smaller, soloists to handle most of the requests. Even for the shorter requests that falls within a single soloist it has the advantage that the identified soloists are much smaller, i.e., solves faster than the huge soloist covering the entire nation. If we observe the blue line in the figure, representing the National case, we see that runtime increase significantly when the request length increase. However, we also see quite some variation in runtime on similar length requests and an increase in runtime that smoothens out and does not increase as rapidly as it could be expected. When observing the red line that represents the County case it is obvious that it scales very well and has a very small increase in runtime when the length of the requests increase. This is mainly due to the fact that the Orchestrator selects the soloists that should be in the route and thereby naturally requires a much smaller search space when finding the optimal path. Another very significant advantage is that the County case is very stable in response time, with a worst case of about 1.5 seconds compared to a worst case for the national case that reaches up to 20 seconds in the worst case. This is an attribute that is crucial if it is to be implemented in the full setup with user interaction, where stable and fast response times are essential. All in all it is clear that the County case outperforms the Nation case in all aspects and show the potential of a distributed Orchestrator approach. Similar results are shown in the figure above where the county case is compared to the regional decomposition of Norway (19 regions). Again, there is a clear advantage in using a finer decomposition, but response times in this case are getting closer.



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Scheduling of maintenance tasks of a large-scale tram network

Alexander Kiefer · Michael Schilde · Karl F. Doerner

1 Introduction

The infrastructure of a public transport network has to be maintained regularly to guarantee a functioning system. The most expensive maintenance measures include the replacement of rails and switches. In addition there are also some minor tasks like grinding and tamping that have to be done periodically too. It is the scheduling of these four tasks that we consider in our research. In particular we seek to generate strategic yearly maintenance plans for a planning horizon of many years. The generation of maintenance plans involves the assignment of tasks to segments of the infrastructure and time periods within the planning horizon. Grouping similar tasks on neighboring segments while performing all tasks in time are the key aspects.

2 Problem description

We consider a large-scale urban tramway network. The network consists of a number of nodes and a set of edges connecting them. Only those edges are incorporated that are actually traversed by lines. In our case, the set of nodes includes all the stations of the network and only a few extra nodes where tramway lines intersect in the absence of a station. In general, one may also consider finer partitions of the network. However, this may significantly affect the size of the problem, eventually making it impossible to handle.

The maintenance scheduling has to be done for the edges, or synonymously segments, in the network. Each task on each segment has its segment-specific maintenance interval, depending on the number of vehicles traversing the segment and its radius. There might be

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tasks that have to be scheduled multiple times on a segment. However, there could be rails that do not have to be replaced within the planning horizon at all.

The infrastructure deteriorates constantly. Even before reaching the end of the maintenance interval there is a point when vehicles cannot traverse an unmaintained segment with the original speed. The reduced velocity causes the need of extra trains to keep the original frequency of the lines. This in turn induces additional costs.

Scheduling the same or similar tasks on neighboring segments grants benefits as some machines may have to be carried to the region only once and the site can be supervised more efficiently. The tasks can be performed in different shifts, i.e. train-free night shifts and day shifts. During the night there is a surcharge for the personnel. However, working during the day requires to block the segment. As a consequence, trains have to turn early leading to an effective blockade of lines on an even larger section. This requires the establishment of costly replacement services with buses along the blocked sections. For now we simply estimate the cost for extra buses on the basis of the originally provided capacity on the segments. This aspect leaves room for more elaborate planning approaches in the future.

The public transport provider has a yearly budget for the maintenance tasks that must not be exceeded. The objective is to minimize the overall costs, which includes maintenance work, extra trains, costs for replacement services, and a penalty for the state of the network at the end of the planning horizon.

3 Solution approach

We developed a MIP model for the problem that extends the one by Budai et al (2006). The schedule is therein represented by four-indexed binary decision variables. These variables indicate assignments for each segment, each task, each year, and each shift type. In total, the model requires 7 groups of mostly binary decision variables and 11 types of constraints. We implemented the model for solving it via CPLEX. Even for intermediate-sized instances the model often cannot be solved to optimality within 1 day. However, in most cases good lower bounds are provided.

Furthermore, we developed a large neighborhood search (LNS) algorithm to solve large instances with a long planning horizon. LNS was invented by Shaw (1998) and is based on repetitively destroying and subsequently repairing parts of the schedule. In every iteration a destroy and a repair operator are randomly selected, as well as a number of assignments to remove. The operators are then applied to the incumbent solution. Newly generated solutions are accepted as new incumbent solution based on a simulated annealing acceptance scheme.

The algorithm incorporates one repair and three destroy operators. The *greedy repair* operator schedules those tasks first that improve the schedule the most. The procedure continues as long as there are improving assignments that can be scheduled feasibly. The destroy operators include the *random destroy* operator that removes assignments at random. Similar to Shaw (1998) we make use of a *related destroy* operator that removes related assignments with a higher probability. The relatedness measure between two assignments is larger if they are scheduled in the same period and if they refer to the same or similar tasks on adjacent segments. Finally, the operator *interval destroy* removes tasks randomly with a bias towards those that are scheduled within a small interval to the previous and the next task of the same type on the same segment relative to the maintenance interval.

The intermediate schedules generated by LNS are fragmented segment-wise and recombined by a set covering (SC) model after the LNS phase. The SC model can be solved via CPLEX in a few seconds and is able to improve the solution quality in almost every case.

4 Numerical tests

The nurmerical tests in this section are carried out on an Intel Xeon E5-2650v2 running at 2.6 GHz. Intermediate-sized instances based on the regions of the Viennese tramway network with a planning horizon of 20 years are solved by the MIP and the LNS approach. The sizes of the instances are described in Table 1. The runtime of CPLEX is limited to 1 day, while LNS has a runtime of 15 minutes. The results corresponding to LNS are averages over 10 runs. The optimality gaps of the solutions generated by CPLEX and the relative differences of the solution qualities of LNS and CPLEX are shown in Table 2. CPLEX finds good solutions with a small optimality gap for all instances, while LNS finds slightly worse solutions. The limitations of CPLEX are revealed when extending the planning horizon together with the size of the network. In particular, the complete network cannot be solved for a planning horizon of 30 years within 1 day. Regarding the effectiveness of the SC phase one has to mention that omitting this feature leads to a deterioration of 0.16% on average.

Instance	Segments	Lines
South	131	11
North	139	14
East	93	8
West	100	10

Table 1: Size of the instances

Instance	CPLEX gap	LNS vs CPLEX
South	0.17%	2.47%
North	0.10%	1.45%
East	0.00%	2.28%
West	0.07%	1.27%

Table 2: Comparison CPLEX vs LNS

The complete network is solved by the LNS approach with a runtime of 1 hour in a single run. The share of the individual cost factors is presented in Table 3. Clearly, the costs for the replacement services and extra trains play inferior roles. The effect of a 10% higher yearly maintenance budget is shown in Table 4. Unsurprisingly, the total penalty can be reduced by slightly relaxing the budget constraint. Increasing the budget leads to a significant decrease of the costs for extra trams and also improves the state of the network at the end of the planning horizon.

maintenance	63.74%
end of horizon	32.97%
extra trams	2.70%
replacement services	0.59%

Table 3: Shares of the cost factors

maintenance	+0.77%
end of horizon	-1.64%
extra trams	-18.19%
replacement services	-14.38%
total	-0.63%

Table 4: Effect of an increased budget

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A parametric priority rule for just-in-time scheduling problem with sequence-dependent setup times

Helio Yochihiro Fuchigami

1 Introduction

This work addresses the minimization of earliness and tardiness of jobs in a single machine scheduling problem with sequence-dependent setup times and individual due dates. This performance measure is directly related to the Just-in-time philosophy, which involves broad concepts related to the elimination of waste and reductions in inventory items.

Just-in-time scheduling consists of obtaining solution that minimizes functions associated with earliness and tardiness of the jobs. In practice, these performance measures are very important because the increase of both leads to higher production costs. Anticipating the completion of jobs increases inventory levels, material handling and product deterioration, and delays in delivery may result in fines, cancellations of orders and even loss of customers. [7] exemplifies some kind of manufacturing and service industries where quick and prompt response to customers' demand is the key to success: health care, nursing, operating theaters, transportation, communication, delivery, apparel, glass, furniture and wood manufacturing.

In addition to its prevalence as a real-world situation, this problem arises as a relaxation of scheduling problems with more complex manufacturing or with additional constraints as in a flow shop and job shop. Yet, it is strongly NP-hard. It means that no efficient algorithm can solve medium or large-sized instances [2]. Many different just-in-time problems can be seen in [11] and [4].

The focused problem consists in scheduling a set of n jobs on a single machine with the objective of minimizing their total earliness and tardiness and is represented in the known three-field notation as $1|s_{ij},d_j|\Sigma E_j+T_j$. Each job has its processing time p_j, due date d_j and sequence-dependent setup times s_{ij}, all deterministic, integer, known in advance and not necessarily equal. The machine is ready at time zero.

Some previous work has addressed single machine problems similar to the one dealt in this research, such as [5], [10], [13], [1] and [7]. Others applied variations of the ATCS rule (apparent tardiness cost with setup) in different productive environments, such as [6], [9] and [2] in parallel machines. However, no paper was found using this rule in the just-in-time scheduling problem with sequence-dependent setup times. Generally, the ATCS rule was applied in tardiness minimization, such as [14]. This is the motivation of the present research.

2 Proposed solution method

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The approach chosen to solve the problem considered is a constructive heuristic based on the parametric priority rule ATCS and an insertion method. Firstly, an initial order is established by the ATCS rule adapted to the present problem, according to the follow index. Whenever the machine becomes free at the instant *t* and considering the last job scheduled as job *i*, the index $I_{ij}(t)$ is calculated for each non-scheduled jobs *j*, i.e., each time a job is chosen, it no longer belongs to the set of candidate jobs. The job with the highest index is chosen to be scheduled next on the machine.

$$I_{ij}(t) = \frac{1}{p_j} e^{\left(-\frac{\max(d_j - p_j - t, 0)}{k_1 \bar{p}}\right)} e^{\left(-\frac{s_{ij}}{k_2 \bar{s}}\right)} \quad (1)$$

The parameters k_1 and k_2 required in the index $I_{ij}(t)$ were used according to previous papers, such as [6] and [2]. The values $\bar{p}$ and $\bar{s}$ are respectively the average processing and setup times of non-scheduled jobs. Next, to refine the solution found, the insertion method's idea of the well-known NEH heuristic [8] was applied to the sequence.

3 Computational results

To evaluate the effectiveness of the proposed method, results were compared with two traditional priority rules: EDD (earliest due date) and MST (minimum slack time). In the computational experiment, 5,600 instances were generated, divided into two groups: small, with 5, 6, 7, 8, 9 and 10 jobs (group 1), and medium and large sizes, with 15, 20, 30, 50, 80, 100, 150, 200 jobs (group 2). For all instances of group 1, the optimal solution was provides by the complete enumeration method.

Intervals of processing times were [1, 99] and setup times [1, 50]. Due dates of jobs were generated in four scenarios, according to the methodology of [12], with tardiness factor in {0.2, 0.4} and due date range in {0.6, 1.2}. For each combination, 100 replicates were generated, totaling 5,600 instances, of which 2,400 were small and 3,200 medium and large sizes.

Table 1 shows the average relative percentage deviations for each group, calculated by: $ARPD = 100.(ET_h - ET_b)/ET_b$, where ET_h is the objective function of proposed heuristic, EDD or MST, and ET_b is the optimal value for the group 1 and the best value found in group 2.

Table 1 – Average relative percentage deviation – group 1 (*a*) and group 2 (*b*)

n	**Proposed**	**EDD**	**MST**
5	2.80	37.31	60.51
6	3.54	41.80	68.15
7	4.02	46.03	74.21
8	4.70	50.18	80.04
9	5.33	53.87	85.63
10	6.02	55.98	87.96
Average	**4.40**	**47.53**	**76.08**

n	**Proposed**	**EDD**	**MST**
15	2.23	60.83	94.85
20	2.43	63.75	98.74
30	1.59	70.11	106.11
50	1.21	77.13	115.36
80	0.82	82.46	121.79
100	0.73	84.87	124.95
150	0.51	89.60	130.62
200	0.38	92.53	133.93
Average	**1.24**	**77.66**	**115.80**

It can be observed, therefore, that the proposed method presents deviations much lower than the EDD and MST rules, in average 4.40% of the optimal solution in the problems with up to 10 jobs (group 1). In all cases in group 2, it also provided the best solution, with improving (decreasing) values with the increase of the problem size. It draws attention the results of the MST rule, whose objective function value was more than the double of the best solution in instances with 30 jobs or more, leading to deviations greater than 100%.

The CPU times in group 1 and for rules EDD and MST in group 2 were practically zero. The proposed rule had on average 7.5 seconds in the instances with medium and large sizes. And the enumeration method consumed 86.6 seconds on average (in group 1 instances).

4 Conclusion

The analysis discussed proves the effectiveness in terms of solution quality, the computational efficiency and the practical applicability of the proposed priority rule in the addressed problem, which, as already mentioned, is quite frequent in real situations.

As a continuation of this research, it is possible to propose more elaborate solution methods, such as mathematical models and metaheuristics and/or include different restrictions on the problem, such as release dates, due windows etc.

Acknowledgements This work was supported by CAPES (Coordination for the Improvement of Higher Education Personnel, Brazil), CNPq (National Council of Technological and Scientific Development, Brazil) and FAPEG (Goias State Research Foundation, Brazil).

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Railway Rolling Stock Maintenance Scheduling

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1 Introduction

Recurring maintenance is necessary in many industries (e.g., power grids, buildings, ships and airplanes). In the railway domain, the vehicles (e.g. locomotives, freight cars and passenger cars) are taken out of their revenue-generating service (transporting goods or passengers) and moved to a maintenance depot. There, maintenance activities are performed before the vehicles are returned to service. Planned maintenance tasks have substantial fixed costs in terms of staff, equipment and vehicle transportation. Therefore, it is very important to perform as many of the maintenance tasks as possible when a vehicle is undergoing preventive maintenance. At the same time, the depot is required to have enough resources available to cover corrective urgent maintenance. Currently, maintenance planning in railways is predominantly done manually and involves solving the following connected planning problems: crew scheduling (assigning maintenance tasks to crew depending on skill sets) and job shop scheduling (assigning time slots for vehicles at certain depot workstations). Ideally, these activities should be synchronized with the vehicles' revenue-generating activities (vehicle owners' commitment to transporting goods or passengers).

Individually, these planning problems are very complex due to the sheer number of tasks and resources involved. Because of this complexity, manually finding solutions is not only time-consuming but highly inefficient both for the maintenance companies and their customers. Due to the substantial fixed costs involved, inefficient solutions are very costly. Few tools exist that help to generate maintenance plans, and they normally focus on myopic parts of the overall problem (e.g. they exclude crew planning or have fewer parameters). Existing approaches try to solve the planning problems independently and then merge the plans, thereby losing the global perspective necessary to achieve the required efficiency. Individually, these planning problems have been studied at length in the scientific literature and in other industries where optimization-based planning tools are used frequently.

This work is performed in collaboration with a Norwegian railway rolling stock maintenance company, Mantena. Together with its customers, Mantena provide real-life data to build models that accurately describe the complexity of the problem. They will also assess and use the output solutions provided.

2 Problem description

All maintenance must be performed under capacity requirements (maintenance crew, depot capacity and the rolling stock being maintained must be at the depot). For each time rolling stock is undergoing maintenance, there is a set of tasks. Some of them are required, and some optional ones can be postponed to the next scheduled maintenance of the rolling stock. Some maintenance tasks are bundled together since they must, or because it is cost favorable to carry them out at the same time. As an example, assume that the preparation work to do task 1 is expensive. The same preparation work is needed for tasks 3 and 5. Therefore, one might want to do tasks 3 and 5 together with task 1. Then, one only needs to do the preparation work a single time. Even if some tasks are bundled, they cannot necessarily be treated as a single task as they might require different crew skill sets. Maintenance tasks require crew with a certain skill set. Crew are allocated to each set of maintenance tasks considering not only their skill set, but also other scheduling aspects (e.g., workload balance, union policies and labor laws). In addition, at the maintenance depot, there are limited equipment and working stations, so the inventory of spare parts must be considered. Also, nowadays, there is a lack of coordination of when locomotives, freight cars, passenger cars, maintenance cars, etc., are available for maintenance, what the crew planning at the depot is, and the possibility of performing optional maintenance tasks. So, it is necessary to plan and adjust the maintenance intervals of different tasks, by proactively performing maintenance tasks that are close to, but not yet on their due dates. This synchronization can postpone the next time equipment must undergo maintenance and generate a cost saving due to reduced setup costs related to performing the tasks.

2 Current practice

The integrated railway maintenance problem is a combination of several complex mathematical optimization problems. All of these problems are individually NP-hard combinatorial problems. For the individual problems, we refer to [1] for a recent review on the literature involving predictive maintenance. In the literature, predictive maintenance is often used to describe the use of sensors to predict maintenance. We refer to proactive maintenance, that has some similarities, but is mainly to perform maintenance before the due date to achieve future cost savings. Much of the focus within maintenance of railways is on infrastructure maintenance, see e.g. [2] for a recent study. In [3] a review of general railway optimization is given. The research is focused on single components of the overall problem. However there exists some work that seek to integrate parts of the process. In [4] a rolling stock rostering problem is designed to consider that the rolling stock is supposed to visit maintenance depots. A similar problem is addressed in [5]. In [6] a rolling stock timetable is designed while considering the crew rostering problem. In [7] the authors consider maintenance appointments in rolling stock rescheduling. Although there is much research on the individual problems, and some research – and interest – on integrating parts of the problems, there is to the best of our knowledge, no work that combines all these problems into a single integrated model. However, there is a clear trend in recent years towards a higher degree of integration. Therefore, this work will help expand the frontier of the current railway research.

This field of railway planning has had the interest of researchers for many years, see [3] for an in-depth overview of the railway timetabling and planning research. When turning to the literature, there is a clear trend to integrate multiple problems into one. This has the clear advantage that less myopic decisions are made. Maintenance at Mantena can be described as a

Job Shop Scheduling Problem (JSSP), but it is necessary to extend the problem by considering optional tasks, task grouping and the additional constraints that arise when the problem is solved in reality. See [8] for a recent survey of the JSSP. Furthermore, there is a "Crew Scheduling Problem" and coordination with the customers, which own the rolling stock (locomotives, freight cars, passenger cars, maintenance cars, etc.), that need to be addressed, see [9]. The combination, coordination and synchronization of these problems will expand the research frontier – as the existing literature has a more myopic focus. It is the combination of these three main problems we refer to as the integrated maintenance problem.

3 Solution approach

Working with applied optimization pose some research challenges when it comes to modelling, scalability, data availability, etc. Problem specification and model development is essential to ensure that the produced mathematical model is fit to represent the maintenance problem – this is a very iterative process with challenges when it comes to represent constraints and objectives in an efficient and realistic way.

We seek to develop tailormade methods to solve the problem. The general approach will be as follows, first we will develop an exact solution method that at the very least, should be able to solve minor instances of the problem. This is essential as it will make it possible to benchmark the performance of heuristic methods. As it can be seen from the literature, e.g. [3, 6,7] mixed integer programming models that are either solved directly or handled by a column generation approach, are widely accepted for solving these type of problems (see [10] for more on column generation). A very important note here is that, by using a column generation approach, good quality feasible solutions can normally be reached earlier in the search phase.

The usage of the railway infrastructure and rolling stock is highly seasonal in the terms that, during the summer period, the demand for railway services fall significantly. Today, this seasonality is not considered when making maintenance plans. However, an approach where additional maintenance is carried out during this period could reduce the total need for rolling stock in the railway system. This could be the case if the maintenance is spread unevenly over the year such that the rolling stock taken out of circulation during their revenue-generating periods are reduced. In this way, the entire system can be serviced by less rolling stock. As a future research step, we seek to investigate to what extent it is possible to extend the models developed in such a way that the seasonal requirement for railway resources are taken into consideration.

4 Summary

The general research trend in railway planning and scheduling is towards more integrated research approaches. However, the integration has not yet reached a state where larger scale integration of maintenance planning can be handled. What we are going to do, is to combine the sub problems into one to ensure that we do not miss globally good solutions (e.g. one efficient solution might force subsequent problems to use very inefficient solutions). This becomes a much harder research challenge (develop models and algorithms that scale with the size of the problem, investigating efficient partition approaches) because solving them as one is scientifically known to be much harder. However, we believe that this goal can be achieved.

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Efficient algorithms for non-preemptive mixed-criticality match-up scheduling problem

A. Novak · P. Sucha · Z. Hanzalek

1 Introduction and problem statement

This paper deals with a mixed-criticality scheduling problem. Each job J_i has a criticality level $L_i \in \mathbb{N}$. The greater is L_i the more critical is J_i. For each job, several estimations of its processing time are considered, one for each level of criticality: each job J_i has L_i different processing time estimations. Optimistic estimations reflect the average, more realistic, behavior of the jobs. Pessimistic (and thus safer) estimations represent worst case executions. In order to achieve the tradeoff between safety guarantees and efficient resource usage, the jobs to execute are selected online depending on the current criticality level of the system. Initially, the criticality level of the system is 1. As soon as some job is executed for longer than its processing time estimation corresponding to the current level of the system, the criticality of the system increases. When the criticality level of the system is equal to some value l, the execution of all the jobs with criticality level at least equal to l must be guaranteed, while jobs with a lower criticality level can be rejected, in order to execute higher criticality jobs. In this paper we propose algorithms for a model [2] that, following the idea of match-up scheduling [1], allows switching back to a lower criticality level, to improve the efficiency of resource usage.

We consider n jobs $J_1, \ldots, J_n$ to be processed on a single machine. For $i = 1, \ldots, n$, job J_i has a release date $r_i \geq 0$ and a *criticality level* $L_i \in \mathbb{N}^*$: the greater is L_i, the more critical is J_i. Let $L = \max_{i=1,\ldots,n} L_i$. Job J_i has L_i possible processing times $0 < p_{i,1} < \ldots < p_{i,L_i}$, as well as a deadline $\tilde{d}_i \geq r_i + p_{i,L_i}$. The actual processing time of job J_i is uncertain (it takes one of the values $p_{i,1}, \ldots, p_{i,L_i}$) and is only known at runtime: we denote it by p_i. If $p_i = p_{i,j}$ we say that j is the *execution level* of J_i, $j \in \{1, \ldots, L_i\}$.

Notice that the criticality level of a job is known in advance, while its execution level is only known at runtime. The execution level of a job is smaller than or equal to its criticality level.

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A *schedule* S is defined as a vector $(s_1, \ldots, s_n)$ where s_i is the starting time of job J_i in S. We define a *feasible schedule* $S = (s_1, \ldots, s_n)$ as a schedule that satisfies the following conditions (see Figure 1):

1. Release dates and deadlines constraints are fulfilled: $\forall i \in \{1, \ldots, n\}$: $s_i \geq r_i$ and $s_i + p_{i,L_i} \leq \tilde{d}_i$;
2. At each level of criticality, jobs do not overlap. Equivalently, jobs do not overlap at their highest common level: $\forall i, j \in \{1, \ldots, n\}$ s.t. $s_i < s_j$, we have: $s_i + p_{i,k} \leq s_j$, with $k = \min(L_i, L_j)$.

A scenario sc is a vector of possible execution levels, one for each job of the instance: $sc = (e_1, \ldots, e_n)$, where $e_i \in \{0, \ldots, L_i\}$, $i = 1, \ldots, n$.

In a scenario $sc = (e_1, \ldots, e_n)$, job J_i is executed during p_{i,e_i} time units if $e_i > 0$, otherwise ($e_i = 0$) J_i is not executed (rejected), $i = 1, \ldots, n$. At runtime, each job J_i starts its execution at its starting time s_i if and only if the machine is idle at time s_i; if the machine is busy at time s_i, then J_i is rejected. If a job J_i is started, it is completed, regardless of its processing time in the realised scenario. Let us consider the schedule of Figure 1 with scenarion (2,1). The dashed line represents the execution levels of the jobs (or equivalently the current level of the system). Each time a job has completed execution, the current level of the system becomes equal to 1.

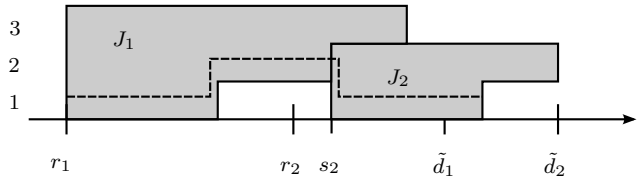

Fig. 1 Feasible schedule of two mixed-criticality jobs and scenario (2,1)

2 Exact solution for the problem without release dates and deadlines

First we concentrate on $1|mc = 2, mu|C_{max}$, i.e., a monoprocessor match-up scheduling with $\mathcal{L}$ criticality levels problem without release dates and deadlines minimizing the makespan. We propose efficient exact algorithms for problems with two and three criticality levels (i.e., $\mathcal{L} \in \{2, 3\}$). The uncertainty about the processing time is modeled using an abstraction based on F-shaped jobs. We will show a new complexity result that establishes the membership of the considered problem into $\mathcal{APX}$ complexity class, and we provide an approximation algorithm.

We study the characterization of the set of optimal solutions for the problem with two criticality levels and we propose very efficient MILP formulation which solves instances with up to 200 jobs, beating the best-known method by a large margin. The formulation is based on a decision variable x_{ij} indicating whether the job J_j on the first criticality level is covered by the job J_i on the second criticality level. The makespan is then given by the sum of lengths of covering blocks and the sum of processing times of all low criticality jobs that are not covered.

Furthermore, we propose efficient exact algorithm for problems with three criticality levels, which solves instances with up to 60 jobs. The algorithm is based on the idea of constructing the schedule in two stages. In the first stage, the relaxed problem considering two levels is solved up to the optimality, which minimizes a lower bound on the optimal makespan of the original problem. The second stage takes the relaxed solution and constructs a locally optimal solution for the original problem. The first stage of the algorithm solves restriction of the given problem instance which omits the highest level; hence it is an instance of $1|mc = 2, mu|C_{max}$. In the second stage, the algorithm defines a new problem instance of the problem $1|mc = 2, mu|C_{max}$ while grouping two lower levels into one and adding the highest level. In general, the algorithm produces suboptimal solutions, however, there are cases when we can verify if the produced schedule is optimal. When we fail to prove optimality, we go to the MILP model for three criticality leves, while using the current solution as a lower bound. The reason for executing heuristic algorithm before solving MILP model is two-fold. First, we have observed the solver struggles to prove optimality when the solution is clearly optimal regarding the critical path. The other observation is that if the problem instance contains the majority of jobs with criticality one and two, then solving its $1|mc = 2, mu|C_{max}$ restriction frequently yields optimal solution since the highest criticality levels are not likely to have impact on C_{max}.

3 Heuristic solution for the problem with release dates and deadlines

The key idea of the heuristic algorithm is to split the solution of the problem into two steps. The purpose of the stage separation is that the determination of the schedulability of $1|r_i, \tilde{d}_i, mc = \mathcal{L}|C_{max}$ problem is $\mathcal{NP}$-hard in the strong sense. The MILP from the previous section is not powerful enough to find any feasible solution even for instances with larger tens of messages in a reasonable time. Therefore, we leverage the finding an initial solution to a heuristic.

In the first stage of the algorithm, an initial feasible solution is obtained. The feasibility stage is inspired by NEH heuristics [3] extended by our local search for reducing infeasibility. In the second stage, the solution is iteratively reoptimized using *Large Neighborhood Search* technique [4]. It is a local search method that iteratively explores a local neighborhood by the MILP. The best solution over each neighborhood is taken and adopted as a starting solution for another iteration. The choice of the neighborhood is problem-dependent and utilizing the structure of the problem is crucial.

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Modeling and Solving the Steelmaking and Casting Planning and Scheduling Problem

Davide Armellini · Paolo Borzone · Sara Ceschia · Luca Di Gaspero · Andrea Schaerf

1 Introduction

One of the most complex production operations is the steelmaking and casting process. Indeed, many technological (physical, chemical, mechanical, ...) and business constraints are involved in the process and the size of the production batches makes the operation of a steelmaking plant quite costly. Consequently, a careful planning and scheduling of the plant with the aim of maximizing the daily/weekly throughput is of crucial importance to ensure productivity and competitiveness.

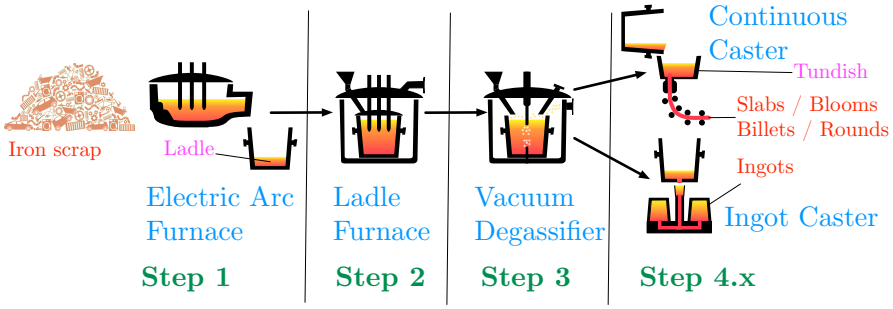

Fig. 1 The steelmaking and casting production process.

A typical steelmaking and casting production plant is composed of several *lines*. A single line is outlined in Figure 1 and consists of four stages. At first the iron scrap is

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melted in an *Electric Arc Furnace* (EAF), then the liquid metal is poured into a *ladle* that will be used to contain the steel in all the following processing steps. The next step is performed in the *Ladle Furnace* (LF), where chemical additives are added to the iron for obtaining the desired alloy. Afterwards, the gas content of the metal has to be reduced in order to reduces hydrogen and nitrogen gases dissolved in the liquid steel in a *Vacuum Degasifier* (VD) unit.

The last production step, called *casting*, might differ based on the type of semifinished product required. Indeed, in the case of "long pieces" (i.e., slabs, rounds, blooms or billets) the production is sent to a *Continuous Casting* (CC) machine, whereas "short pieces" (e.g., ingots) are forged by an *Ingot Casting* (IC) machine. These semifinished products, then, could be subject to further processing (e.g., hot rolling), which might be planned and scheduled somehow independently from the casting one.

The main physical constraint of this initial part of the steelmaking process is the practical impossibility to buffer jobs between different processing steps because of the cooling of the liquid metal in case of waiting. As a consequence, the jobs should be scheduled in a *just-in-time* fashion.

A similar version of the problem was presented by Fanti *et al* [2], who propose an integrated system consisting of a database, an optimization engine, a simulation module and an user interface for the pure scheduling problem. The optimization engine models the scheduling as a hybrid flow shop using a MILP solver.

In our previous work [1], in order to better capture our real-world specification we made the following modifications, with respect to [2]:

1. we considered the possibility that a job switches from one line to another at any stage;
2. we considered the *border data*, coming from the previous scheduling stage (the plant runs for 24h a day); in particular, for each machine, we register the time when it is available, the section of last job and its steel grade; in addition, we consider the status of the ladle with respect to pollutants;
3. we used the *throughput* as objective function, rather than the makespan, as we schedule for a fixed horizon, but flexible number of jobs.

In this work, in order to obtain a model that captures the essential features of the problem and can be shared with other researchers for comparison purposes, we decided to make further modifications. In essence, on the one side, we simplify the problem statement, so as to remove low level details specific for the situation at hand; on the other side, we generalize the problem including features collected from the literature that where not included in the models of [2] and [1].

In detail, the main differences with respect to our previous model in [1] are:

4. we consider possible standstills of single machines for limited times (e.g., for maintenance); as a consequence we have to resort heavily to the possibility that the production is not executed by a single line, but goes through machines belonging to different lines (at a price of longer moving times and possible bottlenecks);
5. we remove the detailed management of ingot wagons and stripping areas for the ingot casting machines, as the number of ingot production is normally limited and it does not have a strong impact on the overall performance;
6. the condition to perform a shorter setup (called *fly-tundish*) for CC machines has been modified; the feasibility of this procedure technically involves the heating and moving times of the tundish, which is an intermediate container that is needed to

Table 1 Instance features.

Feature	Description	Min	Max
jobs	single charge of melted metal	131	318
steel grades	chemical composition of the steel of the job (influences the setup time)	39	98
ladles	container of the molten steel moved through the plant till the end of the process	10	10
appointments	time window for the completion time of a specific job	0	35
standstills	machine stop due to maintenance services or temporary machine breakdown	0	2
pollutants	chemical element present in the fluid metal that pollutes the ladle	2	2
horizon	planning period (in minutes)	1440	4320

pour the content of the ladle; in this work, it is modeled in a simpler, more practical way: there must by a minimum number of jobs between two fly-tundish operations (fixed for each CC);

7. we simplify the cleaning process of the ladles, considering only two levels of cleanness (clean and dirty), but we take into account a generic set of possible pollutants (not just one as in [1]), so that the notion of clean ladle is multidimensional.

Remarkably, point 3 above remains the most peculiar difference with respect to [2] since it involves a change of perspective on the objective function. Given that we have to schedule the production for the next-time horizon, the goal is also to select the jobs, from a large job set, that have to be processed during the horizon, assign them to machines and plan the sequencing and timing (start and end time). Therefore, the problem in its essence is both a planning and scheduling problem.

2 Instances

We collected 59 real-world instances coming from a mid-sized steel-making plant, under different production conditions. The plant comprises two production lines followed by three CC machines, and two IC machines as casters. The main features of the instances are showed in Table 1, in terms of minimum and maximum number of jobs, steel grades, ladles, appointments, standstills, pollutants and horizon length.

We plan to release these instances (properly anonymized) in the near future, along with our best solution, for future comparisons. At the same time, we are preparing a visualization and validation tool that certifies the feasibility of the solution and its costs.

3 Solution Method

In essence, the problem is a complex variant of the *hybrid flow-shop* problem (see, e.g., [3] for a review) with sequence-dependent setup times and heterogeneous processing times. We tackle it by means of local search, modeled as follows:

- The *search space* is an indirect representation of the schedule by means of the sequence (i.e., the permutation) of all the available jobs. Each job has also a set of additional data stating on which machine it has been assigned for each specific processing step. The mapping between the job sequence and the actual execution times of each job is obtained by applying a chronological, right-justification greedy algorithm. All the jobs whose execution time exceed the time-horizon are part of the *tail* of the solution, which consists of the jobs considered as unscheduled. The procedure decides also for the assignment of additional resources (i.e., the ladle and the cooling place in case of ingots), introducing processing delays when no suitable resource is available.
- The *neighborhood* relation is defined as the movement of a job to a new position in the sequence. All jobs in the state within the current position of the candidate job to be moved and its destination are shifted by one (either forward or backward). In addition, the move includes a new machine for each step, which might also be the same of the old ones. The neighborhood is refined by removing moves that are clearly ineffective, such as for example moves that take a job in the tail of the state and reinsert it in a different position still in the tail.
- The *cost function* takes into account the difference between the upper bound of scheduled jobs and the number of jobs actually scheduled in the solution. In addition, it considers the violations of some hard constraints, as a ladle that is not clean as required and a job that misses an appointment.

The experimental analysis with different metaheuristics (hill climbing, simulated annealing, and tabu search) is still ongoing. We will report the results as soon as we complete the tuning and the experimental comparison.

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Solving Tourist Trip Design Problems Using a Virus Optimization Algorithm

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1 Introduction

Tourist Trip Design Problems (TTDP) deal with the task to support tourists in creating a trip consisting of a set or sequence of points-of-interests (POIs) or other items related to travel (Gavalas et al., 2014). Among all POIs, the tourists arrange their trip based on their personal preferences, the time and/or budget constraints. When considering the business hour of each POI, the time window constraints have to be taken into consideration. If tourists arrive a particular POI before its opening time, they will have to wait until it opens. Therefore, TTDP can find its general form in the Orienteering Problem (OP) and its variations such as the Multi-Objective Orienteering Problem with Time Windows (MOOPTW). Recent surveys of orienteering problems can be found in Vansteenwegen et al. (2009) and Gunawan et al. (2016).

Due to its NP-Hard property, research on TTDP has paid lots of attention on metaheuristics in the past decades. Virus Optimization Algorithm (VOA) is a newly developed population-based algorithm and it has been successfully applied to different optimization problems (Cuevas Juarez et al., 2009; Liang and Cuevas Juarez, 2013, 2014 & 2016; Liu et al., 2017). Although the VOA was originally designed for continuous optimization problems, this study focuses on converting the VOA to a structure able to solve combinatorial problems such as the TTDP and also to fit in the "problem-independent" category as described in Sörensen (2015). Different from other existing metaheuristics, the population size of the VOA varies from iteration to iteration, and in order to avoid explosive growth of the viruses, an antivirus mechanism is developed which also has a perfect matching metaphor in virology. Details of the VOA will be discussed in the following sections.

2 Virus Optimization Algorithm

Virus Optimization Algorithm is inspired from the behavior of a virus attacking a host cell, where each virus represents a candidate solution, and the strength of each virus is represented

by the objective function value (Cuevas Juarez et al., 2009; Liang and Cuevas Juarez, 2016). VOA begins the search with a small number of viruses (solutions), where the solution space is symbolized by the host cell, and the nucleolus is the analogy of the global optimal solution. Subsequently, the purpose of the viruses is to explore and exploit the cell's resources so to reproduce new members ever closer to the cell's nucleolus. Once the nucleolus is reached, the cell dies, which by analogy denotes a successful optimization process where the global optimum has been found (*in case it is known*).

The VOA classifies the viruses into two types: strong and common. The strong members are those who are found to have superior objective function values in the population, and it is important to mention that the amount of strong members is defined by the user. As for the common members, they are the remaining population of viruses whose objective function value is not good enough to be considered as strong.

One of the interesting characteristics in VOA is that the population of viruses is dynamic in size; that is to say, it grows in every iteration. The aforementioned is because each strong and common member will replicate or mutate to produce new virus(es), while the predecessor will be combined with the newly generated solutions. Therefore, to control the exponential growth of the virus population, VOA possesses a powerful maintenance mechanism called "Anti-virus" in charge of removing those members in the population that are inferior in performance. The pseudo-code of VOA is shown in the following figure.

```
Virus_population←Generate_initial_population(parameters_values); //Initialization
Viruses_strength←Evaluate_objective_function_value(Virus_population);
While(TRUE)
    [Strong_viruses,  Common_viruses ]←Classification(Virus_population, Viruses_strength);
    [New_Strong, New_Common ]←Replication(Strong_viruses,  Common_viruses); //Replication
    New_members←Storage(New_Strong, New_Common);
    New_members_strength←Evaluate_objective_function_value(New_members);
    Virus_population←Combine(Virus_population, New_members); //Updating
    Apply Antivirus(virus_population); //Maintenance
    Stop←Evaluate_stopping_criterion();
    If (Stop == TRUE) → BREAK WHILE;
End while
```

Figure 1: Pseudo-code of VOA

In this study, when optimizing the TTDP with multiple objectives, VOA will act in accordance with Pareto optimality that takes advantage of the non-dominated solutions found at every step during the optimization process. These non-dominated members are the ones being classified as strong viruses, while the dominated solutions will be considered as common members. Consequently, the number of strong members in the population is going to be dynamic, contrary to how the original (i.e., single-objective) VOA works.

By performing the above approach, VOA provides better chances to balance exploration and exploitation, since strong viruses will exploit those regions where non-dominated solutions are located. In the meantime common members will just explore areas where dominated solutions are distributed. Thus, it is expected that the proposed VOA to have a better equilibrium between convergence and divergence which are considered two key properties to evaluate the performance of a multi-objective optimization algorithm.

Considering TTDP as a combinatorial optimization problem, the encoding of the VOA adopts the permutation of nodes which is different from the conventional real vector solution representation for continuous optimization problems. The proposed VOA consists of several crucial components: Initialization, Replication, and Population Maintenance. The concept of each component will be discussed as follows.

The Initialization step mainly focuses on the generation of initial population (viruses). In order to obtain a better initial population, this study implements the greedy construction

approach suggested by Vansteenwegen et al. (2009) and also detailed in Gunawan et al. (2011). The objective functions of initial population are then evaluated to decide the classification of viruses, i.e., non-dominated solutions as strong viruses and dominated solutions as common ones.

The Replication step proposes a search procedure for new solutions. Considering the permutation encoding, strong (non-dominated) viruses applies the 2-opt heuristics and common viruses implement the order-based crossover to replicate new viruses. In the 2-opt method, two nodes in the path are randomly selected, and the sub-path between the two selected nodes is reversed to create the new solution. On the other hand, to implement the order-based crossover, two common viruses are randomly chosen as parents. Then a sub-path is selected from one parent at random, and produce a proto-child by copying the sub-path into the corresponding positions. The crossover procedure continues by deleting the nodes in the sub-path from the second parent, and placing the remaining nodes into the unfixed positions of the proto-child from the left to the right. Both methods are able to generate feasible solutions without applying any extra repair mechanism.

As mentioned above, different from most of the metaheuristics, the population size of VOA is dynamic. That means it grows with the replication procedure. To better control the size of the population, two parameters are defined as the Population Maintenance mechanism in this study – maximum number of strong viruses and common viruses, respectively. Finally, the VOA terminates when the stopping criterion such as maximum number of iterations is reached.

3 Computational Results

The performance of the proposed VOA is tested on 76 benchmark MOOPTW instances adopted from Chen et al. (2015). Among them, 56 instances with 100 nodes each are categorized into six classes: c1, c2, r1, r2, rc1, and rc2. In c1/2 classes, points in the instances are clustered; in r1/2 classes, points are placed randomly; in rc1/2 classes, random and clustered points are mixed. The instances in the classes with suffix 2 have longer service time windows. For the other 20 instances, pr1-20, the number of nodes ranges from 48 to 288. Note that the original forms of those instances were all single objective. Therefore, the bi-objective versions of instances were generated by taking p_{i1} as $p_{(i+1)2}$, namely the 1st profit of node i is also treated as the 2nd profit of node (i+1). The attributes of the instances are summarized in Table 1.

Table 1. Attributes of Test Instances

Class	Types of Nodes	# of Instances	# of Nodes	Service Time	Max. Service Time Allowed
c1	Clustered	9	100	90	1,236
c2		8			3,390
r1	Random	12	100	10	230
r2		11			1,000
rc1	Mix of Clustered and Random	8	100	10	240
rc2		8			960
pr1-20	Coordinates allows digits and negative values	20	48-288	1-25	1,000

The MO-ACO algorithm proposed in Chen et al. (2015) was implemented by C++ programming language. The computing environment was Windows 7 (64 bit) with Intel Core

i7-4770 CPU and 8 GB RAM. On the other hand, the proposed VOA was coded using Visual Studio 2010 and run in a similar environment with Intel Core i7-4790 CPU and 8 GB RAM.

In order to give a quantitative comparison between the Pareto Frontiers of the MO-ACO algorithm in Chen et al. (2015) and the proposed VOA, the normalized distance ($D1_R$) (Ishibushi et al., 2003) between a Reference Pareto Frontier (RPF) and the outputs of each algorithm is estimated by implementing (1)-(2). Here, Y_{true} and Y_{known} denote the RPF and the output Pareto Frontier by the algorithm respectively, while $d_{w_j v_i}$ is the Euclidean distance, estimated from the RPF to the output provided by the algorithm, which is to be the minimum once it finishes iterating/replicating.

$$D1_R\left(Y_{true}, Y_{known}\right) = \frac{1}{\left|Y_{true}\right|_c} \sum_{v_i \in Y_{true}} \min\left\{d_{w_j v_i} \mid w_j \in Y_{known}\right\} \tag{1}$$

$$d_{w_j v_i} = \sqrt{\left(f_1(v_i) - f_1(w_j)\right)^2 + \left(f_2(v_i) - f_2(w_j)\right)^2 + \cdots + \left(f_n(v_i) - f_n(w_j)\right)^2}$$

$$\forall v_i \in Y_{true} \forall j \mid w_j \in Y_{known} \tag{2}$$

In (1), w_j denotes the j^{th} solution provided by the algorithm and v_i is the i^{th} solution in the RPF, while $f_n(v_i)$ and $f_n(w_j)$ are the n^{th} objective function value obtained from the RPF and the PF provided by VOA or MO-ACO.

Parameter setting plays a critical role for the performance of the algorithms. The optimal parameter values were decided by performing a set of preliminary experiments. The setting of parameter values are as follows: number of initial viruses is set to 20, replication rate of strong viruses is 5, replication rate for common viruses is 3, the upper bound for number of strong viruses is 20, and the upper bound for number of common viruses is set to 100. To compare with the results of MO-ACO, each instance is run 20 times and the stopping criterion is when the number of iterations reaches 300. The Reference Pareto Frontier is obtained by merging the non-dominated solutions generated by all experiments of parameter setting for VOA and the ones provided by MO-ACO in Chen et al. (2015).

Table 2 summarizes the comparison between MO-ACO and VOA over different types of instances. Row of "Win" records the number of instances VOA outperforms MO-ACO while the row of "Lose" stores the number of instances MO-ACO performs better than VOA. The performance of VOA dominates MO-ACO in classes of r2 and rc1. Meanwhile, in rc2 and pr1-20 classes, two competing algorithms perform comparably. In addition, MO-ACO outperforms VOA in the first three classes – c1, c2, and r1.

Table 2. Comparison between MO-ACO and VOA

	c1	c2	r1	r2	rc1	rc2	pr1-20
Win	1	0	3	7	8	4	9
Lose	8	8	9	4	0	4	11

4 Conclusions

This study proposed a Virus Optimization Algorithm to solve the Tourist Trip Design Problem. A permutation encoding is introduced so that the VOA is able to deal with the combinatorial optimization problem. Pareto Optimality is adopted to evaluate the quality of solutions. All solutions are divided into two classes – strong or common where strong ones represent the non-dominated solutions in the Pareto Frontier. In addition, borrowed from the

Traveling Salesman Problem (TSP) and the Genetic Algorithm (GA), two popular methods – 2-opt and order-based crossover are implemented to replicate new solutions.

Computational results show that VOA can perform equally or better than MO-ACO in instances with random data and longer service time (i.e., r2 class), the instances with mix random and clustered data (rc1 and rc2 classes), and the ones with wider range on the number of nodes (the pc class). For those instances VOA is inferior to MO-ACO, VOA can still find good solutions particularly in the middle region of Pareto Frontier. However, VOA is unable to find better solutions on two ends of Pareto Frontier. This observation may lead to future research which focuses on improving diversity of Pareto Frontier.

Finally, note that this study is a preliminary but novel research to investigate the possibilities and validity of applying VOA on network-type combinatorial optimization problems. To further confirm the performance of the VOA, more comprehensive comparison with different metaheuristics using more benchmark instances should be conducted in the future.

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Integrated Scheduling of Machines, Vehicles, and Storage Tasks in Flexible Manufacturing Systems

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This work proposes an integrated scheduling approach for machines, vehicles, and storage tasks in flexible manufacturing systems. Flexible manufacturing system (FMS) is a technology that came into the theory of manufacturing and production systems in late 20th century. It connects production, transportation, storage, and tooling in a way that various parts with different specifications may be processed rapidly and precisely [1], [2]. FMS is important as it allows to produce low volumes of a large variety of products while aiming for high productivity and low costs, as in mass production systems. FMS technology leads to shorter product development cycle, improved equipment utilization, reduced set up times, and lower work-in-process inventories [2], [3]. FMS environments are composed of a set of computer numeric control (CNC) machines connected by a set of automated guided vehicles (AGVs) under the control of a computer system. An automated storage/retrieval system (AS/RS) is used to store and retrieve raw parts, as well as products and/or components.The AS/RS comprises two racks of storage locations, served using a storage/retrieval (S/R) machine. The parts are delivered to the load/ unload (L/U) area by S/R machine, then they are picked up by the AGVs to go through the manufacturing operations required. Once all such operations are completed parts are returned to the L/U area. The S/R machine will then pick them up from the L/U area and transport them back to the storage cell.

Production scheduling refers to sequencing and timing operations on the CNC machines; vehicle scheduling refers to assigning the AGVs to the transportation of parts (or jobs) between the CNC machines, as well as between the CNC machines and the L/U area. Storage and retrieval transfer tasks are assigned to a single S/R machine, thus they need to be scheduled appropriately. Since production scheduling in FMS is highly dependent on the material handling system, the S/R machine and the production operations, as well as AGV scheduling need to be addressed simultaneously. Scheduling in FMS environments is much more difficult than other manufacturing systems due to the higher level of interrelations between various types of equipment used in FMSs [4], [5]. Furthermore, the higher level of automation requires tighter and reliable scheduling.

In FMS environments, a job comprises a set of predefined operations on a specific raw part. The raw parts are stored in predetermined locations of the AS/RS, prior to their starting time of production. A part is retrieved from the storage location to the L/U station by the S/R machine. There, a vehicle picks up the part and move it to the machine performing its first operation. The S/R machine is, usually, assumed to be at the L/U station at the beginning of the scheduling horizon. For the following operations, parts are picked up by the vehicle from the machine where the last operation was performed, waiting for its completion if necessary, and taken to the corresponding machine to have the operation performed. Once the final operation is completed, the part is moved to the L/U station, where the S/R machine picks it up and transfers it back to the storage location (i.e., storage task). The job is then ready to be delivered to the final customer.

Note that when an AGV arrives at a machine, the part is delivered to the buffer area, and the AGV can pursue its next assignment immediately. In such kind of FMS environments, the layout of the machines and the AGV routes are known in advance.

To find solutions to this problem one needs to find the S/R machine schedule (both for storage and retrieval tasks), the AGVs schedule, and the operations schedule. Among the possible solutions, we are interested in one that minimizes the total time required to complete all the jobs, i.e., the makespan.

Most of the reported work considers the simultaneous scheduling of either machines and storage/retrieval tasks or machines and AGVs tasks (with the third equipment being considered as an ever-available resource). The integrated scheduling of machines and AGVs has been considered widely in the last 20 years [2], [3], [6], [7]. In this problem, the sequence of operations on each machine is determined concurrently with the AGVs assignment to transport the parts between the machines and between them and the L/U area. It is assumed that all the parts are available at L/U area whenever an AGV reaches it. Bilge and Ulusoy [6] proposed the first mathematical model, a nonlinear one, for the integrated scheduling of machines and AGVs. Then, Zheng *et al.* [2] developed the first MILP (mixed-integer linear programming) model. However, the former model was never solved and the latter was only used to solve very small example. Ulusoy *et al.* [7], Abdelmaguid *et al.* [3], Gnanavel Babu et al [8], and Zheng *et al.* [2] and many other researchers proposed (meta)heuristic algorithms to solve this problem within reasonable CPU time. More recently, Fontes and Homayouni [9] and [10] have proposed a MILP model and reported good performances solving the usually used benchmark problem instances, first proposed by Bilge and Ulusoy [6]. In these works, the authors assumed that the jobs are in the L/U station ready to be moved by an AGV. Some of the solution methods reported in the literature have obtained "doubtful" results for the above mentioned instances. This fact has been reported by several authors: Abdelmaguid *et al.* [3], Zheng *et al.* (2014), and Fontes and Homayouni [10].

Other authors, recognizing that production scheduling in FMS environment is highly dependent on the availability of the parts at L/U station addressed the simultaneous scheduling of storage/retrieval tasks and production operations [11], [12]. Jawahar *et al.* [11] proposed simultaneous scheduling of AS/RS storage and production in FMS. Later, the work is further developed through developing an adaptive genetic algorithm by Jerald *et al.* [13], and a particle swarm optimization algortihm by Asokan *et al.* [12]. In all these works it is assumed that travelling time between the L/U area and the machines is negligible and that AGVs are available to move the parts whenever needed.

To the best of our knowledge, Jerald *et al.* [14] and Gnanavel Babu *et al.* [15] are the only reported works considering the integrated scheduling of machines, vehicles, and storage/retrieval tasks in FMS. Both works aim at minimizing penalty cost, machine idle time, and distance travelled by the S/R machine. Jerald *et al.* [14] proposed several metaheuristic algorithms such as, genetic algorithm, particle swarm intelligence, and sheep flock heredity algorithm; while Gnanavel Babu *et al.* [15] proposed an artificial immune system. However, no optimal solution methods have ever been proposed for this problem. Here, a mixed integer linear programming model is reported, with which optimal solutions can be found. This is important as a mean to develop alternative heuristics and also to be able to provide a precise quality measure for the solutions found by the (meta)heuristic, at least for small sized instances.

In FMS environments, a set of machines, a set of vehicles, and one S/R machine are in service. There are J jobs, each requiring a set of operations to be done. Each operation is characterized by a machine where it needs to be processed and a processing time. Routes between the L/U area and the machines and between machines are predetermined.

The MILP model we develop extends the one proposed in Fontes and Homayouni [10] by incorporating the AS/RS scheduling and its main novelty is the consideration of a set of chained manufacturing tasks for the machines, a set of chained transportation tasks for the vehicles, and a set of chained storage task for the S/R machine. These three sets of chains are connected through the constraints regarding the completion time of the manufacturing tasks and traveling task, both for vehicles and for the S/R machine. This, in turn results in a lower number of binary

variables, particularly for larger numbers (>2) of vehicles. In addition, our modeling approach differs from current literature in the sense that we model the sequences of tasks for machines, vehicles, and the S/R machine independently; while others, usually, assign machine tasks and vehicles tasks simultaneously, i.e., in the same decision variable (see, e.g., Zheng *et al.* [2]).

For the evaluation purposes, we have used the set of frequently used benchmark instances, first proposed by Bilge and Ulusoy [6]. This set contains 82 instances, considering four different layouts, each of which having of one L/U station, four machines, and two vehicles. The instances have been generated and consist of 10 job sets, each with 13 to 21 operations and 4 to 8 jobs. We have adapted these instances to the problem being addressed by adding the specifications of an AS/RS to the FMS layout, and storage cell for the jobs. The model was implemented in and solved by Gurobi® software. The computational results obtained show that the proposed modeling approach, in addition to being novel, is capable of finding optimal solutions efficiently. Out of the 82 instances considered, 73 were optimally solved and in 47% of them CPU time is under one minute.

The solutions obtained have also shown that the transportation associated the storage and retrieval tasks should not be ignored, since it implies a much larger makespan. For example, for the first instance, the makespan goes up from 96, when only machines and vehicles scheduling is involved, (see, e.g., [2], [3], [7]) to 203 when the scheduling of machines, vehicles, and storage is considered. Furthermore, it could be observed that in more than 95% of the makespan duration, the S/R machine is working to retrieve/store parts, while machines and vehicles have idle times in nearly half of this duration.

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Selection Hyper-heuristics for Solving the Wind Farm Layout Optimisation Problem

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1 Introduction

During the last two centuries, energy consumption from non-renewable sources has reached its peak while demand for energy has increased. Therefore transitioning to renewable energy sources is now recognised. Wind turbine technology is a promising source of renewable energy, and hence efficient wind farm layout is needed so that each turbine produces as much energy as possible. The placement of wind turbines directly impacts the efficiency of the wind farm as turbines are close enough to influence each other's performance due to aerodynamic interactions (wakes) [7].

The wind farm layout optimisation problem is considered a highly complex NP-hard problem, for which exact methods are unsuitable. There is a wide and varied literature on the use of evolutionary algorithms for the optimisation of wind farm layouts [5,6]. These algorithms have proven to be very effective at finding near-optimal solutions to a large number of problems in the energy industry. However, a new breed of optimisation algorithms known as hyper-heuristics is beginning to be applied to these problems. Hyper-heuristics are automated methodologies for *selecting* or *generating* heuristics to solve multiple computationally difficult optimisation problems [1]. They combine simple heuristics to create bespoke algorithms for specific problem domains, and have proven successful on other optimisation problems (see for example [2,3]). This work investigates the use of selection hyper-heuristics to wind farm layout optimisation that could possibly outperform conventional evolutionary approaches in terms of solution quality and run-time. There are two main components in a single-point-based search selection hyper-heuristic: *heuristic selection* and *move acceptance* as identified in [4].

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2 Solution Method

The wind farm layout optimisation problem involves finding the optimal positions of wind turbines in a 2-dimensional plane, such that the cost of energy is minimised taking into account several factors such as wind speed, site characteristics, turbines features, wake effects and existence of obstacles.

Our approach discretises the site into a number of cells, and solutions to the problem are represented as a vector of boolean to decide the absence or presence of turbines in the cells of the grid. The simplest form of a selection hyper-heuristic is a stochastic local search method which combines a simple random heuristic selection method (SR) with an improve or equal acceptance method (IE). The proposed approach, denoted as SR-IE, is implemented in this study using an open source software tool based on a generic API, referred to as WindFLO[1], designed for benchmarking purposes. This tool contains problem domain specific details, such as, the evaluation function computing the cost of energy. Moreover, a set of benchmark problem instances (terrain sizes, obstacles, wind forces, layout shapes, ...) can be downloaded from the WindFLO website. The evaluation function used in this study is as follows:

$$f = \frac{\left(c_t * n + c_s * \lfloor \frac{n}{m} \rfloor\right) + c_{OM} * n}{\left(1 - (1 - r)^{-y}\right)/r} * \frac{1}{8760 * P} + \frac{0.1}{n} \tag{1}$$

where f is the cost of energy, $c_t = \$750,000$ is the turbine cost, $c_s = \$8,000,000$ is the price of a substation, $m = 30$ is the number of turbines per substation, $r = 3\%$ is the interest rate, $y = 20$ years is the farm lifetime in years, $c_{OM} = \$20,000$ per year is the operation and maintenance costs, n is the number of turbines of the layout, P is the layout's energy output reported by the WindFLO API [5].

Selection hyper-heuristics operate by using a prefixed pool of low level heuristics by which a randomly initialised solution is improved over the search time. The low level heuristics used in this study are as follows:

- **LLH1** replace a single cell at random.
- **LLH2** swap two cells at random.
- **LLH3** ruin 10% of cells and rebuild at random.
- **LLH4** ruin 30% of cells and rebuild with all 0s or all 1s.
- **LLH5** is a first improvement hill climbing that searches for the first best solution between adjacent solutions.
- **LLH6** select two rows in a grid and exchange with a crossover rate of 20%.
- **LLH7** select two columns in a grid and exchange with a crossover rate of 20%.

Hence, SR chooses and applies a perturbative low level heuristic with a probability of 86%. Having LLH5 local search method as one of the low level heuristics creates an iterated local search like overall approach [3].

3 Results

The WindFLO API provides an implementation of a genetic algorithm (GA) as a baseline approach whose performance is compared to the proposed method, SR-IE. Both algorithms are applied to three instances each for five trials, and the termination

[1] `https://github.com/d9w/WindFLO`

Table 1 Parameters of the GA

Parameter	Value
Population size	20
Mutation rate	5%
Crossover rate	40%
Selection	4-player tournament with elitism

Table 2 Summary of experimental results. Best values are highlighted in bold

	SR-IE			GA		
Instance	Best	Avg	Std	Best	Avg	Std
Ins-1	**0.001115**	**0.001115**	6.32E-08	0.001181	0.001186	8.38E-06
Ins-2	**0.001474**	**0.001477**	2.22E-06	0.001483	0.001484	1.33E-06
Ins-3	**0.002319**	**0.002326**	5.24E-06	0.002377	0.002388	7.68E-06

criterion is set to 2000 layout evaluations. The performance of each method is measured using the cost of energy provided in Equation 1. Table 1 provides the parameter values for GA; and SR-IE is parameter free method.

Table 2 presents the results, which clearly shows that the SR-IE hyper-heuristic improves significantly on the performance of the GA on all trials.

We will be performing further experiments using more problem instances and additional selection hyper-heuristics and report the results at the conference.

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